Class XII Session 2026-27
Subject - Mathematics
Sample Question Paper - 9

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. 1 1This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculators is not allowed.

Section A

1. If a matrix has 36 elements, the number of possible orders it can have, is: **[1]**

   a) 13 b) 9

   c) 3 d) 5

2. The area of a triangle with vertices (3,0), $(0, k)$ and $(-3, 0)$ is 9 sq units. The value of k is: **[1]**

   a) -9 b) 6

   c) 3 d) 9

3. The value of the determinant $\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$ is: **[1]**

   a) 47 b) 49

   c) -51 d) -79

4. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then det (adj A) equals [1]

a) a^{27} b) a^2

c) a^6 d) a^9

5. A line making angles $45^0 and 60^0$ with the positive directions of the axis of x and y makes with the positive direction of Z axis , an angle of [1]

a) 45^0 b) 60^0

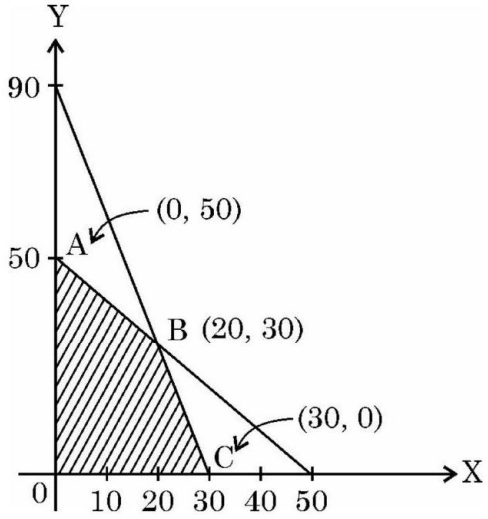
c) $60^0 or 120^0$ d) 120^0

6. The solution of the differential equation $2x \cdot \frac{dy}{dx} - y = 3$ represents a family of [1]

a) straight lines b) ellipses

c) circles d) parabolas

7. The maximum value of $Z = 4x + y$ for a L.P.P. whose feasible region is given below is: [1]



a) 110 b) 170

c) 120 d) 50

8. A unit vector $\hat{a}$ makes equal but acute angles on the co-ordinate axes. The projection of the vector $\hat{a}$ on the vector $\vec{b} = 5\hat{i} + 7\hat{j} - \hat{k}$ is [1]

a) $\frac{11}{15}$ b) $\frac{4}{5}$

c) $\frac{11}{5\sqrt{3}}$ d) $\frac{3}{5\sqrt{3}}$

9. $\int \frac{\tan^2 \sqrt{x}}{\sqrt{x}} dx$ is equal to: [1]

a) $2\sqrt{x} \tan x - x + C$ b) $\sec \sqrt{x} + C$

c) $2 \tan \sqrt{x} - x + C$ d) $2(\tan \sqrt{x} - \sqrt{x}) + C$

10. If A is a 3×4 matrix and B is a matrix such that A'B and AB' are both defined, then the order of the matrix B is: [1]

a) 4×3 b) 3×4

c) 4×4 d) 3×3

11. The feasible region for an LPP is always a **[1]**

a) type of polygon b) convex polygon

c) concave polygon d) Straight line

12. If the angle between the vectors $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$ and $|\vec{a} \times \vec{b}| = 1$, then $\vec{a} \cdot \vec{b}$ is equal to **[1]**

a) -1 b) $\frac{1}{\sqrt{2}}$

c) 1 d) $\sqrt{2}$

13. If $A = \begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$, then A^{-1} exists if. **[1]**

a) $\lambda \neq$ -2 b) None of these

c) $\lambda = 2$ d) $\lambda \neq 2$

14. A family has 2 children and the elder child is a girl. The probability that both children are girls is: **[1]**

a) $\frac{1}{8}$ b) $\frac{3}{4}$

c) $\frac{1}{2}$ d) $\frac{1}{4}$

15. Find the particular solution for $\frac{dy}{dx} - \frac{y}{x} + \text{cosec}\left(\frac{y}{x}\right) = 0; y = 0 \; when \; x = 1$ **[1]**

a) $\cos\left(\frac{y}{x}\right) = \log|ex|$

b) $\cos\left(\frac{2y}{x}\right) = \log|ex|$

c) $\cos\left(\frac{y}{2x}\right) = \log|3ex|$

d) $\cos\left(\frac{y}{x}\right) = \log|2ex|$

16. If the projection of $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ on $\vec{b} = 2\hat{i} + \lambda\hat{k}$ is zero, then the value of λ is: **[1]**

a) 0 b) $\frac{-3}{2}$

c) $\frac{-2}{3}$ d) 1

17. If $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$ $\frac{dy}{dx} = ?$ **[1]**

a) $\tan\theta$ b) $a\cot\theta$

c) $a\tan\theta$ d) $\cot\theta$

18. Find the values of p so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles. **[1]**

a) $p = \frac{71}{13}$ b) $p = \frac{70}{11}$

c) $p = \frac{72}{15}$ d) $p = \frac{70}{12}$

19. **Assertion (A):** The function f(x) = sin x decreases on the interval $(0, \frac{\pi}{2})$. **[1]**

Reason (R): The function f(x) = cos x decreases on the interval $(0, \frac{\pi}{2})$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Let Z be the set of integers. A function $f : Z \to Z$ defined as $f(x) = 3x - 5, \forall x \in Z$ is a bijective. **[1]**

Reason (R): A function is a bijective if it is both surjective and injective.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Evaluate: $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$ **[2]**

OR

Evaluate: $\sin^{-1}(\sin(-600°))$

22. Determine the values of x for which $f(x) = \frac{x-4}{x+1}, x \neq -1$ is an increasing or a decreasing function. **[2]**

23. Find the least value of 'a' so that $f(x) = 2x^2 - ax + 3$ is an increasing function on $[2, 4]$. **[2]**

OR

For the curve $y = 5x - 2x^3$, if x increases at the rate of 2 units/s, then how fast is the slope of the curve changing when $x = 2$?

24. Evaluate: $\int_0^{\frac{\pi}{2}}$ [log (sin x) - log (2 cos x)] dx. **[2]**

25. Find the intervals in which function $f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}}$ is (i) increasing (ii) decreasing. **[2]**

Section C

26. Evaluate: $\int \frac{(3\sin x - 2)\cos x}{13 - \cos^2 x - 7\sin x} dx$ **[3]**

OR

Find: $\int \frac{x^2+x+1}{(x+2)(x^2+1)} dx$

27. Prove that **[3]**

$\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx = \frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$.

28. Find the general solution of the differential equation $x^2\frac{dy}{dx} = x^2 + xy + y^2$. **[3]**

OR

Find the particular solution of the differential equation $x\frac{dy}{dx} + y + \frac{1}{1+x^2}$ = 0, given that y(1) = 0.

29. Draw the graph of the solution set of the system of inequations 2x + 3y $\geq$ 6, x + 4y $\leq$ 4, x $\geq$ 0 and y $\geq$ 0 **[3]**

OR

Solve the following linear programming problem graphically:

Maximise: Z = x + 2y

subject to the constraints:

x + 2y $\geq$100,

2x - y $\leq$ 0,

2x + y $\leq$ 200,

x $\geq$ 0, y $\geq$ 0.

30. Three persons A, B and C apply for a job of Manager in a private company. Chances of their selection (A, B and C are in the ratio 1:2:4. The probabilities that A, B and C can introduce changes to improve profits of the company are 0.8, 0.5 and 0.3, respectively. If the change does not take place, find the probability that it is due to the appointment of C. **[3]**

31. If $y = \sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right)$, that show that $\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}$. **[3]**

Section D

32. Prove that the curves y^2 = 4x and x^2 = 4y divide the area of the square bounded by sides x = 0, x = 4, y = 4 and y = 0 into three equal parts. **[5]**

33. Each of the following defines a relation on N: **[5]**

i. x is greater than y, x, y $\in$ N

ii. x + y = 10, x, y $\in$ N

iii. xy is square of an integer x, y $\in$ N

iv. x + 4y = 10x, y $\in$ N.

Determine which of the above relations are reflexive, symmetric and transitive.

OR

Let A = {1, 2, 3, ...9} and R be the relation in A$\times$A defined by (a, b) R (c, d) if a + d = b + c for (a, b), (c, d) in A$\times$A. Prove that R is an equivalence relation and also obtain the equivalence class [(2, 5)].

34. Find the shortest distance $\overrightarrow{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} - 3\hat{j} + 2\hat{k}\right)$ and $\overrightarrow{r} = \left(4\hat{i} + 5\hat{j} + 6\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + \hat{k}\right)$. **[5]**

OR

Find the length shortest distance between the lines: $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$ and $\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$

35. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 1 & 2 & 0 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equations : **[5]**

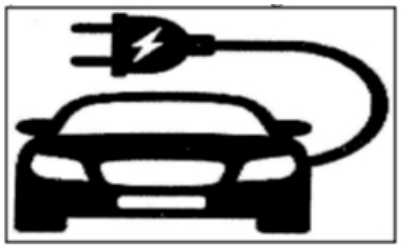
$x + 2y - 3z = 1$

$2x - 3z = 2$

$x + 2y = 3$

Section E

36. Three persons viz. Amber, Bonzi and Comet are manufacturing cars which run on petrol and on battery as well. Their production share in the market is $60\%, 30\%$ and 10% respectively. Of their respective production capacities, $20\%, 10\%$ and 5% cars respectively are electric (or battery operated). Based on the above, answer the following: **[4]**



i. (a) What is the probability that a randomly selected car is an electric car? **(2)**

OR

i. (b) What is the probability that a randomly selected car is a petrol car? **(2)**

ii. A car is selected at random and is found to be electric. What is the probability that it was manufactured by Comet? **(1)**

iii. A car is selected at random and is found to be electric. What is the probability that it was manufactured by Amber or Bonzi? **(1)**

37. **Read the following text carefully and answer the questions that follow:** **[4]**

If two vectors are represented by the two sides of a triangle taken in order, then their sum is represented by the third side of the triangle taken in opposite order and this is known as triangle law of vector addition.

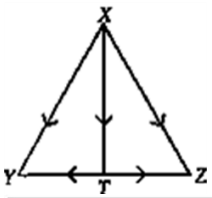
i. If $\vec{p}, \vec{q}, \vec{r}$ are the vectors represented by the sides of a triangle taken in order, then find $\vec{q} + \vec{r}$. (1)

ii. If ABCD is a parallelogram and AC and BD are its diagonals, then find the value of $\overrightarrow{AC} + \overrightarrow{BD}$. (1)

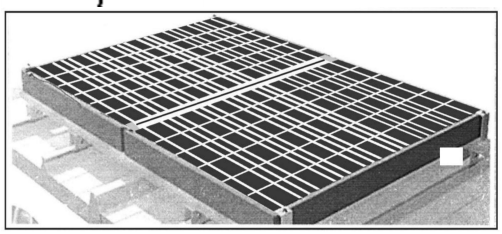
iii. If ABCD is a parallelogram, where $\overrightarrow{AB} = 2\vec{a}$ and $\overrightarrow{BC} = 2\vec{b}$, then find the value of $\overrightarrow{AC} - \overrightarrow{BD}$. (2)

OR

If T is the mid point of side YZ of $\triangle$XYZ, then what is the value of $\overrightarrow{XY} + \overrightarrow{XZ}$. (2)



38. **[4]**



A technical company is designing a rectangular solar panel installation on a roof using 300 metres of boundary material. The design includes a partition running parallel to one of the sides dividing the area (roof) into two sections.

Let the length of the side perpendicular to the partition be x metres and with parallel to the partition be y metres.

Based on this information, answer the following questions:

i. Write the equation for the total boundary material used in the boundary and parallel to the partition in terms of x and y. **(1)**

ii. Write the area of the solar panel as a function of x. **(1)**

iii. a. Find the critical points of the area function. Use second derivative test to determine critical points at the maximum area. Also, find the maximum area. **(2)**

OR

b. Using first derivative test, calculate the maximum area the company can enclose with the 300 metres of boundary material, considering the parallel partition. **(2)**

Solution

Section A

1.

(b) 9
Explanation:
9

2.

(c) 3
Explanation:
3

3. **(a)** 47
Explanation:
47

$$\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

= 2(1-8)-7(1-10)+1(8-10)
= 2(-7) - 7(-9) +1(-2)
= -14 +63-2
= -16 +63
= 47

4.

(c) a^6
Explanation:

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$$

$|A| = a^3$

$|\text{adj } A| = |A|^{3-1} = |A|$

$|\text{adj } A| = (a^3)^2 = a^6$

5.

(c) $60^0 or 120^0$
Explanation:
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
$\alpha = 45, \beta = 60$
put the values in above equation
$(1/\sqrt{2})^2 + (1/2)^2 + \cos^2\gamma = 1$
$\cos\gamma = \pm 1/2$

6.

(d) parabolas
Explanation:

Given equation can be written as

$\frac{2dy}{y+3} = \frac{dx}{x}$

$\Rightarrow$ 2log (y + 3) = log x + log c

$\Rightarrow$ $(y + 3)^2$ = cx which represents the family of parabolas

7.

(c) 120

Explanation:

120

8. **(a)** $\frac{11}{15}$

Explanation:

$\frac{11}{15}$

9.

(d) $2(\tan\sqrt{x} - \sqrt{x}) + C$

Explanation:

$2(\tan\sqrt{x} - \sqrt{x}) + C$

10.

(b) 3×4

Explanation:

We have to find: Order of A = 3×4

Order of A' = 4×3

As A^TB and BA^T are both defined, so the number of columns in B should be equal to the number of rows in A' for BA' and also the number of columns in A' should be equal to the number of rows in A' for BA'.

Therefore, the order of matrix B is 3×4.

11.

(b) convex polygon

Explanation:

Feasible region for an LPP is always a convex polygon.

12.

(c) 1

Explanation:

1

13.

(b) None of these

Explanation:

We have,

$$A = \begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$$

A^{-1} exists if $|A| \neq 0$

Now $|A| = 2(6 - 5) - \lambda(-5) - 3(-2) = 8 + 5\lambda \neq 0$

$\Rightarrow 5\lambda \neq$ -8

$\Rightarrow \lambda \neq \frac{-8}{5}$

So, A^{-1} exists if and only if $\neq \frac{-8}{5}$

14.

(c) $\frac{1}{2}$

Explanation:

$\frac{1}{2}$

15. **(a)** $\cos\left(\frac{y}{x}\right) = \log|ex|$

Explanation:

Let y = vx

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

Question becomes $v + x\frac{dv}{dx} = v - cosecv$

$x\frac{dv}{dx} = -cosecv$

$-\int sinvdv = \int \frac{dx}{x}$

$cosv = logx + logc$

$cos(\frac{y}{x}) = logx + logc$

when x = 1 and y = 0

$cos(\frac{0}{1}) = log1 + logc$ { log c =1 }

$c = e$

$cos(\frac{y}{x}) = logx + loge$

$cos(\frac{y}{x}) = log|ex|$

16.

(c) $\frac{-2}{3}$

Explanation:

$\frac{\vec{a}\cdot\vec{b}}{|b|}$ = 0

$\vec{a} \cdot \vec{b}$ = 0

$(\hat{i} - 2\hat{j} + 3\hat{k}) \cdot (2\hat{\mathrm{i}} + \lambda\hat{\mathrm{k}})$ = 0

$2 \times 1 + 3\lambda = 0$

$2 + 3\lambda = 0$

3λ = -2

$\lambda = \frac{-2}{3}$

17. **(a)** $\tan\theta$

Explanation:

$x = a(\cos\theta + \theta\sin\theta)$,we get

$\therefore \frac{dx}{d\theta} = a(-\sin\theta + \sin\theta + \theta\cos\theta)$

$\Rightarrow \frac{d\theta}{dx} = \frac{1}{a\theta\cos\theta}$

$y = a(\sin\theta - \theta\cos\theta)$,we get

$\therefore \frac{\mathrm{d}y}{\mathrm{d}\theta} = a(\cos\theta - (\cos\theta + \theta(-\sin\theta))$

$\Rightarrow \frac{dy}{d\theta} = \mathrm{a}\cos\theta - \mathrm{a}\cos\theta + \theta\,\mathrm{a}\sin\theta$

$\Rightarrow \frac{\mathrm{dy}}{\mathrm{d}\theta} = \mathrm{a}\theta\sin\theta$

$\Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\mathrm{dy}}{\mathrm{d}\theta} \times \frac{\mathrm{d}\theta}{\mathrm{dx}}$

$\Rightarrow \frac{dy}{dx} = a\theta\sin\theta \times \frac{1}{a\theta\cos\theta}$

$\Rightarrow \frac{dy}{dx} = \tan\theta$

18.

(b) $p = \frac{70}{11}$

Explanation:

Given lines are: $\frac{x-1}{-3} = \frac{y-2}{2p/7} = \frac{z-3}{2}$ and $\frac{x-1}{-3p/7} = \frac{y-5}{1} = \frac{z-6}{-5}$

The D.R's of the lines are -3, 2p/7, 2 and -3p/7, 1, -5.

$\therefore \cos 90^o = \frac{(-3)\times\left(\frac{-3p}{7}\right)+\left(\frac{2p}{7}\right)\times 1+2\times(-5)}{\sqrt{9+\frac{4p^2}{49}+4}\sqrt{\frac{9p^2}{49}+1+25}}$

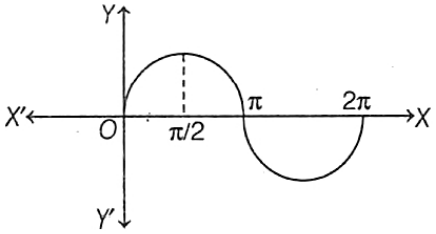
$\Rightarrow \frac{9p}{7} + \frac{2p}{7} - 10 = 0 \Rightarrow p = \frac{70}{11}$

19.

(d) A is false but R is true.

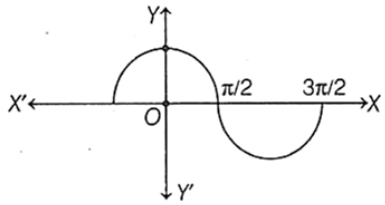
Explanation:

Assertion: Given, function f(x) = sin x



From the graph of sin x, we observe that f(x) increases on the interval $(0, \frac{\pi}{2})$.

Reason: Given function is f(x) = cos x.



From the graph of cos x, we observe that, f(x) decreases on the interval $(0, \frac{\pi}{2})$.

Hence, Assertion is false and Reason is true.

20.

(d) A is false but R is true.

Explanation:

Assertion (A) is false, but Reason (R) is true.

Section B

21. $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$

$= \tan^{-1}\left[2\sin\left(2\times\frac{\pi}{6}\right)\right] = \tan^{-1}\left[2\sin\frac{\pi}{3}\right]$

$= \tan^{-1}\left[2\times\frac{\sqrt{3}}{2}\right] = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

OR

$\sin^{-1}(\sin(-600^\circ)) = \sin^{-1}\{\sin(-600 \times \frac{\pi}{180})\}$

$= \sin^{-1}\{\sin(-\frac{10\pi}{3})\} = \sin^{-1}(-\sin\frac{10\pi}{3})$

$= \sin^{-1}\{-\sin(3\pi + \frac{\pi}{3})\} = \sin^{-1}\{-(-\sin\frac{\pi}{3})\}$

$= \sin^{-1}(\sin\frac{\pi}{3}) = \frac{\pi}{3}$

22. $f'(x) = \frac{x+1-x+4}{(x+1)^2} = \frac{5}{(x+1)^2} > 0$

Hence f is increasing in its domain.

23. $f(x) = 2x^2 - ax + 3 \Rightarrow f'(x) = 4x - a$

Now $2 \le x \le 4 \Rightarrow 8 - a \le 4x - a \le 16 - a$

For f to be an increasing function, $f'(x) \ge 0$

$\Rightarrow 8 - a \ge 0 \Rightarrow a \le 8$

$\therefore$ Least value of a does not exist.

OR

$y = 5x - 2x^3$

Given $\frac{dx}{dt} = 2$ units $/s$

slope of the curve $= \frac{dy}{dx} = 5 - 6x^2 = m$

$\frac{dm}{dt} = -12x\frac{dx}{dt} = -12x(2) = -24x$

at $x = 2$, $\frac{dm}{dt} = -24(2) = -48$

Hence, slope of curve is decreasing at the rate of 48

24. Let $I = \int_0^{\frac{\pi}{2}} [\log \sin x - \log (2 \cos x)]\, dx = \int_0^{\frac{\pi}{2}} \log\left(\frac{\tan x}{2}\right) dx$

Using property $\int_0^a f(x)\, dx = \int_0^a f(a - x) dx$

We get, $I = \int_0^{\frac{\pi}{2}} \log\left(\frac{\cot x}{2}\right) dx$

$\therefore 2I = \int_0^{\frac{\pi}{2}} \log\left(\frac{\tan x}{2} \times \frac{\cot x}{2}\right) dx = \int_0^{\frac{\pi}{2}} \log\left(\frac{1}{4}\right) dx$

$2I = \log\left(\frac{1}{4}\right)x|_0^{\frac{\pi}{2}} = \frac{\pi}{2}\log\frac{1}{4}$

$I = \frac{\pi}{4}\log\frac{1}{4}$ OR $-\frac{\pi}{2}\log 2$

////////

25. $f(x) = 5x^{3/2} - 3x^{5/2} \Rightarrow f'(x) = \frac{15}{2}\sqrt{x}(1 - x)$

For increasing/decreasing, put $f'(x) = 0$

$\Rightarrow x = 0, 1$

i. When $x \in [0, 1]$, $f'(x) \ge 0$. So, f is increasing when $x \in [0, 1]$
(The intervals $(0, 1)$, $[0, 1)$ or $(0, 1]$ can also be considered.)

ii. When $x \in [1, \infty)$, $f'(x) \le 0$.So, f is decreasing when $x \in [1, \infty)$
(The interval($1, \infty$)can also be considered.)

Section C

26. Let the given integral be,

$I = \int \frac{(3\sin x - 2)\cos x}{13 - \cos^2 x - 7\sin x} dx$

$= \int \frac{(3\sin x - 2)\cos x}{13 - (1 - \sin^2 x) - 7\sin x} dx$ ($\because \cos^2 x = 1 - \sin^2 x$)

$= \int \frac{(3\sin x - 2)\cos x}{\sin^2 2x - 7\sin x + 12} dx$

$= \int \frac{(3\sin x - 2)\cos x}{\sin^2 2x - 4\sin x - 3\sin x + 12} dx$

$= \int \frac{(3\sin x - 2)\cos x}{\sin x(\sin x - 4) - 3(\sin x - 4)} dx$

$= \int \frac{(3\sin x - 2)\cos x}{(\sin x - 3)(\sin x - 4)} dx$

Let sin x = t

$=$ cos x dx = dt

$\therefore\ I = \int \frac{(3t-2)}{(t-3)(t-4)} dt$

Using partial fraction, we get

$\frac{(3t-2)}{(t-3)(t-4)} = \frac{A}{(t-3)} + \frac{B}{(t-4)} = \frac{A(t-4)+B(t-3)}{(t-3)(t-4)}$

$=$ 3t - 2 = (A + B)t - 4A - 3B

Comparing coefficients, we get

A = -7 and B = 10

So,

I = $-7\int \frac{1}{(t-3)} dt + 10\int \frac{1}{(t-4)} dt$

= -7 In $|t - 3|$ + 10 In $|t - 4|$ + c

$\therefore$ I = -7 In $|\sin x - 3|$ + 10 In $|\sin x - 4|$ + c

OR

Let, $I = \int \frac{x^2+x+1}{(x+2)(x^2+1)} dx$

By partial fractions

$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$

$x^2 + x + 1 = A(x^2 + 1) + Bx + C(x + 2)$

$= Ax^2 + A + Bx^2 + 2Bx + Cx + 2Cx^2 + x + 1$

$= x^2(A + B) + x(2B + C) + A + 2C$

On comparing coefficients of equation (i), we get

1 = A + B and 1 = 2B + C

On solving above three equations, we get

$A = \frac{3}{5}, B = \frac{2}{5}$ and $C = \frac{1}{5}$

Hence, $\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{3}{5(x+2)} + \frac{\frac{2}{5}x+\frac{1}{5}}{x^2+1}$

Therefore, $\int \frac{(x^2+x+1)dx}{(x+2)(x^2+1)} = \frac{3}{5}\int \frac{1}{x+2} dx + \frac{2}{5}\int \frac{x}{x^2+1} dx + \frac{1}{5}\int \frac{1}{(x^2+1)} dx$

$= \frac{3}{5}\log|x+2| + \frac{1}{5}\int \frac{2x}{x^2+1} + \frac{1}{5}\int \frac{dx}{x^2+1}$

$= \frac{3}{5}\log|x+2| + \frac{1}{5}\log|x^2+1| + \frac{1}{5}\tan^{-1} x + c$

27. According to the question, $I = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx$ ...(i)

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^2\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$

$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$

$\Rightarrow \quad I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos x + \sin x} dx$ ...(ii)

Adding Equations (i) and (ii),

$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x + \cos^2 x}{\sin x + \cos x} dx$

$\Rightarrow \quad 2I = \int_0^{\frac{\pi}{2}} \frac{1}{\sin x + \cos x} dx$

$[\because sin^2 x + cos^2 x = 1\,]$

$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \frac{1}{\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} + \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}} dx$

$\left[\because \sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} \text{ and } \cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right]$

$= \int_0^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}}{2\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2}} dx$

Put $\tan\frac{x}{2} = t \Rightarrow \sec^2\frac{x}{2} \cdot \frac{1}{2} dx = dt \Rightarrow \sec^2\frac{x}{2} dx = 2dt$

Lower limit when x = 0, then t = tan 0 = 0

Upper limit when x = $\frac{\pi}{2}$, then t = tan $\frac{\pi}{4} = 1$.

$\therefore \quad 2I = \int_0^1 \frac{2dt}{2t+1-t^2} dt = 2\int_0^1 \frac{dt}{-\left[t^2-2t-1\right]} dt$

$= 2\int_0^1 \frac{dt}{-\left[(t-1)^2-1-1\right]} dt = 2\int_0^1 \frac{dt}{(\sqrt{2})^2-(t-1)^2}$

$= \left[\frac{2}{2\sqrt{2}} \log\left|\frac{\sqrt{2}+t-1}{\sqrt{2}-t+1}\right|\right]_0^1$

$\left[\because \int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C\right]$

$= \frac{1}{\sqrt{2}}\left[\log\frac{\sqrt{2}+1-1}{\sqrt{2}-1+1} - \log\frac{\sqrt{2}+0-1}{\sqrt{2}-0+1}\right]$

$= \frac{1}{\sqrt{2}}\left[\log 1 - \log\frac{\sqrt{2}-1}{\sqrt{2}+1}\right]$

$= -\frac{1}{\sqrt{2}}\log\left[\frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}\right]$ [$\because$log 1 = 0]

$= \frac{-1}{\sqrt{2}}\log\frac{2-1}{(\sqrt{2}+1)^2}$ [$\because$(a - b) (a + b) = a^2 - b^2]

$= \frac{-1}{\sqrt{2}}\log\frac{1}{(\sqrt{2}+1)^2}$

$\Rightarrow \quad 2I = \frac{2}{\sqrt{2}}\log(\sqrt{2}+1) \Rightarrow I = \frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$

28. $\frac{dy}{dx} = \frac{y^2}{x^2} + \frac{y}{x} + 1$

Put $\frac{y}{x} = v$ i.e. $y = vx$

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Equation becomes

$v + x\frac{dv}{dx} = v^2 + v + 1$

$\frac{1}{v^2+1} dv = \frac{dx}{x}$

Integrating, we get

$\tan^{-1} v = \log|x| + C \Longrightarrow \tan^{-1}\frac{y}{x} = \log|x|$ + C

OR

$x\frac{dy}{dx} + y + \frac{1}{1+x^2} = 0$

$x\frac{dy}{dx} + y = -\frac{1}{1+x^2}$

$\frac{dy}{dx} + \left(\frac{1}{x}\right) y = -\frac{1}{x(1+x^2)}$

So it a linear differential equation

P = $\frac{1}{x}$ and Q = $-\frac{1}{x(1+x^2)}$

$IF = e^{\int p dx} = e^{\int \frac{1}{x} dx} = e^{\log x} = x$

General solution

$y.IF = \int Q.IF dx + C$

$y.x = -\int \frac{1}{x(1+x^2)} . x \, dx + C$

$y.x = -\int \frac{1}{(1+x^2)} \, dx + C$

$y.x = -\tan^{-1} x + C$ ...(i)

At x =1, y = 0

$0 = -\tan^{-1}(1) + C$

$C = \frac{\pi}{4}$

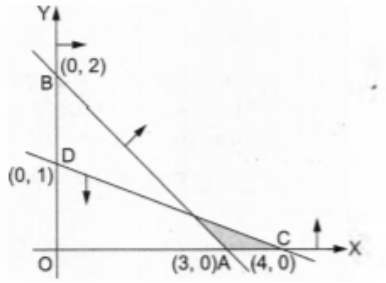
Putting in eqn (i)

$y.x = -\tan^{-1} x + \frac{\pi}{4}$

29. Here, given equations

$2x + 3y = 6, x + 4y = 4 , x = 0 \; and \; y = 0$

Now, $2x + 3y = 6 \Rightarrow \frac{x}{3} + \frac{y}{2} = 1$



Now the line meets the axes at A(3,0) and B(0,2). Join these points and draw a thick line. Clearly, the portion not containing (0, 0) represents the solution set of the inequation 2x + 3y > 6

Again, $x + 4y = 4 \Rightarrow \frac{x}{4} + \frac{y}{1} = 1$

This line meets the axes at C(4, 0) and D(0, 1).

Join these points and draw a thick line. Clearly, the portion containing (0, 0) represents the solution set of the inequation x + 4y < 4

Clearly, x > 0 is represented by the y-axis and the portion on its right-hand side

Also, y > 0 is represented by the x-axis and the portion above the x-axis.

Therefore, the shaded region represents the solution set of the given inequations.

OR

Our problem is to mimmise and maximise the given objective function given as Z = x + 2 y .....(i)

Subject to the given constraints,

x + 2y $\geq$100 .....(ii)

2x - y $\leq$ 0 .....(iii)

2x + y $\leq$200 .....(iv)

x$\geq$ 0, y$\geq$ 0 .....(v)

Table for line x + 2y = 100 is

x	0	100
y	50	0

So, the line x + 2y = 100 is passing through the points with coordinates (0, 50) and (100, 0).

On replacing the coordinates of the origin O (0, 0) in the inequality x + 2y $\geq$100, we get

$2 \times 0 + 0 \geq 100$

$\Rightarrow \quad 0 \geq 100$(which is False)

So, the half plane for the inequality of the line (ii) is away from the origin, which means that the point O(0,0) does not lie in the feasible region of the inequality of (ii)

Table for the line (iii) 2x - y = 0 is given as follows.

x	0	10
y	0	20

So, the line 2x - y = 0 is passing through the points (0, 0) and (10, 20).

On replacing the point (5, 0) in the inequality 2x - y$\leq$ 0, we get

$2 \times 5 - 0 \leq 0$

$\Rightarrow \quad 10 \leq 0$ (which is False)

So, the half plane for the inequality of (iii) is towards Y-axis.

Table of values for line 2x + y = 200 is given as follows.

x	0	100
y	200	0

So, the line 2x + y = 200 is passing through the points with coordinates (0, 200) and (100, 0).

On replacing O (0, 0) in the inequality 2 + y $\leq$ 200, we get

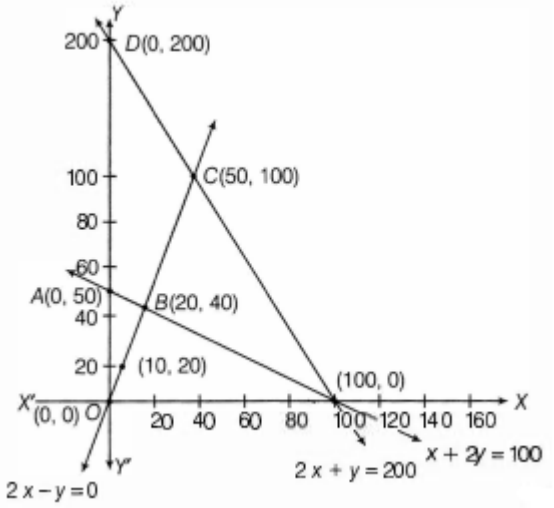
$2 \times 0 + 0 \leq 200$

$\Rightarrow \quad 0 \leq 200$ (which is true)

So, the half plane for the inequality of the line (iv) is towards the origin, which means that the point O (0,0) is a point in the feasible region.

Aslo, x, y$\geq$ 0

So, the region lies in the I quadrant only.



On solving equations 2x - y = 0 and x + 2y = 100, we get the point of intersection as B(20, 40).

Again, solving the equations 2x - y = 0 and 2x + y = 200, we get C(50, 100).

$\therefore$ Feasible region is ABCDA, which is a bounded feasible region.

The coordinates of the corner points of the feasible region are A(0, 50), B(20, 40), C(50, 100) and D(0, 200).

The values of Z at corner points are given below:

Corner points	**Z = x + 2y**
A(0, 50)	Z = 0 + 2$\times$50 = 100
B(20, 40)	Z = 20 + 2$\times$40 = 100
C(50, 100)	Z = 50 + 2$\times$100 = 250
D(0, 200)	Z = 0 + 2$\times$200 = 400

The maximum value of Z is 400 at D(0, 200) and the minimum value of Z is 100 at all the points on the line segment joining A(0, 50) and B(20, 40).

30. Let us define the following events

A = selecting person A

B = selecting person B

C = selecting person C

P(A) = $\frac{1}{1+2+4}$, P(B) = $\frac{2}{1+2+4}$

and P(C) = $\frac{4}{1+2+4}$

P(A) = $\frac{1}{7}$, P(B) = $\frac{2}{7}$

and P(C) = $\frac{4}{7}$

Let E = Event to introduce the changes in their profit.

Also given $P\left(\frac{E}{A}\right)$ = 0.8, $P\left(\frac{E}{B}\right)$ = 0.5 and $P\left(\frac{E}{C}\right)$ = 0.3

$\Rightarrow P\left(\frac{\overline{E}}{A}\right)$ = 1 - 0.8 = 0.2, $P\left(\frac{\overline{E}}{B}\right)$ = 1 - 0.5 = 0.5

and $P\left(\frac{\overline{E}}{C}\right)$ = 1 - 0.3 = 0.7

The probability that change does not take place by the appointment of C,

$P\left(\frac{C}{E}\right) = \frac{P(C)\cdot P\left(\frac{\overline{E}}{C}\right)}{P(A)\times P\left(\frac{\overline{E}}{A}\right)+P(B)\times P\left(\frac{\overline{E}}{B}\right)+P(C)\times P\left(\frac{\overline{E}}{C}\right)}$

$= \frac{\frac{4}{7}\times 0.7}{\frac{1}{7}\times 0.2+\frac{2}{7}\times 0.5+\frac{4}{7}\times 0.7}$

$= \frac{2.8}{0.2+1.0+2.8} = \frac{2.8}{4}$ = 0.7

31. **To prove:** $\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}$

Given: $y = \sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right)$

Put x = cos2θ

$y = \sin^{-1}\left[\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{2}\right]$

$y = \sin^{-1}\left[\frac{\sqrt{1+2\cos^2\theta-1}+\sqrt{1-\left(1-2\sin^2\theta\right)}}{2}\right]$ [cos 2θ = 2cos$^2\theta$ - 1 = 1 - 2sin$^2\theta$]

$y = \sin^{-1}\left[\frac{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}{2}\right]$

$y = \sin^{-1}\left[\frac{1}{\sqrt{2}}\cos\theta+\frac{1}{\sqrt{2}}\sin\theta\right]$

$y = \sin^{-1}\left[\sin\frac{\pi}{4}\cos\theta+\cos\frac{\pi}{4}\sin\theta\right]$

[Using sin A cos B + cos A sin B = sin (A + B)]

$y = \sin^{-1}\left[\sin\left(\frac{\pi}{4}+\theta\right)\right]$

$y = \frac{\pi}{4}+\theta$

Put $\theta = \frac{1}{2}\cos^{-1}x$ [x = cos 2θ]

$y = \frac{\pi}{4}+\frac{1}{2}\cos^{-1}x$

Differentiate above equation w.r.t x:

$\frac{dy}{dx} = 0+\frac{1}{2}\times\frac{-1}{\sqrt{1-x^2}}$

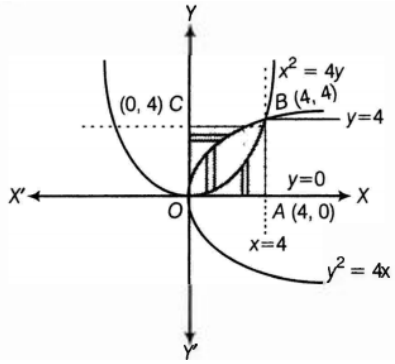
$\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}$

Section D

32. The given curves are $y^2 = 4x$ and $x^2 = 4y$

Let OABC be the square whose sides are represented by following equations

Equation of OA is y = 0
Equation of AB is x = 4
Equation of BC is y = 4
Equation of CO is x = 0



On solving equations $y^2 = 4x$ and $x^2 = 4y$, we get A(0, 0) and B(4, 4) as their points of intersection.
The Area bounded by these curves

$$= \int_0^4 \left[y_{\left(\text{ parabola } y^2=4x \right)} - y_{\left(\text{ parabola } x^2=4y \right)} \right] dx$$

$$= \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4} \right) dx$$

$$= \left[2 \cdot \frac{2}{3} x^{3/2} - \frac{x^3}{12} \right]_0^4$$

$$= \left[\frac{4}{3} x^{3/2} - \frac{x^3}{12} \right]_0^4$$

$$= \frac{4}{3} \cdot (4)^{3/2} - \frac{64}{12}$$

$$= \frac{4}{3} \cdot \left(2^2\right)^{3/2} - \frac{64}{12}$$

$$= \frac{4}{3} \cdot (2)^3 - \frac{64}{12}$$

$$= \frac{32}{3} - \frac{16}{3}$$

$$= \frac{16}{3} \text{ sq units}$$

Hence, area bounded by curves $y^2 = 4x$ and $x = 4y$ is $\frac{16}{3}$ sq units .....(i)

Area bounded by curve $x^2 = 4y$ and the lines x = 0, x = 4 and X-axis

$$= \int_0^4 y_{\left(\text{ parabola } x^2=4y \right)} dx$$

$$= \int_0^4 \frac{x^2}{4} dx$$

$$= \left[\frac{x^3}{12} \right]_0^4$$

$$= \frac{64}{12}$$

$$= \frac{16}{3} \text{ sq units .....(ii)}$$

The area bounded by curve $y^2 = 4x$, the lies y = 0, y = 4 and Y-axis

$$= \int_0^4 x_{\left(\text{ parabola } y^2=4x \right)} dy$$

$$= \int_0^4 \frac{y^2}{4} dy$$

$$= \left[\frac{y^3}{12} \right]_0^4$$

$$= \frac{64}{12}$$

$$= \frac{16}{3} \text{ sq units .....(iii)}$$

From Equations. (i), (ii) and (iii), area bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$ divides the area of square into three equal parts.

33. i. x is greater than y, $x, y \in N$

For xRx x > x is not true for any $x \in N$.

Therefore, R is not reflexive.

Let $(x, y) \in R \Rightarrow xRy$

x > y

but y > x is not true for any $x, y \in N$

Thus, R is not symmetric.

Let xRy and yRz

x > y and y > z $\Rightarrow$ x > z

$\Rightarrow$ xRz

So, R is transitive.

ii. $x + y = 10, x, y \in N$

$R=\{(x,y) : x+y = 10 ,x,y \in R\}$

$R = \{(1, 9), (2, 8), (3, 7), (4, 6), (5, 5), (6, 4), (7, 3), (8, 2), (9, 1)\}$ $(1, 1) \notin R$

So, R is not reflexive.

$(x, y) \in R \Rightarrow (y, x) \in R$

Therefore, R is symmetric.

$(1, 9) \in R, (9, 1) \in R \Rightarrow (1, 1) \notin R$

Hence, R is not transitive.

iii. Given xy, is square of an integer $x, y \in N$

$\Rightarrow R = \{(x, y) : xy$ is a square of an integer $x, y \in N\}$

$(x, x) \in R, \forall x \in N$

As x^2 is square of an integer for any $x \in N$

Hence, R is reflexive.

If $(x, y) \in R \Rightarrow (y, x) \in R$

Therefore, R is symmetric.

If $(x, y) \in R$ $(y, z) \in R$

So, xy is square of an integer and yz is square of an integer.

Let $xy = m^2$ and $yz = n^2$ for some $m, n \in Z$

$x = \frac{m^2}{y}$ and $z = \frac{x^2}{y}$

$xz = \frac{m^2n^2}{y^2}$, Which is square of an integer.

So, R is transitive.

iv. $x + 4y = 10, x, y \in N$

$R = \{(x, y) : x + 4y = 10, x, y \in N\}$

$R\{(2, 2), (6, 1)\}$

$(1, 1), (3, 3) ..... \notin R$

Thus, R is not reflexive.

$(6, 1) \in R$ but $(1, 6) \notin R$

Hence, R is not symmetric.

$(x, y) \in R \Rightarrow x + 4y = 10$ but $(y, z) \in R$

$y + 4z = 10 \Rightarrow (x, z) \in R$

So, R is transitive.

OR

Given that $A = \{1, 2, 3, ...9\}$ (a, b) R (c, d) a + d = b + c for $(a, b) \in A \times A$ and $(c, d) \in A \times A$.

Let (a, b) R (a, b)

$\Rightarrow a + b = b + a, \forall a, b \in A$

Which is true for any $a, b \in A$

Hence, R is reflexive.

Let (a, b) R (c, d)

a+d = b+c

c + b = d + a $\Rightarrow$ (c, d) R (a, b)

So, R is symmetric.

Let (a, b) R (c, d) and (c, d) R (e, f)

a + d = b + c and c + f = d + e

a + d = b + c and d + e = c + f(a + d) - (d + e) = (b + c) - (c + f)

(a - e) = b - f

a + f = b + e

(a, b) R (e, f)

So, R is transitive.

Hence R is an equivalence relation.

Let (a,b) R (2,5),then

a+5=b+2

a=b-3

If b<3 ,then a does not belong to A.

Therefore,possible values of b are >3.

For b=4,5,6,7,8,9

a=1,2,3,4,5,6

Therefore, equivalence class of (2,5) is

{(1,4),(2,5),(3,6),(4,7),(5,8),(6,9).

34. $\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b}_1 = \hat{i} - 3\hat{j} + 2\hat{k}$

$\vec{a}_2 = 4\hat{i} + 5\hat{j} + 6\hat{k}$

$\vec{b}_2 = 2\hat{i} + 3\hat{j} + \hat{k}$

$\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j} + 3\hat{k}$

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$

$= -9\hat{i} + 3\hat{j} + 9\hat{k}$

$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (3\hat{i} + 3\hat{j} + 3\hat{k})(-9\hat{i} + 3\hat{j} + 9\hat{k}) = -27 + 9 + 27 = 9$

$d = \left| \frac{(\vec{a}_2 - \vec{a}_2) \cdot (\vec{b}_1 \times \vec{b}_2)}{\left| \vec{b}_1 \times \vec{b}_2 \right|} \right|$

$= \left| \frac{9}{3\sqrt{19}} \right| = \frac{3}{\sqrt{19}}$

OR

Here,it is given equations of lines

$L_1 : \frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$

$L_2 : \frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$

Direction ratios of L_1 and L_2 are (2, 1, -3) are (2, -7, 5) respectively.

Suppose, general point on line L_1 is P = (x_1, y_1, z_1)

x_1 = 2s - 1, y_1 = s + 1, z_1 = -3s + 9

and let general point on line L_2 is Q = (x_2, y_2, z_2)

x_2 = 2t + 3, y_2 = -7t - 15, z_2 = 5t + 9 then ,we get

$\therefore \vec{PQ} = (x_2 - x_1)\,\hat{i} + (y_2 - y_1)\,\hat{j} + (z_2 - z_1)\,\hat{k}$

$= (5t + 9 - 2s + 1)\hat{i} + (-7t - 15 \ \ -s - 1)\hat{j} + (5t + 9 + 3s - 9)\hat{k}$

$\therefore \overrightarrow{PQ} = (5t - 2s + 10)\hat{i} + (-7t - s - 16)\hat{j} + (5t + 3s)\hat{k}$

Direction ratios of $\overrightarrow{PQ}$ are ((5t - 2s + 10), (-7t - s - 16), (5t + 3s))

PQ will be the shortest distance if it is perpendicular to both the given lines

Thus, by the condition of perpendicularity,

2(5t - 2s + 10) + 1(-7t - s - 16) - 3(5t - 3s) = 0 and

2(5t - 2s + 10) -7(-7t - s - 16) + 5(5t + 3s) = 0

$\Rightarrow$ 10t - 4s + 20 - 7t - s - 16 - 15t - 9s = 0 and

10t - 4s + 20 + 49t + 7s + 112 + 25t + 15s = 0

$\Rightarrow$ -12t - 14s = -4 and 84t + 18s = -132

Solving above two equations, we obtain

t = -2 and s = 2

thus,

P = (3, 3, 3) and Q = (-1, -1, -1)

Now, distance between points P and Q is

$d = \sqrt{(3+1)^2 + (3+1)^2 + (3+1)^2}$

$= \sqrt{(4)^2 + (4)^2 + (4)^2}$

$= \sqrt{16 + 16 + 16}$

$= \sqrt{48}$

$= 4\sqrt{3}$

Thus, the shortest distance between two given lines is

$d = 4\sqrt{3}$ units

Now, equation of line passing through points P and Q is

$\frac{x-x_1}{x_1-x_2} = \frac{y-y_1}{y_1-y_2} = \frac{z-z_1}{z_1-z_2}$

$\therefore \frac{x-3}{3+1} = \frac{y-3}{3+1} = \frac{z-3}{3+1}$

$\therefore \frac{x-3}{4} = \frac{y-3}{4} = \frac{z-3}{4}$

$\therefore$ x - 3 = y - 3 = z - 3

$\Rightarrow$ x = y = z

Therefore, equation of line of shortest distance between two given lines is x = y = z

35. $\|A\| = 1(6) - 2(3) - 3(4) = -12 \neq 0, \therefore A^{-1}$ exist

$\text{adj } A = \begin{bmatrix} 6 & -6 & -6 \\ -3 & 3 & -3 \\ 4 & 0 & -4 \end{bmatrix}$

$\therefore A^{-1} = -\frac{1}{12}\begin{bmatrix} 6 & -6 & -6 \\ -3 & 3 & -3 \\ 4 & 0 & -4 \end{bmatrix}$

The given system of equations can be written as $AX = B, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{12}\begin{bmatrix} 6 & -6 & -6 \\ -3 & 3 & -3 \\ 4 & 0 & -4 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1/2 \\ 2/3 \end{bmatrix}$

$\therefore$The solution of the given system of equations is: $x = 2, y = \frac{1}{2}, z = \frac{2}{3}$

Section E

36. i. (i) (a) Let $A = Amber$ manufactures the car

$B =$ Bonzi manufactures the car

$C =$ Comet manufactures the car

$E =$ The selected car is electric

$P(A) = \frac{60}{100}, P(B) = \frac{30}{100}, P(C) = \frac{10}{100}$

$P(E) = P(A) \times P\left(\frac{E}{A}\right) + P(B) \times P\left(\frac{E}{B}\right) + P(C) \times P\left(\frac{E}{C}\right)$

$= \frac{60}{100} \times \frac{20}{100} + \frac{30}{100} \times \frac{10}{100} + \frac{10}{100} \times \frac{5}{100}$

$= \frac{155}{1000}$ or $\frac{31}{200}$

OR

i. (b) Let $A =$ Amber manufactures the car

$B =$ Bonzi manufactures the car

$C =$ Comet manufactures the car

$E =$ The selected car is a petrol car

$P(A) = \frac{60}{100}, P(B) = \frac{30}{100}, P(C) = \frac{10}{100}$

$P(E) = P(A) \times P\left(\frac{E}{A}\right) + P(B) \times P\left(\frac{E}{B}\right) + P(C) \times P\left(\frac{E}{C}\right)$

$= \frac{60}{100} \times \frac{80}{100} + \frac{30}{100} \times \frac{90}{100} + \frac{10}{100} \times \frac{95}{100}$

$= \frac{845}{1000}$ or $\frac{169}{200}$

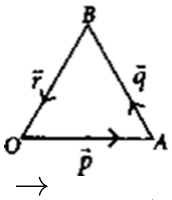
ii. $P\left(\frac{C}{E}\right) = \frac{P(C) \times P\left(\frac{E}{C}\right)}{P(E)}$

$= \frac{\frac{10}{100} \times \frac{5}{100}}{\frac{60}{100} \times \frac{20}{100} + \frac{30}{100} \times \frac{10}{100} + \frac{10}{100} \times \frac{5}{100}}$

$= \frac{\frac{50}{10000}}{\frac{1550}{10000}} = \frac{1}{31}$

iii. $P\left(\frac{A \text{ or } B}{E}\right) = 1 - P\left(\frac{C}{E}\right) = 1 - \frac{1}{31} = \frac{30}{31}$

37. i. Let OAB be a triangle such that



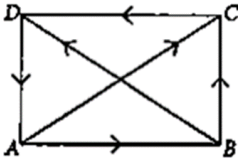
$\overrightarrow{AO} = -\vec{p}, \overrightarrow{AB} = \vec{q}, \overrightarrow{BO} = \vec{r}$

Now, $\vec{q} + \vec{r} = \overrightarrow{AB} + \overrightarrow{BO}$

$= \overrightarrow{AO} = -\vec{p}$

ii. From triangle law of vector addition,

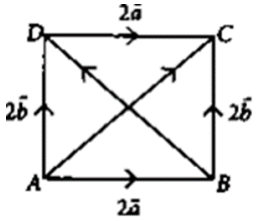
$\overrightarrow{AC} + \overrightarrow{BD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{BC} + \overrightarrow{CD}$



$= \overrightarrow{AB} + 2\overrightarrow{BC} + \overrightarrow{CD}$

$= \overrightarrow{AB} + 2\overrightarrow{BC} - \overrightarrow{AB} = 2\overrightarrow{BC}$ $[\because \overrightarrow{AB} = -\overrightarrow{CD}]$

iii. In $\triangle$ABC, $\overrightarrow{AC} = 2\vec{a} + 2\vec{b}$ ...(i)



and in $\triangle$ABD, $2\vec{b} = 2\vec{a} + \overrightarrow{BD}$ ...(ii) [By triangle law of addition]

Adding (i) and (ii), we have $\overrightarrow{AC} + 2\vec{b} = 4\vec{a} + \overrightarrow{BD} + 2\vec{b}$

$\Rightarrow \overrightarrow{AC} - \overrightarrow{BD} = 4\vec{a}$

OR

Since T is the mid point of YZ

So, $\overrightarrow{YT} = \overrightarrow{TZ}$

Now, $\overrightarrow{XY} + \overrightarrow{XZ} = (\overrightarrow{XT} + \overrightarrow{TY}) + (\overrightarrow{XT} + \overrightarrow{TZ})$ [By triangle law]

$= 2\overrightarrow{XT} + \overrightarrow{TY} + \overrightarrow{TZ} = 2\overrightarrow{XT}$ $[\because \overrightarrow{TY} = -\overrightarrow{YT}]$

38. i. $2x + 3y = 300$

ii. $A = xy = \frac{x}{3}(300 - 2x)$

iii. a. $A = \frac{x}{3}(300 - 2x) = \frac{1}{3}\left(300x - 2x^2\right)$

$\Rightarrow \frac{dA}{dx} = \frac{1}{3}(300 - 4x)$

For critical points, put $\frac{dA}{dx} = 0 \Rightarrow x = 75$

Also, $\frac{d^2A}{dx^2} = -\frac{4}{3} < 0$. So, A is maximum at $x = 75$

Also, maximum area is $A = \frac{75}{3}(300 - 150) = 3750m^2$

OR

b. $A = \frac{x}{3}(300 - 2x) = \frac{1}{3}\left(300x - 2x^2\right)$

$\Rightarrow \frac{dA}{dx} = \frac{1}{3}(300 - 4x)$

For critical points, put $\frac{dA}{dx} = 0 \Rightarrow x = 75$

As $\frac{dA}{dx}$ changes its sign from positive to negative as x passes through $x = 75$ from left to right, which means $x = 75$ is the point of maximum.

Also, maximum area is $A = \frac{75}{3}(300 - 150) = 3750m^2$

Note: Full credit to be given if the student takes equation as $2x + 2y = 300$ or $2x + 4y = 300$ or $4x + 4y = 300$ or $4x + 3y = 300$

The solutions of sub-parts will differ and marks may be given accordingly.