Class XII Session 2026-27
Subject - Mathematics
Sample Question Paper - 8

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If A = $\begin{bmatrix} 2 & -1 & 3 \\ -4 & 5 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 3 \\ 4 & -2 \\ 1 & 5 \end{bmatrix}$ then **[1]**

   a) only BA is defined
   b) AB and BA both are defined
   c) AB and BA both are not defined
   d) only AB is defined

2. If $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{bmatrix}$, then the value of $\det\left(A^{-1}\right)$ is: **[1]**

   a) -1
   b) 2
   c) 0
   d) 1

3. The integrating factor of the differential equation $\frac{dy}{dx} + \frac{2}{x}y = 0, x \neq 0$ is: **[1]**

   a) $e^{\frac{2}{x}}$
   b) $e^{\log(2x)}$
   c) $\frac{2}{x}$
   d) x^2

4. The derivative of 5^x w.r.t. e^x is: **[1]**

   a) $\left(\frac{5}{e}\right)^x \frac{1}{\log 5}$
   b) $\left(\frac{e}{5}\right)^x \frac{1}{\log 5}$
   c) $\left(\frac{e}{5}\right)^x \log 5$
   d) $\left(\frac{5}{e}\right)^x \log 5$

5. The shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ is **[1]**

a) $3\sqrt{30}$ b) $5\sqrt{30}$

c) $\sqrt{30}$ d) $2\sqrt{30}$

6. The integrating factor of the differential equation $(x + 3y^2)\frac{dy}{dx} = y$ is **[1]**

a) $-\frac{1}{y}$ b) $\frac{1}{y}$

c) y d) -y

7. Objective function of an LPP is **[1]**

a) a function to be optimized b) a constraint

c) a relation between the variables d) a function between the variables

8. Domain of $f(x) = \cos^{-1} x + \sin x$ is: **[1]**

a) ϕ b) $(-1, 1)$

c) R d) $[-1, 1]$

9. $\int_{\pi}^{2\pi} |\sin x| dx = ?$ **[1]**

a) 1 b) 0

c) 3 d) 2

10. If $A = \begin{bmatrix} 0 & -3 & 8 \\ 3 & 0 & 5 \\ -8 & -5 & 0 \end{bmatrix}$, then A is a: **[1]**

a) null matrix b) skew-symmetric matrix

c) diagonal matrix d) symmetric matrix

11. A linear programming problem deals with the optimization of a/an: **[1]**

a) linear function b) logarithmic function

c) quadratic function d) exponential function

12. Let g(x) = $\begin{cases} e^{2x}, & x < 0 \\ e^{-2x}, & x \geq 0 \end{cases}$ then g(x) does not satisfy the condition **[1]**

a) not differentiable at x = 0 b) continuous $\forall$ x $\in$ R and non differentiable at x = ± 1

c) continuous $\forall$ x $\in$ R d) differentiable at x = 0

13. The Cartesian equation of the line passing through the point $(1, -3, 2)$ and parallel to the line : **[1]**

$\vec{r} = (2 + \lambda)\hat{i} + \lambda\hat{j} + (2\lambda - 1)\hat{k}$ is

a) $\frac{x+1}{1} = \frac{y-3}{1} = \frac{z+2}{2}$ b) $\frac{x-1}{2} = \frac{y+3}{0} = \frac{z-2}{-1}$

c) $\frac{x+1}{2} = \frac{y-3}{0} = \frac{z+2}{-1}$ d) $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}$

14. From each of the four married couples, one of the partners is selected at random. The probability that those selected are of the same sex is **[1]**

a) $\frac{1}{4}$ b) $\frac{1}{8}$

c) $\frac{1}{16}$ d) $\frac{1}{2}$

15. The general solution of the differential equation x dy - $(1 + x^2)$ dx = dx is: **[1]**

a) $y = \frac{x^2}{2} + C$ b) $y = 2x + \frac{x^3}{3} + C$

c) $y = 2 \log x + \frac{x^2}{2} + C$ d) $y = 2 \log x + \frac{x^3}{3} + C$

16. The minimum value of the function $f(x) = 2x^3 - 21x^2 + 36x - 20$ is **[1]**

a) -126 b) -128

c) -120 d) -124

17. If $y = \sqrt{x + \sqrt{x + \sqrt{x+}}} \ldots \infty$ then $\frac{dy}{dx} = ?$ **[1]**

a) $\frac{2y}{(y^2-1)}$ b) $\frac{1}{(y^2-1)}$

c) $\frac{1}{(2y-1)}$ d) $\frac{y}{(y^2-1)}$

18. Equation of a line passing through point (1, 1, 1) and parallel to z-axis is **[1]**

a) $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-1}{1}$ b) $\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{1}$

c) $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$ d) $\frac{x}{0} = \frac{y}{0} = \frac{z-1}{1}$

19. **Assertion (A):** Two lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ are parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$. **[1]**

Reason (R): Two lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ are perpendicular if $a_1a_2 + b_1 b_2 + c_1c_2 = 0$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** $\int \left(\frac{1}{1+x^2}\right) dx = \tan^{-1}(x^2) + c$ **[1]**

Reason (R): $\int \frac{1}{1+x^2} dx = \tan^{-1}x + c$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Find values of K if area of triangle is 35 square Units and vertices are (2, -6), (5, 4), (K, 4) **[2]**

OR

Using determinant show that the (1, -1), (2,1) and (4,5) points are collinear.

22. Evaluate: $\int \frac{1}{a^2-b^2x^2} dx$ **[2]**

23. Find the intervals in which the function $f(x) = x^4 - 4x^3 + 4x^2 + 15$ is increasing or decreasing. **[2]**

OR

The radius r of a right circular cone is decreasing at the rate of 3 cm/minute and the height h is increasing at the rate of 2 cm/minute. When r = 9 cm and h = 6 cm, find the rate of change of its volume.

24. A bag contains 3 red and 4 white balls. Three balls are drawn at random, one-by-one without replacement from the bag. If the first ball drawn is red in colour, then find the probability that the remaining two balls drawn are also red in colour. **[2]**

25. Show that the function ϕ defined by ϕ(x) = cos x (x $\in$ R); satisfies the initial value problem $\frac{d^2y}{dx^2}$ + y = 0, y(0) = 1, y'(0) = 0 **[2]**

Section C

26. Evaluate: $\int \frac{(2x-1)}{(x-1)(x+2)(x-3)} dx$. **[3]**

27. Differentiate the function with respect to x: $\sin^{-1}(2x^2 - 1)$, 0 < x < 1. **[3]**

28. Evaluate: $\int \frac{x}{\sqrt{x^2+x+1}} dx$ **[3]**

OR

Prove that: $\int_0^1 \frac{\log(1+x)}{(1+x^2)} dx = \frac{\pi}{8}(\log 2)$.

29. In the differential equation show that it is homogeneous and solve it: $y^2 + (x^2 - xy)\frac{dy}{dx} = 0$. **[3]**

OR

Show that the differential equation $(x^2 + xy)\,dy = (x^2 + y^2)dx$ is homogeneous and solve it.

30. Find the angle between the lines $\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ and $\vec{r} = 7\hat{i} - 6\hat{j} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$. **[3]**

OR

If $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$, then find the value of λ, so that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular vectors.

31. Solve the following problem graphically: **[3]**

Minimise or Maximise Z = 3x + 9y subject to the constraints:

$x + 3y \leq 60$

$x + y \geq 10$

$x \leq y$

$x \geq 0, y \geq 0$

Section D

32. Draw a rough sketch of the curve $y = \sqrt{x}$. Using integration, find the area of the region bounded by the curve $y = \sqrt{x}, x = 4$ and x-axis, in the first quadrant. **[5]**

33. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three mutually perpendicular vectors of the same magnitude, then prove that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined with the vectors $\vec{a}, \vec{b}$ and $\vec{c}$. **[5]**

OR

If $\vec{a}, \vec{b}, \vec{c}$ are unit vector such that $\vec{a} + \vec{b} + \vec{c} = 0$ find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ **[5]**

34. If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$.

35. A point on the hypotenuse of a right triangle is at distances a and b from the sides of the triangle. Show that the minimum length of the hypotenuse is $(a^{\frac{2}{3}} + b^{\frac{2}{3}})^{\frac{3}{2}}$. **[5]**

OR

A square piece of tin of side 18 cm is to be made into a box without the top, by cutting a square piece from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum? Also, find the maximum volume of the box.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0.65. The probability that many are not present and still the work gets completed on time is 0.35. The probability that work will be completed on time when all workers are present is 0.80.

Let: E_1: represent the event when many workers were not present for the job;

E_2: represent the event when all workers were present; and

E: represent completing the construction work on time.

i. What is the probability that all the workers are present for the job? (1)

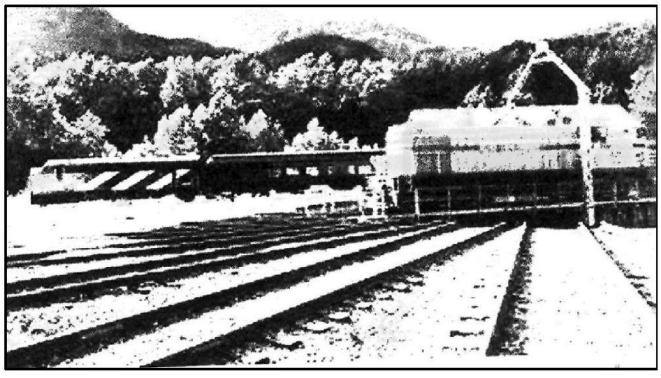
ii. What is the probability that construction will be completed on time? (1)

iii. What is the probability that many workers are not present given that the construction work is completed on time? (2)

OR

What is the probability that all workers were present given that the construction job was completed on time? (2)

37. Students of a school are taken to a railway museum to learn about railways heritage and its history. **[4]**



An exhibit in the museum depicted many rail lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by

$R = \{(l_1, l_2) : l_1$ is parallel to $l_2\}$

On the basis of the above information, answer the following questions :

i. Find whether the relation R is symmetric or not. **(1)**

ii. Find whether the relation R is transitive or not. **(1)**

iii. a. If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$, then find the set of rail lines in R related to it. **(2)**

Or

b. Let S be the relation defined by $S = \{(l_1, l_2) : l_1$ is perpendicular to $l_2\}$ check whether the relation S is symmetric and transitive. **(2)**

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Let f(x) be a real valued function. Then its

- Left Hand Derivative (L.H.D.): $Lf'(a) = \lim_{h \to 0} \frac{f(a-h)-f(a)}{-h}$
- Right Hand Derivative (R.H.D.): $Rf'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$

Also, a function f(x) is said to be differentiable at x = a if its L.H.D. and R.H.D. at x = a exist and both are equal.

For the function $f(x) = \begin{cases} |x-3|, x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, x < 1 \end{cases}$

i. What is R.H.D. of f(x) at x = 1? (1)

ii. What is L.H.D. of f(x) at x = 1? (1)

iii. Check if the function f(x) is differentiable at x = 1. (2)

OR

Find f'(2) and f'(-1). (2)

Solution

Section A

1. **(b)** AB and BA both are defined
   Explanation:
   In given matrix
   order of A = 2×3
   order of B $= 3 \times 2$
   AB will be defined if the number of column in A is equal to the number of rows in B
   so, $(A_{2\times3})\,(B_{3\times2}) = AB_{2\times2}$
   Similarly $(B_{3\times2})\,(A_{2\times3}) = BA_{3\times3}$
   Thus, Both AB and BA are defined.

2. **(a)** -1
   Explanation:
   -1

3. **(d)** x^2
   Explanation:
   x^2

4. **(d)** $\left(\frac{5}{e}\right)^x \log 5$
   Explanation:
   $\left(\frac{5}{e}\right)^x \log 5$

5. **(a)** $3\sqrt{30}$
   Explanation:
   Use formula for shortest distance between two skew lines.

6. **(b)** $\frac{1}{\text{y}}$
   Explanation:
   $\frac{1}{\text{y}}$

7. **(a)** a function to be optimized
   Explanation:
   a function to be optimized
   The objective function of a linear programming problem is either to be maximized or minimized i.e. objective function is to be optimized.

8. **(d)** $[-1, 1]$
   Explanation:
   $[-1, 1]$

9.

(d) 2

Explanation:

The given integral is $\int_{\pi}^{2\pi} |\sin x| dx$

sin x = 0

$as\ x = 0, \pi, 2\pi$ ...

So $\pi, 2\pi$ are the limits so no breaking points for the integral,

$\therefore \int_{\pi}^{2\pi} -\sin x dx = -\cos x(\pi \text{ to } 2\pi)$

= 2

10.

(b) skew-symmetric matrix

Explanation:

skew-symmetric matrix

11. **(a)** linear function

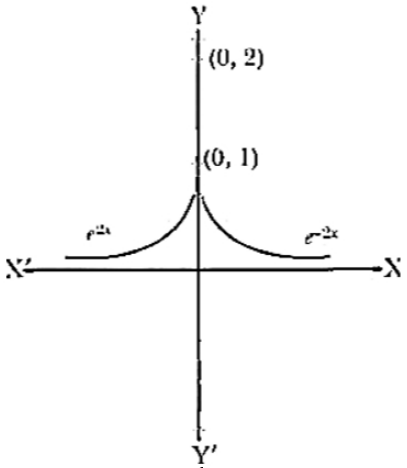
Explanation:

linear function

12.

(b) continuous $\forall$ x $\in$ R and non differentiable at x = ± 1

Explanation:



Given g(x) = $\begin{cases} e^{2x}, & x < 0 \\ e^{-2x}, & x \geq 0 \end{cases}$

g'(x) = $\begin{cases} 2e^{2x}, & x < 0 \\ -2e^{-2x}, & x \geq 0 \end{cases}$

$\therefore$ LHD at x = 0, g'(0) = $2e^{2 \times 0} = 2e^0 = 2$

RHD at x = 0, g'(0) = $-2e^0 = -2 \times 1 = -2$

As LHD $\neq$ RHD at x = 0

$\therefore$ g(x) is not differentiable at x = 0

Again RHL = $\lim_{x \to 0^+} g(x) = \lim_{x \to 0^+} e^{-2x} = e^0 = 1$

LHL = $\lim_{x \to 0^-} g(x) = \lim_{x \to 0^-} e^{2x} = e^0 = 1$

g(0) = $e^0 = 1$

As LHL = RHL = f(0)

$\therefore$ g(x) is continuous $\forall$ x $\in$ R

13.

(d) $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}$

Explanation:

$\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}$

14.

(b) $\frac{1}{8}$

Explanation:

Here, s = {(M M M M), (F F F F), ...}

Clearly, n (s) = 16

$\therefore$ Required proability = P [(M M M M) or (F F F F)]

= P [(M M M M) + (F F F F)]

= $\frac{2}{16} + \frac{2}{16} = \frac{4}{16} = \frac{1}{8}$

15.

(c) y = 2 log x + $\frac{x^2}{2} + C$

Explanation:

y = 2 log x + $\frac{x^2}{2} + C$

16.

(b) -128

Explanation:

f(x) = $2x^3$ - $21x^2$ + 36x - 20

$\Rightarrow$ f'(x) = $6x^2$ - 42x + 36

For local maxima or minima

$6x^2$ - 42x + 36 = 0

x^2 - 7x + 6 = 0

$\Rightarrow$ x = 1 or x = 6

f''(x) = 12x - 42

$\Rightarrow$ f''(1) = -30 < 0

also, f''(6) = 30 > 0

function has minima at x = 6

$\Rightarrow$ f(6) = -128

17.

(c) $\frac{1}{(2y-1)}$

Explanation:

Given:

$\Rightarrow y = \sqrt{x + \sqrt{x + \sqrt{x+}}} \ldots \infty$

We can write it as

$\Rightarrow y = \sqrt{x+y}$

Squaring we get

$\Rightarrow$ y^2 = x + y

Differentiating with respect to x,we get

$\Rightarrow 2y\frac{dy}{dx} = 1 + \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{(2y-1)}$

18.

(b) $\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{1}$

Explanation:

$\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{1}$

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

If two lines have direction ratios a_1, b_1, c_1 and a_2, b_2, c_2, respectively.

Then, condition for parallel lines: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

and condition for perpendicular lines: $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

20.

(d) A is false but R is true.

Explanation:

$\int \frac{dx}{1+x^2}$

Let x = $\tan\theta$

dx = $\sec^2\theta\, d\theta$

$= \int \frac{\sec^2\theta\, d\theta}{(1+\tan^2\theta)}$

$= \int \frac{(1+\tan^2\theta)d\theta}{(1+\tan^2\theta)}$ {$\because \sec^2\theta = 1 + \tan^2\theta$}

$= \theta$

$= \tan^{-1}x + c$

Section B

21. $area = \Delta = \frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ K & 4 & 1 \end{vmatrix}$

$= \frac{1}{2}[2(4-4) + 6(5-K) + 1(20-4K)]$

$= \frac{1}{2}[50 - 10K]$

= 25 - 5 K

Area =35 units (given)

25 - 5K = 35

K = 2

OR

Here, (1, -1), (2, 1) and (4, 5) are three points ...(given)

Now, we know that,

Vertices of a triangle are (x_1, y_1), (x_2, y_2) and (x_3, y_3), then the area of the triangle is given by:

$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

Now,

Substituting given value in above formula

$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ 4 & 5 & 1 \end{vmatrix} = 0$$

take LHS

$$\frac{1}{2}\begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ 4 & 5 & 1 \end{vmatrix}$$

Expanding along R_1

$$= \frac{1}{2}\left[1\begin{vmatrix} 1 & 1 \\ 5 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 4 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 4 & 5 \end{vmatrix}\right]$$

$= \frac{1}{2}$ [1 - 5 + 2 - 4 + 10 - 4]

$= \frac{1}{2}[0]$

$= 0$

= RHS

Since, Area of triangle is zero.

Hence, points are collinear

22. Let I = $\int \frac{1}{a^2-b^2x^2}dx$, then we have

$$I = \frac{1}{b^2}\int \frac{1}{\frac{a^2}{b^2}-x^2}dx$$

$$= \frac{1}{b^2}\int \frac{1}{\left(\frac{a}{b}\right)^2-x^2}dx$$

$$I = \frac{1}{b^2} \times \frac{1}{2\times\left(\frac{a}{b}\right)}\log\left|\frac{\frac{a}{b}+x}{\frac{a}{b}-x}\right| + c \quad \left[\text{ since } \int \frac{1}{a^2-x^2}dx = \frac{1}{2a}\log\left|\frac{x+a}{x-a}\right| + c\right]$$

$$I = \frac{1}{2ab}\log\left|\frac{a+bx}{a-bx}\right| + c$$

23. Given: $f(x) = x^4 - 4x^3 + 4x^2 + 15$

$\Rightarrow$ f'(x) = $4x^3 - 12x^2 + 8x$

To find critical point of f(x), we must have

$\Rightarrow$ f'(x) = 0

$\Rightarrow 4x^3 - 12x^2 + 8x = 0$

$\Rightarrow 4(x^3 - 3x^2 + 2x) = 0$

$\Rightarrow x(x^2 - 3x + 2) = 0$

$\Rightarrow x(x^2 - 2x - x + 2) = 0$

$\Rightarrow x(x - 2)(x - 1)$

$\Rightarrow x = 0, 1, 2$

clearly, f'(x) > 0 if 0 < x < 1 and x > 2 and f'(x) < 0 if x < 0 and 1 < x <

Thus, f(x) increases on $(0, 1) \cup (2, \infty)$

and f(x) is decreasing on interval $(-\infty, 0) \cup (1, 2)$

OR

We know that Volume of right circular cone $= \frac{\pi r^2 h}{3}$

$$\frac{\partial V}{\partial t} = \frac{\pi}{3}\left(2rh\frac{\partial r}{\partial t} + r^2\frac{\partial h}{\partial t}\right)$$

$$\frac{\partial v}{\partial t} = \frac{\pi}{3}(108 \times -3 + 81 \times 2)$$

$\frac{\partial v}{\partial t} = \frac{\pi}{3}(-162) = -54\pi \text{cm}^2/\text{min}$

Therefore Volume is decreasing at rate $54\pi \text{cm}^2$/min..

24. Total balls = 3 + 4 = 7

Now if 1st ball was red then red balls left = 2

Total balls = 6

P(B_2) = $\frac{^2C_1}{^6C_1}$

And now for 3rd ball, red ball left = 1

Total balls= 5

P(B_3) = $\frac{^1C_1}{^5C_1}$

Now, required probability= $\frac{^2C_1}{^6C_1} \times \frac{^1C_1}{^5C_1} = \frac{2}{6} \times \frac{1}{5} = \frac{1}{15}$

25. We have, ϕ(x) = cos x

$\Rightarrow \phi'(x) = -\sin x \text{ and } \phi''(x) = -\cos x$

$\Rightarrow \phi'(x) = -\sin x \text{ and } \phi''(x) = -\phi(x)$

Clearly, if we replace ϕ by y in ϕ"(x) = -ϕ(x), we obtain

$\frac{d^2y}{dx^2} = -y$ or, $\frac{d^2y}{dx^2} + y$ = 0,

Thus, ϕ(x) satisfies the given differential equation for all x $\in$ R

Also, ϕ(0) = cos 0 = 1 and ϕ'(0) = -sin 0 = 0

$\Rightarrow$ y(0) = 1 and y'(0) = 0

So, ϕ satisfies the initial conditions. Hence, ϕ satisfies the initial value problem.

Section C

26. Let I, $= \int \frac{(2x-1)}{(x-1)(x+2)(x-3)} dx$

Using partial fractions we have,

$\frac{(2x-1)}{(x-1)(x+2)(x-3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x-3}$ .... (i)

Which implies,

A(x + 2)(x - 2) + B(x - 1)(x - 3) + C(x - 1)(x + 2) = 2x - 1

Now put x + 2 = 0

Therefore, x = -2

A(0) + B(-2 - 1)(-2 - 3) + C(0) = 2x - 2 - 1

B(-3)(-5) = -5

$B = -\frac{1}{3}$

Now put x - 3 = 0

Therefore, x = 3

A(0) + B(0) + C(2) (5) = 5

$C = \frac{1}{2}$

Now put x - 1 = 0

Therefore, x = 1

A(3) (-2) + B(0) + C(0) = 1

$A = -\frac{1}{6}$

Now From equation (i) we get,

$\frac{(2x-1)}{(x-1)(x+2)(x-3)} = \frac{-1}{6} \times \frac{1}{x-1} - \frac{1}{3} \times \frac{1}{x+2} + \frac{1}{2} \times \frac{1}{x-3}$

$\int \frac{(2x-1)}{(x-1)(x+2)(x-3)} dx = \frac{-1}{6} \int \frac{1}{x-1} dx - \frac{1}{3} \int \frac{1}{x+2} dx + \frac{1}{2} \int \frac{1}{x-3} dx$

$= \frac{-1}{6} \log|x-1| - \frac{1}{3} \log|x+2| + \frac{1}{2} \log|x-3| + c$

27. Let y = $\sin^{-1}[2x^2 - 1]$

Put x = cos θ

$y = \sin^{-1}\{2\cos^2\theta - 1\}$

$y = \sin^{-1}(\cos 2\theta)$

$y = \sin^{-1}\left\{\sin\left(\frac{\pi}{2} - 2\theta\right)\right\}$ ...(i)

Here, $0 < x < 1$

$\Rightarrow 0 < \cos\theta < 1$

$\Rightarrow 0 < \theta < \frac{\pi}{2}$

$\Rightarrow 0 < 2\theta < \pi$

$\Rightarrow 0 > -2\theta > -\pi$

$\Rightarrow \frac{\pi}{2} > \left(\frac{\pi}{2} - 2\theta\right) > -\frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{2} < \left(\frac{\pi}{2} - 2\theta\right) < \frac{\pi}{2}$

So, from equation (i),

$y = \frac{\pi}{2} - 2\theta \left[\text{ Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$

$\Rightarrow y = \frac{\pi}{2} - 2\cos^{-1} x [\text{ Since }, x = \cos\theta]$

Differentiating it with respect to x,

$\frac{dy}{dx} = 0 - 2\frac{d}{dx}\left(\cos^{-1} x\right)$

$\Rightarrow \frac{dy}{dx} = -2\left(-\frac{1}{\sqrt{1-x^2}}\right)$

$\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$

28. Let the given integral be, $I = \int \frac{x}{\sqrt{x^2+x+1}}dx$

Let $x = \lambda\frac{d}{dx}(x^2 + x + 1) + \mu$

$= \lambda(2x + 1) + \mu$

$\Rightarrow x = (2\lambda)x + \lambda + \mu$

Comparing the coefficients of like powers of x,

$2\lambda = 1 \Rightarrow \lambda = \frac{1}{2}$

$\lambda + \mu = 0$

$\Rightarrow \left(\frac{1}{2}\right) + \mu = 0$

$\Rightarrow \mu = -\frac{1}{2}$

So, $I = \int \frac{\frac{1}{2}(2x+1)-\frac{1}{2}}{\sqrt{x^2+x+1}}dx$

$= \frac{1}{2}\int \frac{(2x+1)}{\sqrt{x^2+x+1}}dx - \frac{1}{2}\int \frac{1}{\sqrt{x^2+2x\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1}}dx$

$= \frac{1}{2}\int \frac{2x+1}{\sqrt{x^2+x+1}}dx - \frac{1}{2}\int \frac{1}{\sqrt{\left(x+\frac{1}{2}\right)^2-\left(\frac{\sqrt{3}}{2}\right)^2}}dx$

$= \frac{1}{2} \times 2\sqrt{x^2+x+1} - \frac{1}{2}\log\left|x + \frac{1}{2} + \sqrt{\left(x+\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2}\right| + c$ [Since,

$\int \frac{1}{\sqrt{x}}dx = 2\sqrt{x} + c, \int \frac{1}{\sqrt{x^2-a^2}}dx = \log|x + \sqrt{x^2-a^2}| + c]$

$\Rightarrow I = \sqrt{x^2+x+1} - \frac{1}{2}\log\left|x + \frac{1}{2} + \sqrt{x^2+x+1}\right| + c$

OR

Let $x = \tan t$

$\Rightarrow dx = \sec^2 t\, dt$

At $x = 0$, $t = 0$

At $x = 1$, $t = \frac{\pi}{4}$

Therefore the given integral becomes,

$y = \int_0^{\frac{\pi}{4}} \frac{\log(1+\tan t)}{1+\tan^2 t}\sec^2 t dt$

$y = \int_0^{\frac{\pi}{4}} \log(1+\tan t)dt$ .... (i)

Use King theorem of definite integral

$\int_a^b f(t)dt = \int_a^b f(a+b-t)dt$

$y = \int_0^{\frac{\pi}{4}} \log\left(1+\tan\left(\frac{\pi}{4}-t\right)\right)dt$

$y = \int_0^{\frac{\pi}{4}} \log\left(1+\frac{1-\tan t}{1+\tan t}\right)dt$

$y = \int_0^{\frac{\pi}{4}} \log\left(\frac{2}{1+\tan t}\right)dt$ ..... (ii)

Adding eq.(i) and (ii)

$2y = \int_0^{\frac{\pi}{4}} \log(1+\tan t)dt + \int_0^{\frac{\pi}{4}} \log\left(\frac{2}{1+\tan t}\right)dt$

$2y = \int_0^{\frac{\pi}{4}} \log(1+\tan t)\left(\frac{2}{1+\tan t}\right)dt$

$2y = \int_0^{\frac{\pi}{4}} \log 2 dt$

$y = \frac{\pi}{8}\log 2$

29. The given differential equation is,

$y^2 + (x^2 - xy)\frac{dy}{dx} = 0$

$\frac{dx}{dy} = \frac{xy-x^2}{y^2} = \frac{x}{y} - (\frac{x}{y})^2$

$\Rightarrow \frac{dx}{dy} = f\left(\frac{x}{y}\right)$

$\Rightarrow$ the given differential equation is a homogenous equation.

The solution of the given differential equation is:

Put x = vy

$\Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$

$\Rightarrow v + y\frac{dv}{dy} = \frac{vy}{y} - \left(\frac{vy}{y}\right)^2$

$\Rightarrow y\frac{dv}{dy} = v - v^2 - v$

$\Rightarrow y\frac{dv}{dy} = -v^2$

$\Rightarrow \frac{dv}{v^2} = -\frac{dy}{y}$

Integratin both the sides we get

$\Rightarrow \int \frac{dv}{v^2} = -\int \frac{dy}{y} + c$

$\Rightarrow \frac{-1}{v} = -\ln|y| + c$

$\Rightarrow \frac{y}{x} = -(\ln|y| + c)$

$\Rightarrow y = -x(\ln|y| + c)$

OR

We can rewrite the given differential equation as,

$\frac{dy}{dx} = \frac{x^2+y^2}{x^2+xy}$

On dividing the Nr and Dr of RHS of (i) by x^2, we get

$\frac{dy}{dx} = \frac{\left\{1+\left(\frac{y}{x}\right)^2\right\}}{\left\{1+\frac{y}{x}\right\}} = f\left(\frac{y}{x}\right)$

Therefore, the given differential equation is homogeneous.

Put y = vx and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ in (i), we get

$v + x\frac{dv}{dx} = \frac{x^2+v^2x^2}{x^2+vx^2} = \frac{1+v^2}{1+v}$

$\Rightarrow \quad x\frac{dv}{dx} = \frac{(1+v^2)}{(1+v)} - v = \frac{1+v^2-v-v^2}{(1+v)} = \frac{(1-v)}{(1+v)}$

$\Rightarrow \frac{(1+v)}{(1-v)}dv = \frac{1}{x}dx$

$\Rightarrow \int \frac{(1+v)}{(1-v)}dv = \int \frac{1}{x}dx$ [on integrating both sides]

$\Rightarrow \int \frac{|2-(1-v)\}}{(1-v)}dv = \int \frac{1}{x}dx$

$\Rightarrow \int \left\{ \frac{2}{(1-v)} - 1 \right\} dv = \int \frac{1}{x}dx$

$\Rightarrow$ -2 log | 1 - v| -v = log | x| + log | C| where C is an arbitrary constant

$\Rightarrow$ log | x| + log | C| + 2 log | 1 - v| = -v

$\Rightarrow$ log | Cx $(1 - v)^2$| = -v

$\Rightarrow$ | C x$(1-v)^2$| = e^{-v}

$\Rightarrow$ | Cx $\left(1 - \frac{y}{x}\right)^2$ | = $e^{-y/x}$ [$\because$ v = $\frac{y}{x}$]

$\Rightarrow$ C | $(x - y)^2$ = | x| $e^{-y/x}$, which is the required solution.

30. If vector form of lines are $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$ then angle between them is

$\cos\theta = \frac{\left|\overrightarrow{b_1}.\overrightarrow{b_2}\right|}{\left|\overrightarrow{b_1}\right|\left|\overrightarrow{b_2}\right|}$

Given equations of lines are

$\vec{r} = (2\hat{i} - 5\hat{j} + \hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ .....(i)

and $\vec{r} = (7\hat{i} - 6\hat{j} - 6\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$ .....(ii)

On comparing Equations , (i) and (ii) with vector form of equation of line i.e.

$\vec{r} = \vec{a} + \lambda\vec{b}$, we get,

$\overrightarrow{a_1} = 2\hat{i} - 5\hat{j} + \hat{k}, \overrightarrow{b_1} = \hat{3}\hat{i} + 2\hat{j} + 6\hat{k}$

and $\overrightarrow{a_2} = \hat{7}\hat{i} - 6\hat{j} - 6\hat{k}, \vec{b}_2 = \hat{i} + 2\hat{j} + 2\hat{k}$

We know that, the angle between two lines is given by

$\cos\theta = \frac{\left|\overrightarrow{b_1}\cdot\overrightarrow{b_2}\right|}{\left|\overrightarrow{b_1}\right|\left|\overrightarrow{b_2}\right|}$

$\cos\theta = \frac{(3\hat{i}+2\hat{j}+6\hat{k})\cdot(\hat{i}+2\hat{j}+2\hat{k})}{\sqrt{(3)^2+(2)^2+(6)^2}\cdot\sqrt{(1)^2+(2)^2+(2)^2}}$

$\Rightarrow \cos\theta = \left|\frac{4+12}{3\sqrt{49}\times\sqrt{9}}\right|$

$\Rightarrow \cos\theta = \left|\frac{19}{7\times3}\right| \Rightarrow \cos\theta = \frac{19}{21}$

Hence, the angle between given two lies is

$\theta = \cos^{-1}\left(\frac{19}{21}\right)$

OR

According to the question ,

Given vectors are , $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$ and

$\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$

Now,

$\vec{a} + \vec{b} = (\hat{i} - \hat{j} + 7\hat{k}) + (5\hat{i} - \hat{j} + \lambda\hat{k})$

$= 6\hat{i} - 2\hat{j} + (7+\lambda)\hat{k}$

Now,

$\vec{a} - \vec{b} = (\hat{i} - \hat{j} + 7\hat{k}) - (5\hat{i} - \hat{j} + \lambda\hat{k})$

$= -4\hat{i} + (7-\lambda)\hat{k}$

Since, $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ are perpendicular vectors, then dot product of these vectors will be zero, i.e. $(\vec{a}+\vec{b}) \cdot (\vec{a}-\vec{b}) = 0$

$\Rightarrow \quad [6\hat{i} - 2\hat{j} + (7+\lambda)\hat{k}] \cdot [-4\hat{i} + (7-\lambda)\hat{k}] = 0$

$\Rightarrow \quad -24 + (7+\lambda)(7-\lambda) = 0$

$\Rightarrow \quad 49 - \lambda^2 = 24$

$\Rightarrow \quad \lambda^2 = 25$

$\Rightarrow \quad \lambda = \pm 5$

$\therefore \quad \lambda = \pm 5$

31. The linear inequations or constraints

$x + 3y \leq 60$

$x + y \geq 10$

$x \leq y$

$x \geq 0, y \geq 0$ and objective functions is max or min $(Z) = 3x + 9y$

Reducing the above inequations into equations and finding their point of intersection i.e.,

$x + 3y = 60$ ... (i)

$x + y = 10$ ... (ii)

$x = y$ ... (iii)

$x = 0, y = 0$... (iv)

Equations	**Point of Intersection**
(i) and (ii)	$x = -15, y = 25$
	Point is $\Rightarrow (-15, 25)$
(i) and (iii)	x = 15 $\Rightarrow$ y = 15
	Point is $\Rightarrow (15, 15)$
(ii) and (iii)	$x = 5, y = 5$
	Point is $(5, 5)$
(i) and (iv)	when x = 0 $\Rightarrow$ y = 20,
	Point is$(0, 20)$
	when y = 0 $\Rightarrow$ x = 60,
	Point is $(60, 0)$
(ii) and (iv)	when x = 0 $\Rightarrow$ y = 10,
	Point is $(0, 10)$
	when y = 0 $\Rightarrow$ x = 10,
	Point is $(10, 0)$

Now for feasible region,

For $x + 3y \leq 60$, putting x = 0 and y = 0, we have

$0 + 0 \geq 10$ i.e., Not true

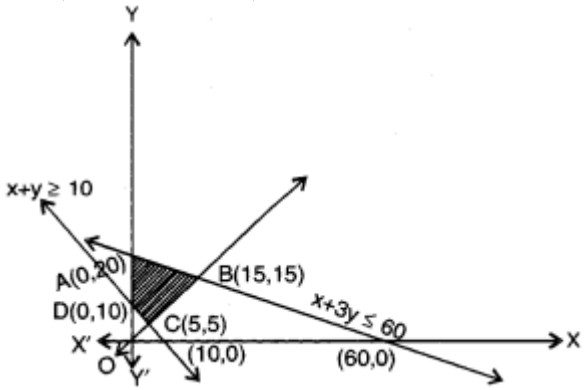
$\Rightarrow$ The shaded region will be away from origin.

Also, we have $x \geq 0$, and $y \geq 0$ indicates that the shaded part will exist in first quadrant only. Here feasible region will be ABCDA, having corner points as $A(0, 20), B(15, 15), C(5, 5)$ *and* $D(0, 10)$.

For optimal point substituting the value of all corner points in objective function $Z = 3x + 9y$

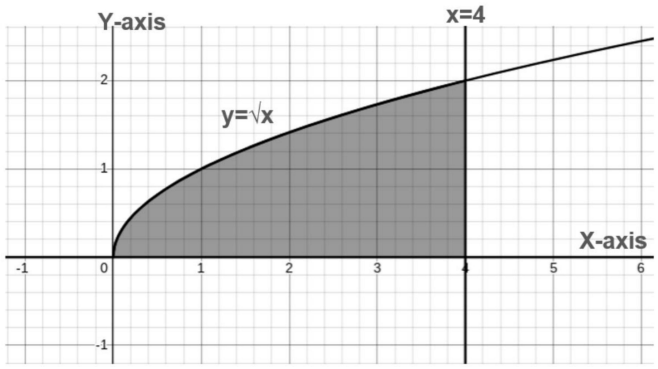
Corner points	**Z**	
$A(0, 20)$	180	Maximum
$B(15, 15)$	180	
$C(5, 5)$	60	Minimum
$D(0, 10)$	90	

So that the minimum value of Z is 60 at C (5, 5) of the feasible region and the maximum value at A (0, 20) and B(15, 15) is Z = 180.



Section D

32.



Required area $= \int_0^4 \sqrt{x} dx$

$= \frac{2}{3}\left[x^{3/2}\right]_0^4$

$= \frac{2}{3} \times 8 = \frac{16}{3}$

33. According to the question, $|\vec{a}| = |\vec{b}| = |\vec{c}| = \lambda$(say) ...(i)

and $\vec{a} \cdot \vec{b} = 0, \vec{b}.\vec{c} = 0$ and $\vec{a}.\vec{c} = 0$ ...(ii)

Now, $|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$

$= \lambda^2 + \lambda^2 + \lambda^2 + 2(0 + 0 + 0) = 3\lambda^2$

$\Rightarrow \quad |\vec{a} + \vec{b} + \vec{c}| = \sqrt{3}\lambda$

[length cannot be negative]

Let $(\vec{a} + \vec{b} + \vec{c})$ is inclined at angles θ_1, θ_2

and θ_3 respectively with vector $\vec{a}, \vec{b}$ and $\vec{c}$,

$(\vec{a} + \vec{b} + \vec{c}) \cdot \vec{a} = |\vec{a} + \vec{b} + \vec{c}||\vec{a}| \ cos\ \theta_1$

$[\because \vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta]$

$\Rightarrow \quad |\vec{a}|^2 + \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c} = \sqrt{3}\lambda \times \lambda\cos\theta_1$

$\Rightarrow \lambda^2 + 0 + 0 = \sqrt{3}\lambda^2\cos\theta_1$

[from Equations (i) and (ii)]

$\therefore \quad \cos\theta_1 = \frac{1}{\sqrt{3}}$

$(\vec{a}+\vec{b}+\vec{c})\cdot\vec{b} = \left|\vec{a}+\vec{b}+\vec{c}\right|\left|\vec{b}\right|\cos\theta_2$

$\Rightarrow \quad \vec{a}\cdot\vec{b} + |\vec{b}|^2 + \vec{c}\cdot\vec{b} = \sqrt{3}\lambda\cdot\lambda\cos\theta_2$

$\Rightarrow \quad \theta + \lambda + \theta = \sqrt{3}\lambda_2\cos\theta_2$

[from Equation. (i) and (ii)]

$\Rightarrow \quad \cos\theta_2 = \frac{1}{\sqrt{3}}$

Similarly, $(a+\vec{b}+\vec{c})\cdot\vec{c} = \left|\vec{a}+\vec{b}+\vec{c}\right||\vec{c}\,|\cos\theta_3|$

$\Rightarrow \quad \cos\theta_1 = \frac{1}{\sqrt{3}}$

Thus, $\cos\theta_1 = \cos\theta_2 = \cos\theta_3 = \frac{1}{\sqrt{3}}$

$\therefore \overrightarrow{(a+\vec{b}+\vec{c})}$ is equally inclined with the vectors $\vec{a}, \vec{b}$ and $\vec{c}$.

OR

$|\vec{a}| = 1, \left|\vec{b}\right| = 1, |\vec{c}| = 1$

$\vec{a}+\vec{b}+\vec{c} = 0$ (Given)

$\vec{a}.\left(\vec{a}+\vec{b}+\vec{c}\right) = 0$

$\vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{a}.\vec{c} = 0$

$|\vec{a}|^2 + \vec{a}.\vec{b} + \vec{a}.\vec{c} = 0$

$1 + \vec{a}.\vec{b} + \vec{a}.\vec{c} = 0$

$\vec{a}.\vec{b} + \vec{a}.\vec{c} = -1$ ...(i)

Similarly,

$\vec{b}.\vec{a} + \vec{b}.\vec{c} = -1$ ...(ii)

again

$\vec{c}.\vec{a} + \vec{c}.\vec{b} = -1$ ...(iii)

adding (i), (ii) and (iii)

$2\left(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}\right) = -3 \quad \left[\because \vec{a}.\vec{b} = \vec{b}.\vec{a}\right]$

$\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -\frac{3}{2}$

34. L.H.S. = $A^3 - 6A^2 + 7A + 2I$

$$= \begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix} - 6\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix} + 7\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix} + 2\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$= \begin{bmatrix}1+0+4 & 0+0+0 & 2+0+6\\0+0+2 & 0+4+0 & 0+2+3\\2+0+6 & 0+0+0 & 4+0+9\end{bmatrix}\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix} - 6\begin{bmatrix}1+0+4 & 0+0+0 & 2+0+6\\0+0+2 & 0+4+0 & 0+2+3\\2+0+6 & 0+0+0 & 4+0+9\end{bmatrix} + \begin{bmatrix}7&0&14\\0&14&7\\14&0&21\end{bmatrix} + \begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}$$

$$= \begin{bmatrix} 5+0+16 & 0+0+0 & 10+0+24 \\ 2+0+10 & 0+8+0 & 4+4+15 \\ 8+0+26 & 0+0+0 & 16+0+39 \end{bmatrix}$$

$$- \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7+2 & 0+0 & 14+0 \\ 0+0 & 14+2 & 7+0 \\ 14+0 & 0+0 & 21+2 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 14 \\ 0 & 16 & 7 \\ 14 & 0 & 23 \end{bmatrix} = \begin{bmatrix} 21-30 & 0-0 & 34-48 \\ 12-12 & 8-24 & 23-30 \\ 34-48 & 0-0 & 55-78 \end{bmatrix}$$

$$+ \begin{bmatrix} 9 & 0 & 14 \\ 0 & 16 & 7 \\ 14 & 0 & 23 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & 0 & -14 \\ 0 & -16 & -7 \\ -14 & 0 & -23 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 14 \\ 0 & 16 & 7 \\ 14 & 0 & 23 \end{bmatrix} = \begin{bmatrix} -9+9 & 0+0 & -14+14 \\ 0+0 & -16+16 & -7+7 \\ -14+14 & 0+0 & -23+23 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0 \text{ (Zero matrix)}$$

= R.H.S. Proved.

35. Let P be a point on the hypotenuse AC of right-angled ΔABC, Such that $PL \perp AB$ and PL=a and $PM \perp BC$ and PM=b.

Let $\angle APL = \angle ACB = \theta$[say]

Then, $AP = a\sec\theta, PC = b\ cosec\theta$

Let l be the length of the hypotenuse, then

l = AP+ PC

$\Rightarrow l = a\sec\theta + b\ cosec\theta, 0 < \theta < \frac{\pi}{2}$

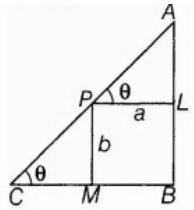
On differentiating both sides w.r.t. θ, we get, $\frac{dl}{d\theta} = a\sec\theta\tan\theta - b\ cosec\theta\cot\theta$.....(i)

For maxima or minima, put $\frac{dl}{d\theta} = 0$

$\Rightarrow a\sec\theta\tan\theta = b\ cosec\theta\cot\theta$

$\Rightarrow \quad \frac{a\sin\theta}{\cos^2\theta} = \frac{b\cos\theta}{\sin^2\theta}$

$\Rightarrow \quad \tan\theta = \left(\frac{b}{a}\right)^{1/3}$



Again, on differentiating both sides of Eq.(i) w.r.t. θ we get

$\frac{d^2l}{d\theta^2} = a\left(\sec\theta\times\sec^2\theta + \tan\theta\times\sec\theta\tan\theta\right) - b\left[\ cosec\theta\left(-\ cosec^2\theta\right)\right.$

$\left. + \cot\theta(-\ cosec\theta\cot\theta)\right]$

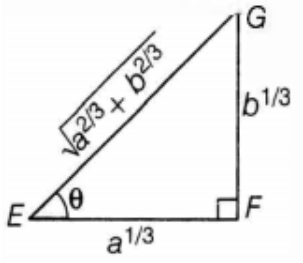
$= a\sec\theta\left(\sec^2\theta + \tan^2\theta\right) + b\ cosec\theta\left(\ cosec^2\theta + \cot^2\theta\right)$

For $0 < \theta < \frac{\pi}{2}$, all trigonometric ratios are positive

Also, a > 0 and b > 0

$\because \frac{d^2l}{d\theta^2}$ is positive.

Thus, l is least when $\tan\theta = \left(\frac{b}{a}\right)^{\frac{1}{3}}$



$\because$ Least value of,

$l = a\sec\theta + b\ cosec\theta$

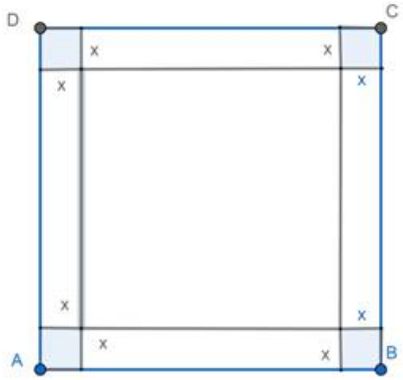
$= a\frac{\sqrt{a^{2/3}+b^{2/3}}}{a^{1/3}} + b\frac{\sqrt{a^{2/3}+b^{2/3}}}{b^{1/3}}$

$= \sqrt{a^{2/3}+b^{2/3}}\left(a^{2/3}+b^{2/3}\right) = \left(a^{2/3}+b^{2/3}\right)^{3/2}$

$\left[\because \text{ in } \Delta EFG, \tan\theta = \frac{b^{1/3}}{a^{1/3}}, \sec\theta = \frac{\sqrt{a^{2/3}+b^{2/3}}}{a^{1/3}} \text{and } \cos ec\theta = \frac{\sqrt{a^{2/3}+b^{2/3}}}{b^{1/3}}\right]$

OR

Given, the Side of the square piece is 18 cms.



Let us consider,

'x' be the length and breadth of the piece cut from each vertex of the piece.

Side of the box now will be (18 - 2x)

The height of the newly formed box will also be 'x'.

Let the volume of the newly formed box is :

$V = (18 - 2x)^2 \times (x)$

$V = 4x^3 - 72x^2 + 324x$ ...(i)

For finding the maximum/ minimum of a given function, we can find it by differentiating it with x and then equating it to zero. This is because if the function V(x) has a maximum/minimum at a point c then V'(c) = 0.

Differentiating the equation (i) with respect to x:

$\frac{dV}{dx} = \frac{d}{dx}\left[4x^3 - 72x^2 + 324x\right]$

$\frac{dV}{dx} = 12x^2 - 144x + 324$ ...(ii)

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

To find the critical point, we need to equate equation (ii) to zero.

$\frac{dV}{dx} = 12x^2 - 144x + 324 = 0$

$x^2 - 12x + 27 = 0$

$x = \frac{-(-12)\pm\sqrt{(-12)^2-4(1)(27)}}{2(1)}$

$= \frac{12 \pm \sqrt{144-108}}{2} = \frac{12 \pm \sqrt{36}}{2}$

$x = \frac{12 \pm 6}{2}$

x = 9 or x = 3

x = 2

[as x = 9 is not a possibility, because 18 - 2x = 18 - 18 = 0]

Now to check if this critical point will determine the maximum area of the box, we need to check with second differential which needs to be negative.

Consider differentiating the equation (ii) with x:

$\frac{d^2V}{dx^2} = \frac{d}{dx}\left[12x^2 - 144x + 324\right]$

$\frac{d^2v}{dx^2} = 24x - 144$ ....(iv)

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

Now let us find the value of

$\left(\frac{d^2V}{dx^2}\right)_{x=3}$ = 24(3) - 144 = 72 - 144

= -72

As $\left(\frac{d^2V}{dx^2}\right)_{x=3} = -72 < 0$, so the function V is maximum at x = 3cm

Now substituting x = 3 in 18 - 2x, the side of the considered box:

Side = 18 - 2x = 18 - 2(3) = 18 - 6 = 12cm

Therefore side of wanted box is 12cms and height of the box is 3cms.

Now, the volume of the box is

$V = (12)^2 \times 3 = 144 \times 3 = 432 cm^3$

Hence maximum volume of the box formed by cutting the given 18cms sheet is $432 cm^3$ with 12cms side and 3cms height.

Section E

36. i. $P(E_2) = 1 - P(E_1) = 1 - 0.65 = 0.35$

ii. $P(E) = P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right)$

$= 0.65 \times 0.35 + 0.35 \times 0.8$

$= 0.35 \times 1.45$

$= 0.51$

iii. $P\left(\frac{E_1}{E}\right) = \frac{P(E_1) \cdot P\left(\frac{E}{E_1}\right)}{P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right)} = \frac{0.65 \times 0.35}{0.51} = 0.45$

OR

$P\left(\frac{E_2}{E}\right) = \frac{P(E_2) \cdot P\left(\frac{E}{E_2}\right)}{P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right)} = \frac{0.35 \times 0.8}{0.51} = 0.55$

37. i. Let $(l_1, l_2) \in R \Rightarrow l_1 \backslash | l_2 \Rightarrow l_2 \backslash | l_1 \Rightarrow (l_2, l_1) \in R, \therefore R$ is a symmetric relation

ii. Let $(l_1, l_2), (l_2, l_3) \in R \Rightarrow l_1 \| l_2, l_2 \| l_3 \Rightarrow l_1 \| l_3 \Rightarrow (l_1, l_3) \in R, \therefore R$ is a transitive relation

iii. a. The set is $\{1 : 1$ is a line of type $y = 3x + c, c \in R\}$

Or

b. Let $(l_1, l_2) \in R \Rightarrow l_1 \perp l_2 \Rightarrow l_2 \perp l_1 \Rightarrow (l_2, l_1) \in R, \therefore R$ is a symmetric relationLet $(l_1, l_2), (l_2, l_3) \in R \Rightarrow l_1 \perp l_2, l_2 \perp l_3 \Rightarrow l_1 \| l_3 \Rightarrow (l_1, l_3) \notin R, \therefore R$ is not a transitive relation

38. i. R.H.D. of f(x) at x = 1 = $\lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$

$= \lim_{\boldsymbol{h\to 0}} \frac{|1+h-3|-|-2|}{h} = \lim_{\boldsymbol{h\to 0}} \frac{2-h-2}{h}$ = -1

ii. L.H.D. of f(x) at x = 1 = $\lim_{\mathrm{h}\to 0} \frac{f(1-h)-f(1)}{-h}$

$= \lim_{h\to 0} \frac{\left[\frac{(1-h)^2}{4} - \frac{3(1-h)}{2} + \frac{13}{4} - 2\right]}{-h}$

$= \lim_{h\to 0} \left[\frac{h^2-2h+1-6+6h+13-8}{-4h}\right]$

$= \lim_{h\to 0} \left[\frac{h^2+4h}{-4h}\right]$ = -1.

iii. Since L.H.D. of f(x) at x = 1
is same as R.H.D. of f(x) at x = 1,
f(x) is differentiable at x = 1.

OR

$$f(x) = \begin{cases} x-3, x \geq 3 \\ 3-x, 1 \leq x < 3 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, x < 1 \end{cases}$$

$[f'(x)]_{x=2}$ = 0 - 1 = -1

$[f'(x)]_{x=-1} = \frac{2(-1)}{4} - \frac{3}{2}$ = -2