Class XII Session 2026-27
Subject - Mathematics
Sample Question Paper - 7

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculators is not allowed.

Section A

1. Let P be a skew-symmetric matrix of order 3. If $\det(P) = \alpha$, then $(2025)^{\alpha}$ is **[1]**

   a) 0 b) 2025

   c) 1 d) $(2025)^3$

2. If $\begin{vmatrix} -1 & 2 & 4 \\ 1 & x & 1 \\ 0 & 3 & 3x \end{vmatrix} = -57$, the product of the possible values of x is: **[1]**

   a) 24 b) 16

   c) -24 d) -16

3. Let A be a square matrix of order 3. If $|A| = 5$, then $|\text{adj } A|$ is: **[1]**

   a) 25 b) 125

   c) 5 d) -5

4. If A, B are two n $\times$ n non - singular matrices, then what can you infer about AB? **[1]**

a) $(AB)^{-1}$ does not exist

b) AB is non-singular

c) AB is singular

d) $(AB)^{-1} = A^{-1}B^{-1}$

5. The line $x = 1 + 5\mu, y = -5 + \mu, z = -6 - 3\mu$ passes through which of the following point? **[1]**

a) $(1, -5, 6)$

b) $(-1, -5, 6)$

c) $(1, 5, 6)$

d) $(1, -5, -6)$

6. The number of solutions of the differential equation $\frac{dy}{dx} = \frac{y+1}{x-1}$, when y(1) = 2, is: **[1]**

a) infinite

b) zero

c) one

d) two

7. The graph of the inequality 2x + 3y > 6 is **[1]**

a) whole XOY – plane excluding the points on the line 2x + 3y = 6.

b) half plane that neither contains the origin nor the points of the line 2x + 3y = 6.

c) entire XOY plane.

d) half plane that contains the origin.

8. The projection vector of vector $\vec{a}$ on vector $\vec{b}$ is **[1]**

a) $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

b) $\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right)\vec{b}$

c) $\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$

d) $\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\right)\vec{b}$

9. The primitive of $\frac{2}{1+\cos 2x}$ is **[1]**

a) $2\sec^2 x \tan x$

b) $\sec^2 x$

c) tan x

d) -cot x

10. Which of the following can be both a symmetric and skew-symmetric matrix? **[1]**

a) Diagonal Matrix

b) Unit Matrix

c) Null Matrix

d) Row Matrix

11. The corner points of the feasible region determined by the system of linear inequalities are (0, 0), (4, 0), (2, 4), and (0, 5). If the maximum value of z = ax + by, where a, b > 0 occurs at both (2, 4) and (4, 0), then: **[1]**

a) 2a = b

b) 3a = b

c) a = b

d) a = 2b

12. If $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along with three mutually perpendicular directions, then: **[1]**

a) $\hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = 1$

b) $\hat{i} \cdot \hat{k} = 0$

c) $\hat{\mathrm{i}} \times \hat{\mathrm{k}} = 0$

d) $\hat{\mathrm{i}} \times \hat{\mathrm{j}} = 1$

[1]

13. Let $A = \begin{vmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{vmatrix}$ then A^TA is

a) $\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}$

b) $\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$

c) $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

d) $\begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix}$

14. If A and B are two events such that $P\left(\frac{A}{B}\right) = 2 \times P\left(\frac{B}{A}\right)$ and P(A) + P(B) = $\frac{2}{3}$, then P(B) is equal to **[1]**

a) $\frac{4}{9}$

b) $\frac{7}{9}$

c) $\frac{5}{9}$

d) $\frac{2}{9}$

15. The number of arbitrary constants in the general solution of a differential equation of fourth order are: **[1]**

a) 4

b) 3

c) 1

d) 2

16. The value of p for which the vectors $2\hat{i} + p\hat{j} + \hat{k}$ and $-4\hat{i} - 6\hat{j} + 26\hat{k}$ are perpendicular to each other, is: **[1]**

a) 3

b) -3

c) $-\frac{17}{3}$

d) $\frac{17}{3}$

17. The derivative of log(cos e^x) is **[1]**

a) -tan(e^x)

b) -tan(e) - tan (e^x)

c) e^x tan e^x

d) -e^x tan(e^x)

18. If the projections of $\overrightarrow{PQ}$ on OX, OY, OZ are respectively 1, 2, 3 and 4, then the magnitude of $\overrightarrow{PQ}$ is **[1]**

a) 169

b) 144

c) 19

d) 13

19. **Assertion (A):** The absolute maximum value of the function $2x^3$ - 24x in the interval [1, 3] is 89. **[1]**

Reason (R): The absolute maximum value of the function can be obtained from the value of the function at critical points and at boundary points.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The possible number of reflexive relations of a set A whose n(A) = 4 is 2^{12}. **[1]**

Reason (R): Number of reflexive relation on a set contain n elements is 2^{n^2-n}.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Find value of k if **[2]**

$\sin^{-1}\left[k\tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right] = \frac{\pi}{3}$

OR

Find the domain of $\sin^{-1}\left(x^2-3\right)$.

22. If $f(x) = x + \frac{1}{x}, x \geq 1$, show that f is an increasing function. **[2]**

23. Amongst all pairs of positive numbers with product 256, find those whose sum is the least. **[2]**

OR

The volume of a sphere is increasing at the rate of 3 cubic centimeter per second. Find the rate of increase of its surface area, when the radius is 2 cm.

24. Evaluate: $\int_{-4}^{0} |x+2| dx$ **[2]**

25. Find the least value of 'a' so that $f(x) = 2x^2 - ax + 3$ is an increasing function on $[2, 4]$. **[2]**

Section C

26. Find: $\int \frac{5x-3}{\sqrt{1+4x-2x^2}} dx$ **[3]**

OR

Find : $\int \frac{2x+1}{(x+1)^2(x-1)} dx$

27. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{5\sin x + 3\cos x}{\sin x + \cos x}\ dx$ **[3]**

28. Find the general solution of the differential equation $\frac{dy}{dx} = xy\ \log x\ \log y$. **[3]**

OR

Find the general solution of the differential equation $ydx - xdy + (x\log x)dx = 0$

29. Solve the following linear programming problem graphically: **[3]**

Maximise z = 3x + 9y

subject to constraints

$x + 3y \leq 60$

$x + y \geq 10$

$x \leq y$

$x, y \geq 0$

OR

Solve the following linear programming problem graphically:

Maximize $z = x + y$

subject to constraints

$2x + 5y \leq 100$

$8x + 5y \leq 200$

$x \geq 0, y \geq 0.$

30. A card from a pack of 52 playing cards is lost. From the remaining cards of the pack, **[3]**

two cards are drawn at random (without replacement) and both are found to be spades. Find the probability of the lost card being a spade.

31. Differentiate $\log\left(x^x + \text{cosec}^2\, x\right)$ with respect to x. **[3]**

Section D

32. Using integration, find the area of the region enclosed between the curve $y = \sqrt{4 - x^2}$ and the lines $x = -1, x = 1$ and the x-axis. **[5]**

33. Let $A = R - \{5\}$ and $B = R - \{1\}$. Consider the function $f : A \to B$, defined by $f(x) = \frac{x-3}{x-5}$. Show that f is one-one and onto. **[5]**

OR

Check whether the relation S in the set of all real numbers $(\mathbb{R})$ defined by $S = \{(a, b) : a \leq b^3\}$ is reflexive, symmetric or transitive.

34. Find the equations of the line passing through the point (3, 0, 1) and parallel to the planes x + 2y = 0 and 3y - z = 0. **[5]**

OR

Find the image A' of the point $A(2, 1, 2)$ in the line $l : \vec{r} = 4\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - \hat{j} - \hat{k})$. Also, find the equation of line joining AA'. Find the foot of perpendicular from point A on the line l.

35. If $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the given system of equations **[5]**

$3x + 4y + 7z = 14; 2x - y + 3z = 4; x + 2y - 3z = 0$.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

In a bilateral cricket series between India and South Africa, the probability that India wins the first match is 0.6. If India wins any match, then the probability that it wins the next match is 0.4, otherwise, the probability is 0.3. Also, it is given that there is no tie in any match.



i. Find the probability that India won the second match, if India has already loose the first match. (1)

ii. Find the probability that India losing the third match, if India has already lost the first two matches. (1)

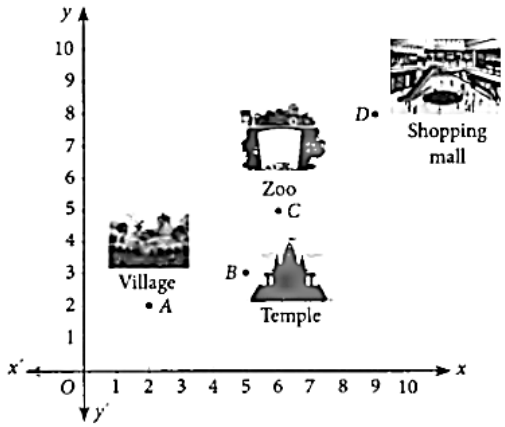
iii. Find the probability that India losing the first two matches. (2)

OR

Find the probability that India winning the first three matches. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Girish left from his village on weekend. First, he travelled up to temple. After this, he left for the zoo. After this he left for shopping in a mall. The positions of Girish at different places is given in the following graph.



i. Find position vector of B (1)

ii. Find position vector of D (1)

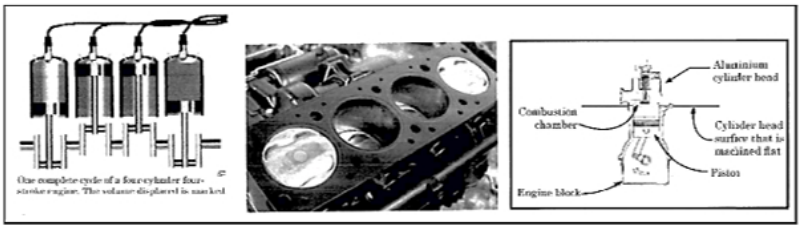
iii. Find the vector $\overrightarrow{BC}$ in terms of $\hat{i}$, $\hat{j}$. (2)

OR

iv. Find the length of vector $\overrightarrow{AD}$. (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine. The piston moves inside the cylinder bore



The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area 75π cm^2.

i. If the radius of cylinder is r cm and height is h cm, then write the volume V of cylinder in terms of radius r. (1)

ii. Find $\frac{dV}{dr}$. (1)

iii. Find the radius of cylinder when its volume is maximum. (2)

OR

For maximum volume, h > r. State true or false and justify. (2)

Solution

Section A

1.

(c) 1
Explanation:
1

2.

(c) -24
Explanation:
-24

3. **(a)** 25
Explanation:
25

4.

(b) AB is non-singular
Explanation:
If A and B are non - singular then $|AB| \neq 0$
= AB is non - singular matrix,
As $|AB = |A||B|$

5.

(d) $(1, -5, -6)$
Explanation:
$(1, -5, -6)$

6.

(c) one
Explanation:
one

7.

(b) half plane that neither contains the origin nor the points of the line 2x + 3y = 6.
Explanation:
The inequality 2x + 3y > 6 represents a half-plane where points satisfy the condition of being above the line 2x + 3y = 6. The line 2x + 3y = 6 is not included in the solution, since the inequality is strict (">" and not "geq").
To determine which half-plane, we can test a point not on the line. Testing the origin (0, 0) : Substitute x = 0 and y = 0 into the inequality: 2(0) + 3(0) = 0 Since $0 \not> 6$, the origin is not in the solution region.
Thus, the solution is a half-plane that does not contain the origin nor the points on the line.
The correct option is: half-plane that neither contains the origin nor the points of the line 2x + 3y = 6.

8.

(b) $\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2}\right)\vec{b}$

Explanation:
$\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right)\vec{b}$

9.

(c) tan x
Explanation:
tan x

10.

(c) Null Matrix
Explanation:
Null Matrix

11.

(d) a = 2b
Explanation:
The maximum value of 'z' occurs at (2, 4) and (4, 0)
$\therefore$ Value of z at (2, 4) = value of z at (4, 0)
a(2) + b(4) = a(4) + b(0)
2a + 4b = 4a + 0
4b = 4a - 2a
4b = 2a
a = 2b

12.

(b) $\hat{i}\cdot\hat{k}=0$
Explanation:
Dot product of the mutually perpendicular vector is always zero.
here $\hat{i}$ and $\hat{k}$ are perpendicular to each other
hence, $\hat{i}\cdot\hat{k}=0$

13.

(c) $\begin{vmatrix}1 & 0\\0 & 1\end{vmatrix}$
Explanation:
$A=\begin{vmatrix}\cos\alpha & \sin\alpha\\-\sin\alpha & \cos\alpha\end{vmatrix}$
$A^TA=\begin{vmatrix}\cos\alpha & -\sin\alpha\\\sin\alpha & \cos\alpha\end{vmatrix}\begin{vmatrix}\cos\alpha & \sin\alpha\\-\sin\alpha & \cos\alpha\end{vmatrix}$
$=\begin{vmatrix}\cos^2\alpha+\sin^2\alpha & \cos\alpha\sin\alpha-\sin\alpha\cos\alpha\\\sin\alpha\cos\alpha-\sin\alpha\cos\alpha & \sin^2\alpha+\cos^2\alpha\end{vmatrix}$
$=\begin{vmatrix}1 & 0\\0 & 1\end{vmatrix}$

14.

(d) $\frac{2}{9}$
Explanation:

$\frac{2}{9}$

15. **(a)** 4

Explanation:

4, because the no. of arbitrary constants is equal to order of the differential equation.

16. **(a)** 3

Explanation:

3

17.

(d) -e^x tan(e^x)

Explanation:

Let y = log (cos e^x)

On differentiating both sides w.r.t. x, we get

$\frac{dy}{dx} = \frac{d}{dx}[\log\{\cos(e^x)\}]$

$= \frac{1}{\cos(e^x)} \frac{d}{dx}\{\cos(e^x)\}$ [using chain rule]

$= \frac{1}{\cos(e^x)}\{-\sin(e^x)\} \frac{d}{dx}(e^x)$[using chain rule]

= -tan (e^x)·e^x = -e^x tan (e^x)

18.

(d) 13

Explanation:

Let $\overrightarrow{PQ} = x\hat{i} + y\hat{j} + z\hat{k}$

Since $OX = \hat{i} OY = \hat{j} OZ = \hat{k}$

Projection of PQ on OX = $\frac{\overrightarrow{PQ}.\overrightarrow{OX}}{|OX|}$

$12 = \frac{x}{1}$

x = 12

Similarly

Projection of PQ on OY = $\frac{\overrightarrow{PQ}.\overrightarrow{OY}}{|OY|}$

$3 = \frac{y}{1}$

y = 3

Similarly

Projection of PQ on OZ = $\frac{\overrightarrow{PQ}.\overrightarrow{OZ}}{|OZ|}$

$4 = \frac{z}{1}$

z = 4

Hence $\overrightarrow{PQ} = 12\hat{i} + 3\hat{j} + 4\hat{k}$

$|PQ| = \sqrt{12^2 + 3^2 + 4^2} = 13$

19.

(d) A is false but R is true.

Explanation:

Let f(x) = $2x^3$ - 24x

$\Rightarrow$ f'(x) = $6x^2$ - 24 = 6(x^2 - 4)

= 6(x + 2) (x - 2)

For maxima or minima put f'(x) = 0.

$\Rightarrow$ 6(x + 2)(x - 2) = 0

$\Rightarrow$ x = 2, -2

We first consider the interval [1, 3].

So, we have to evaluate the value of f at the critical point x = 2 $\in$ [1, 3] and at the end points of [1, 3].

At x = 1, f(1) = 2×1^3 - 24×1 = -22

At x = 2, f(2) = 2×2^3 - 24×2 = -32

At x = 3, f(3) = 2×3^3 - 24×3 = -18

$\therefore$ The absolute maximum value of f(x) in the interval [1, 3] is -18 occurring at x = 3.

Hence, Assertion is false and Reason is true.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Given, n(A) = 4

Total number of reflexive relations = $2^{n(n-1)}$

$= 2^{4\times3} = 2^{12}$

Section B

21. $k\tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right) = \sin\frac{\pi}{3}$

$\Rightarrow k tan\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\Rightarrow k\sqrt{3} = \frac{\sqrt{3}}{2} \therefore k = \frac{1}{2}$

OR

Domain of $\sin^{-1} x$ is $[-1, 1]$

$-1 \leq x^2 - 3 \leq 1 \Rightarrow 2 \leq x^2 \leq 4$

$\Rightarrow$ Domain $= [-2, -\sqrt{2}] \cup [\sqrt{2}, 2]$

22. $f(x) = x + \frac{1}{x} \Rightarrow f'(x) = 1 - \frac{1}{x^2} = \frac{x^2-1}{x^2}$

Now $\frac{x^2-1}{x^2} \geq 0$ for all $x \geq 1$

$\Rightarrow f'(x) \geq 0 \Rightarrow f$ is an increasing function.

23. Let the required numbers be x and y. Then,

xy = 256 ...(i)

Let S = x + y. Then,

S = x + $\frac{256}{x}$

$\Rightarrow \frac{dS}{dx}$ = 1 - $\frac{256}{x^2}$ and, $\frac{d^2S}{dx^2} = \frac{512}{x^3}$

The critical points of S are given by $\frac{dS}{dx}$ = 0.

$\because \frac{dS}{dx}$ = 0 $\Rightarrow 1 - \frac{256}{x^2}$ = 0 $\Rightarrow x^2$ = 256 $\Rightarrow$ x = 16

Now, $\left(\frac{d^2S}{dx^2}\right)_{x=16}$ = $\frac{512}{(16)^3} = \frac{1}{8} > 0$

Thus, S is minimum when x = 16. Putting x = 16 in (i) we get y = 16.

Hence, the required numbers are both equal to 16.

OR

Let r be the radius of sphere and V be its volume.

Then V = $\frac{4}{3}\pi r^3$.....(i)

Given, $\frac{dV}{dt}$ = 3 cm^3/s

Differentiating (i) both sides w.r.t x,we get,

$\frac{dV}{dt} = \frac{4}{3}\pi\left(3r^2\right)\frac{dr}{dt}$

$\Rightarrow \quad 3 = \frac{4}{3}(3\pi r^2)\frac{dr}{dt}$

$\Rightarrow \quad \frac{dr}{dt} = \frac{3}{4\pi r^2}$.....(ii)

Now, let S be the surface area of sphere, then

S = $4\pi r^2$

$\Rightarrow \quad \frac{dS}{dt} = 4\pi(2r)\frac{dr}{dt}$

$\Rightarrow \quad \frac{dS}{dt} = 8\pi r\left(\frac{3}{4\pi r^2}\right)$[using Eq.(ii)]

$\Rightarrow \quad \left(\frac{dS}{dt}\right) = \frac{6}{r}$

when r = 2, then

$\frac{dS}{dt} = \frac{6}{2}$ = 3 cm^2/s

Therefore,the rate of inrcrease of the surface area of sphere is 3 cm^2/s when it's radius is 2 cm.

24. $\int_{-4}^{0}|x+2|dx = \int_{-4}^{-2}-(x+2)dx + \int_{-2}^{0}(x+2)dx$

$= -\frac{(x+2)^2}{2}\Big]_{-4}^{-2} + \frac{(x+2)^2}{2}\Big]_{-2}^{0} = 2+2 = 4$

25. $f(x) = 2x^2 - ax + 3 \Rightarrow f'(x) = 4x - a$

Now $2 \le x \le 4 \Rightarrow 8 - a \le 4x - a \le 16 - a$

For f to be an increasing function, $f'(x) \ge 0$

$\Rightarrow 8 - a \ge 0 \Rightarrow a \le 8$

$\therefore$ Least value of a does not exist.

Section C

26. $\int \frac{5x-3}{\sqrt{1+4x-2x^2}}dx = -\frac{5}{4}\int \frac{-4x+4}{\sqrt{1+4x-2x^2}}dx + \sqrt{2}\int \frac{1}{\sqrt{\left(\left(\sqrt{\frac{3}{2}}\right)^2 - (x-1)^2\right)}}dx$

$= -\frac{5}{2}\sqrt{1+4x-2x^2} + \sqrt{2}\sin^{-1}\left(\sqrt{\frac{2}{3}}(x-1)\right) + c$

OR

$\int \frac{2x+1}{(x+1)^2(x-1)}dx = -\frac{3}{4}\int \frac{1}{x+1}dx + \frac{1}{2}\int \frac{1}{(x+1)^2}dx + \frac{3}{4}\int \frac{1}{x-1}dx$

$= -\frac{3}{4}\log|x+1| - \frac{1}{2(x+1)} + \frac{3}{4}\log|x-1| + C$

or, $= \frac{3}{4}\log\left|\frac{x-1}{x+1}\right| - \frac{1}{2(x+1)} + C$

27. $I = \int_0^{\pi/2} \frac{5\sin x + 3\cos x}{\sin x + \cos x}dx$ ...(i)

$I = \int_0^{\pi/2} \frac{5\sin\left(\frac{\pi}{2}-x\right)+3\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}dx = \int_0^{\pi/2} \frac{5\cos x + 3\sin x}{\cos x + \sin x}dx$ ...(ii)

Adding (i) and (ii), we get

$2I = \int_0^{\pi/2} 8dx \Rightarrow I = 4x\Big]_0^{\frac{\pi}{2}} = 2\pi$

28. Separating the variables and integrating, $\int \frac{dy}{y\log y} = \int x\log x dx$

$\Rightarrow \log|\log y| = \log x\left(\frac{x^2}{2}\right) - \int \frac{1}{x}\cdot\frac{x^2}{2}dx + c$

$\Rightarrow \log|\log y| = \frac{x^2}{2}\log x - \frac{x^2}{4} + c$

OR

The given differential equation can be written as, $\frac{dy}{dx} - \frac{1}{x}y = \log x$

$\therefore P = -\frac{1}{x}, Q = \log x$

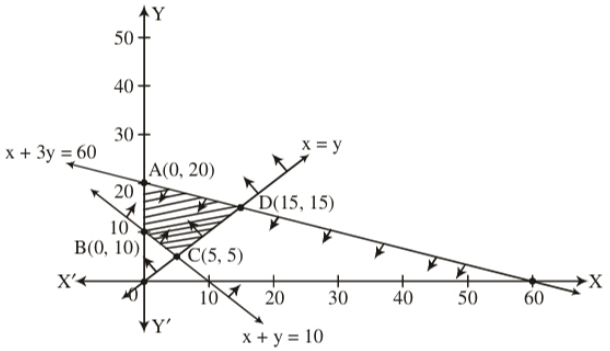
I.F. $= e^{\int -\frac{1}{x}dx} = e^{-\log x} = e^{\log \frac{1}{x}} = \frac{1}{x}$

The solution of the differential equation is:

$y \cdot \frac{1}{x} = \int \log x \cdot \frac{1}{x} dx$

$\Rightarrow \frac{y}{x} = \frac{(\log x)^2}{2} + c,$ or $y = \frac{1}{2}x \cdot (\log x)^2 + cx$

29.



Correct graph

Value of z at corner points

z(A) = 3(0) + 9(20) = 180

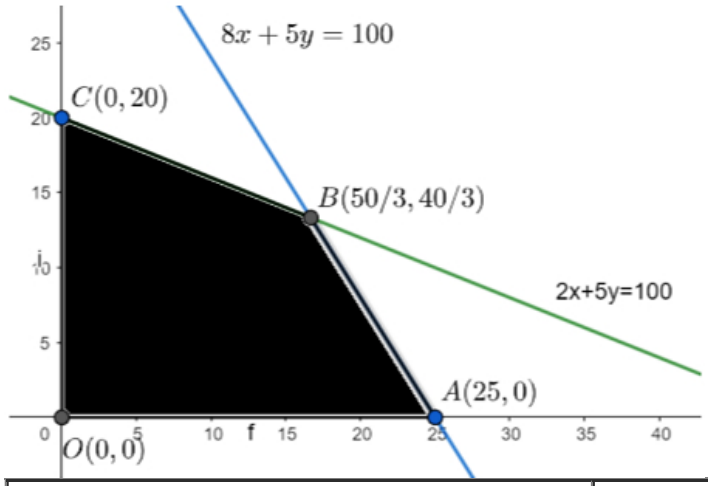
z(B) = 0 + 90 = 90

z(C) = 15 + 45 = 60

z(D) = 45 + 45 = 180

Max(z) = 180 at any point on AD

OR



Corner Point	Value of z = 4x+3y
O(0,0)	0
A(25,0)	25
$B\left(\frac{50}{3}, \frac{40}{3}\right)$	30
C(0,20)	20

$z_{\max} = 30$ when $x = \frac{50}{3}, y = \frac{40}{3}$

30. Let E_1, E_2, E_3 and E_4 be the events of losing a card of spades, clubs, hearts and diamonds respectively.

Therefore, we have,

$P(E_1) = P(E_2) = P(E_3) = P(E_4) = \frac{13}{52} = \frac{1}{4}$

Let E be the event of drawing 2 spades from the remaining 51 cards. Therefore,we have,

$P\left(\frac{E}{E_1}\right)$ = probability of drawing 2 spades, given that a card of spades is missing

$= \frac{^{12}C_2}{^{51}C_2} = \frac{(12\times11)}{2!} \times \frac{2!}{(51\times50)} = \frac{22}{425}$

$P\left(\frac{E}{E_2}\right)$ = probability of drawing 2 spades, given that a card of clubs is missing

$= \frac{^{13}C_2}{51} = \frac{(13\times12)}{2!} \times \frac{2!}{(51\times50)} = \frac{26}{425}$

$P\left(\frac{E}{E_3}\right)$ = probability of drawing 2 spades, given that a card of hearts is missing

$= \frac{^{13}C_2}{^{51}C_2} = \frac{26}{425}$

$P\left(\frac{E}{E_4}\right)$ = probability of drawing 2 spades, given that a card of diamonds is missing

$= \frac{^{13}C_2}{^{51}C_2} = \frac{26}{425}$

Therefore, by Baye's theorm,

$P\left(\frac{E}{E_1}\right)$ = probability of the lost card being a spade, given that 2 spades are drawn from the remaining 51 cards

$$= \frac{P\left(\frac{E}{E_1}\right)\cdot P(E_1)}{P\left(\frac{E}{E_1}\right)\cdot P(E_1)+P\left(\frac{E}{E_2}\right)\cdot P(E_2)+P\left(\frac{E}{E_3}\right)\cdot P(E_3)+p(\frac{E}{E_4})\cdot P(E_4)}$$

$$= \frac{\left(\frac{1}{4}\times\frac{22}{425}\right)}{\left(\frac{1}{4}\times\frac{22}{425}\right)+\left(\frac{1}{4}\times\frac{26}{425}\right)+\left(\frac{1}{4}\times\frac{26}{425}\right)+\left(\frac{1}{4}\times\frac{26}{425}\right)}$$

$= \frac{22}{100} = 0.22$

Hence, the required probability is 0.22.

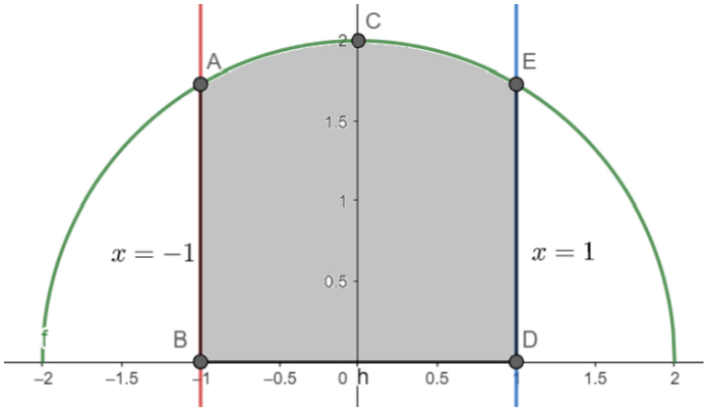
31. $\frac{d}{dx}\log\left(x^x + \operatorname{cosec}^2 x\right) = \frac{1}{x^x+\operatorname{cosec}^2 x}\frac{d}{dx}\left(e^{x\log x} + \operatorname{cosec}^2 x\right) \quad \left(\because x^x = e^{x\log x}\right)$

$= \frac{1}{x^x+\operatorname{cosec}^2 x}\left[e^{x\log x}(1+\log x) - 2\operatorname{cosec}^2 x\cot x\right]$

$= \frac{1}{x^x+\operatorname{cosec}^2 x}\left[x^x(1+\log x) - 2\operatorname{cosec}^2 x\cot x\right]$

Section D

32.



The limits of integration are changed.

When $x = -1, \sin\theta = -\frac{1}{2}$, so $\theta = -\frac{\pi}{6}$.

The integral becomes $A = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sqrt{4-(2\sin\theta)^2}\,(2\cos\theta)d\theta.$

The expression under the square root is simplified:

$\sqrt{4-4\sin^2\theta} = \sqrt{4\left(1-\sin^2\theta\right)} = \sqrt{4\cos^2\theta} = 2\cos\theta$

The integral is now $A = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (2\cos\theta)(2\cos\theta)d\theta = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} 4\cos^2\theta d\theta.$

The identity $\cos^2\theta = \frac{1+\cos(2\theta)}{2}$ is used. The integral becomes

$A = \int\limits_{-\frac{\pi}{6}}^{\frac{\pi}{6}} 4\left(\frac{1+\cos(2\theta)}{2}\right)d\theta = \int\limits_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (2+2\cos(2\theta))d\theta$

The integral is evaluated: $A = [2\theta+\sin(2\theta)]_{-\frac{\pi}{6}}^{\frac{\pi}{6}}$.

The limits are applied: $A = \left(2\left(\frac{\pi}{6}\right)+\sin\left(2\left(\frac{\pi}{6}\right)\right)\right) - \left(2\left(-\frac{\pi}{6}\right)+\sin\left(2\left(-\frac{\pi}{6}\right)\right)\right)$

33. Let $f(x_1) = f(x_2)$, for some $x_1, x_2 \in A$

$\Rightarrow \frac{x_1-3}{x_1-5} = \frac{x_2-3}{x_2-5}$

$\Rightarrow (x_1-3)(x_2-5) = (x_2-3)(x_1-5)$

$\Rightarrow x_1 = x_2$, So f is one-one Function.

Let $y = f(x) = \frac{x-3}{x-5} \Rightarrow y(x-5) = x-3$

$\Rightarrow yx-5y = x-3$

$\Rightarrow x = \frac{5y-3}{y-1}$, We observe that x is defined for all values of y except $y = 1$,

So, Range $= R-\{1\}$ and Co-domain is Given $R-\{1\}$ [As, $f: A \to B$]

Since, Range $= Co$-domain, f is onto Function.

Thus, f is one-one & onto function.

OR

Reflexive: Let $\frac{1}{2} \not\leq \left(\frac{1}{2}\right)^3$,

$\therefore$ S is not reflexive.

Symmetric: $1 \leq 2^3 \Rightarrow (1,2) \in S$ but $2 \not\leq 1^3 \Rightarrow (2,1) \notin S$.

$\therefore S$ is not symmetric.

Transitive: $10 \leq 6^3 \Rightarrow (10,6) \in S \& 6 \leq 2^3$

$\Rightarrow (6,2) \in S$ but $10 \not\leq 2^3 \Rightarrow (10,2) \notin S$

$\therefore S$ is not Transitive.

34. Equation of the two planes are x + 2y = 0 and 3y - z = 0.

Let $\vec{n}_1$ and $\vec{n}_2$ be the normal to the two planes, respectively.

$\therefore \vec{n}_1 = \hat{i}+2\hat{j}$ and $\vec{n}_2 = 3\hat{j}-\hat{k}$

Since required line is parallel to the given two planes.

Therefore, $\vec{b} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 0 \\ 0 & 3 & -1 \end{vmatrix}$

$= \hat{i}(-2) - \hat{j}(-1) + \hat{k}(3)$

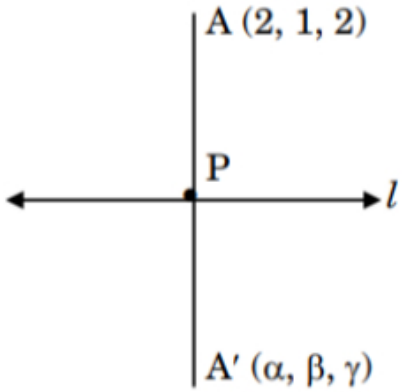
$= -2\hat{i}+\hat{j}+3\hat{k}$

So, the equation of the line through the point (3, 0, 1) and parallel to the given two planes is

$(x-3)\hat{i}+(y-0)\hat{j}+(z-1)\hat{k}+\lambda(-2\hat{i}+\hat{j}+3\hat{k})$

$= (x-3)\hat{i}+y\hat{j}+(z-1)\hat{k}+\lambda(-2\hat{i}+\hat{j}+3\hat{k})$

OR



Let the image of A in the line be $A'(\alpha, \beta, \gamma)$

The point P , which is the point of intersection of the lines l and AA', will have coordinates ($\lambda+4, -\lambda+2, -\lambda+2$) for some λ.

Drs of AP are $< \lambda+2, -\lambda+1, -\lambda >$

$AP \perp l$

$(\lambda+2)-(-\lambda+1)-(-\lambda)=0$

$\Rightarrow \lambda = -\frac{1}{3}$

Therefore, the coordinates of P are $\left(\frac{11}{3}, \frac{7}{3}, \frac{7}{3}\right)$

P is the mid-point of AA'

$\Rightarrow \frac{2+\alpha}{2} = \frac{11}{3}, \frac{1+\beta}{2} = \frac{7}{3}, \frac{2+\gamma}{2} = \frac{7}{3}$

$\Rightarrow \alpha = \frac{16}{3}, \beta = \frac{11}{3}, \gamma = \frac{8}{3}$

The coordinates of the image are $\left(\frac{16}{3}, \frac{11}{3}, \frac{8}{3}\right)$

The equation of AA' is

$\frac{x-2}{\frac{10}{3}} = \frac{y-1}{\frac{8}{3}} = \frac{z-2}{\frac{2}{3}}$

or,

$\frac{3(x-2)}{5} = \frac{3(y-1)}{4} = \frac{3(z-2)}{1}$

35. $|A| = 3(-3)-2(-26)+1(19) = 62 \neq 0 \Rightarrow A^{-1}$ exists.

cofactor Matrix $= \begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}$

adj $A = \begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix}$

$A^{-1} = \frac{1}{62}\begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix}$

Given system of equations can be written as $A' \cdot X = B$

where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$

Now, $A'.X = B \Rightarrow X = (A')^{-1}.B$

$\Rightarrow X = \left(A^{-1}\right)' \cdot B = \frac{1}{62}\begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}\begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$

$= \frac{1}{62}\begin{bmatrix}62\\62\\62\end{bmatrix} = \begin{bmatrix}1\\1\\1\end{bmatrix}$

$\Rightarrow x = 1, y = 1, z = 1$

Section E

36. i. It is given that if India loose any match, then the probability that it wins the next match is 0.3.

∴ Required probability = 0.3

ii. It is given that, if India loose any match, then the probability that it wins the next match is 0.3.

∴ Required probability = 1 - 0.3 = 0.7

iii. Required probability = P(India losing first match) · P(India losing second match when India has already lost first match) = 0.4 × 0.7 = 0.28

OR

Required probability = P(India winning first match) · P(India winning second match if India has already won first match) · P(India winning third match if India has already won first two matches) = 0.6 × 0.4 × 0.4 = 0.096

37. i. Here (5, 3) are the coordinates of B.

∴ Position vector of B = $5\hat{i} + 3\hat{j}$

ii. Here (9, 8) are the coordinates of D.

∴ Position vector of D = $9\hat{i} + 8\hat{j}$

iii. Position vector of B = $5\hat{i} + 3\hat{j}$ and Position vecto of C = $6\hat{i} + 5\hat{j}$

∴ $\overrightarrow{BC}$ = (6 - 5)$\hat{i}$ + (5 - 3)$\hat{j}$ = $\hat{i} + 2\hat{j}$

OR

Since P.V. of A = $2\hat{i} + 2\hat{j}$, P.V. of D = $9\hat{i} + 8\hat{j}$

∴ $\overrightarrow{AD}$ = (9 - 2)$\hat{i}$ + (8 - 2)$\hat{j}$ = $7\hat{i} + 6\hat{j}$

$|\overrightarrow{AD}|^2 = 7^2 + 6^2 = 49 + 36 = 85$

$\Rightarrow |\overrightarrow{AD}| = \sqrt{85}$ units

38. i. $\pi r^2 + 2\pi rh = 75\pi \Rightarrow$ h = $\frac{75-r^2}{2r}$, ∴ V = $\pi r^2 h = \frac{\pi}{2}$(75r - r^3)

ii. $\frac{dV}{dr} = \frac{\pi}{2}(75 - 3r^2)$

iii. $\frac{dV}{dr} = 0 \Rightarrow r = 5, \left.\frac{d^2V}{dr^2}\right]_{r=5} = \frac{\pi}{2}(-6r) < 0$ ∴ volume is maximum when r = 5

OR

False,

$\frac{dV}{dr} = 0 \Rightarrow r = 5, \left.\frac{d^2V}{dr^2}\right]_{r=5} = \frac{\pi}{2}(-6r) < 0$ ∴ volume is maximum when r = 5

As volume is maximum at r = 5 $\Rightarrow$ h = $\frac{75-5^2}{2(5)}$ = 5 $\Rightarrow$ h = r