Class XII Session 2026-27
Subject - Mathematics
Sample Question Paper - 6

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculators is not allowed.

Section A

1. For what value of x, the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ is skew-symmetric matrix? **[1]**

   a) x = 3 b) x = 2

   c) x = -2 d) x = 1

2. The vertices of a a ABC are A(-2, 4), B(2, -6) and C(5, 4). The area of a ABC is **[1]**

   a) 28 sq units b) 32 sq units

   c) 35 sq units d) 17.5 sq units

3. If $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix} = 0$ then x = ? **[1]**

   a) -6 b) 6

c) 0 d) 9

4. If f(x) = $\begin{cases} kx+5, & \text{when } x \leq 2 \\ x+1, & \text{when } x > 2 \end{cases}$ is continuous at x = 2 then k = ? **[1]**

a) 2 b) -1

c) -3 d) -2

5. The sum of the direction cosines of Z-axis is **[1]**

a) 1 b) 2

c) 0 d) 3

6. Consider the following statements in respect of the differential equation $\frac{d^2y}{dx^2}$ + cos $\left(\frac{dy}{dx}\right)$ = 0 **[1]**

i. The degree of the differential equation is not defined.

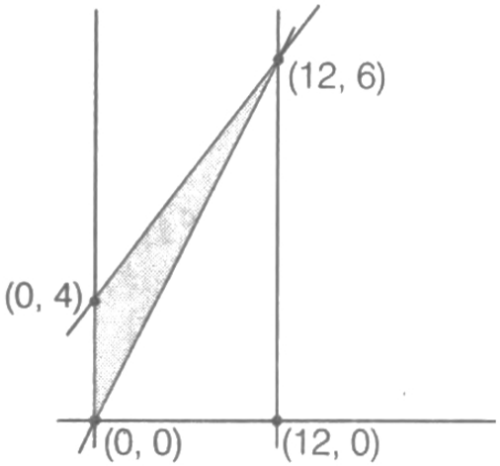
ii. The order of the differential equation is 2.

Which of the above statement(s) is/are correct?

a) Neither (i) nor (ii) b) Both (i) and (ii)

c) Only (ii) d) Only (i)

7. The feasible region for an LPP is shown in the following figure. **[1]**



Let F = 3x - 4y be the objective function. Maximum value of F is

a) 12 b) -18

c) 0 d) 8

8. Two adjacent sides of a parallelogram are represented by the vectors $\vec{a} = (3\hat{i} + \hat{j} + 4\hat{k})$ and $\vec{b} = (\hat{i} - \hat{j} + \hat{k})$ the area of the parallelogram is **[1]**

a) $\sqrt{40}$ b) $\sqrt{35}$

c) $\sqrt{42}$ d) 6 sq units

9. $\int e^{\cos^2 x} \sin 2x dx = ?$ **[1]**

a) $e^{\cos^2 x} + C$ b) $-e^{\cos^2 x} + C$

c) $-e^{\sin^2 x} + C$ d) $e^{\sin^2 x} + C$

[1]

10. A matrix $A = [a_{ij}]_{3\times 3}$ is defined by $a_{ij} = \begin{cases} 2i+3j & , \quad i<j \\ 5 & , \quad i=j \\ 3i-2j & , \quad i>j \end{cases}$

The number of elements in A which are more than 5, is 4:

a) 3 b) 4

c) 5 d) 6

11. By graphical method solution of LLP maximize Z = x + y subject to x + y $\leq$ 2x; y $\geq$ 0 obtained at **[1]**

a) only two points b) at infinite number of points

c) at definite number of points d) only one point

12. The number of vectors of unit length perpendicular to the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$ **[1]**

a) one b) two

c) infinite d) three

13. If A = $\begin{vmatrix} 0 & \sin\theta \\ \cos\theta & 1 \end{vmatrix}$ and A = A^T then θ is equal to **[1]**

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$

c) $\frac{\pi}{6}$ d) $\frac{\pi}{4}$

14. If $P\left(\frac{A}{B}\right)$ = 0.3, P(A) = 0.4 and P(B) = 0.8, then $P\left(\frac{B}{A}\right)$ is equal to: **[1]**

a) 0.06 b) 0.6

c) 0.3 d) 0.4

15. The solution of the DE $\frac{dy}{dx}$ = 1 - x + y - xy is **[1]**

a) $e^y = x - \frac{x^2}{2} + C$ b) $\log(1+y) = x - \frac{x^2}{2} + C$

c) $e^{(1+y)} = x - \frac{x^2}{2} + C$ d) $e^y = x + \frac{x^2}{4} + C$

16. Find the angle between the two vectors $\vec{a}$ and $\vec{b}$ with magnitude 2 and $\sqrt{3}$ respectively and $\vec{a}.\vec{b} = 4$. **[1]**

a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$

c) $\cos^{-1}\frac{2}{\sqrt{3}}$ d) $\cos^{-1}\frac{\sqrt{2}}{3}$

17. If $f(x) = \begin{cases} \frac{\log(1+ax)+\log(1-bx)}{x}, & \text{for } x \neq 0 \\ k & , \text{ for } x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is: **[1]**

a) $a + b$ b) b

c) a d) $a - b$

18. If the directions cosines of a line are k,k,k, then **[1]**

a) $k = \frac{1}{\sqrt{3}}$ or $k = -\frac{1}{\sqrt{3}}$ b) $0 < k < 1$

c) k = 1 d) $k > 0$

19. **Assertion (A):** f(x) = tan x - x always increases. **[1]**

Reason (R): Any function y = f(x) is increasing if $\frac{dy}{dx} > 0$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A)** Let $f : R \to R$ be defined by $f(x) = x^2 + 1$. Then, pre-images of 17 are ± 4. **[1]**

Reason (R): A function $f : A \to B$ is called a one-one function, if distinct elements of A have distinct images in B.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Simplify $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$, $0 < x < \frac{1}{\sqrt{2}}$. **[2]**

OR

Write the interval for the principal value of function and draw its graph: $\cos^{-1} x$.

22. Show that $f(x) = (x - 1)e^x + 1$ is an increasing function for all $x > 0$. **[2]**

23. The volume of a spherical balloon is increasing at the rate of 25 cm^3/sec. Find the rate of change of its surface area at the instant when radius is 5 cm. **[2]**

OR

Find the rate of change of the area of a circle with respect to its radius r when

a. r = 3 cm

b. r = 4 cm

24. Evaluate $\int \frac{x^3+x}{x^4-9} dx$ **[2]**

25. The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference? **[2]**

Section C

26. Evaluate $\int (x+2)\sqrt{3x+5}\, dx$ **[3]**

OR

Evaluate the definite integral $\int_0^{\pi/2} \sqrt{1+\sin x}\, dx$

27. Evaluate $\int_2^8 |x-5| dx$ **[3]**

28. Solve the following differential equation : **[3]**

$x^2 dy + y(x+y)dx = 0$

OR

Solve the following differential equation $(1 + x^2)dy + 2xy\, dx = \cot x\, dx$, where $x \neq 0$.

29. Minimise Z = 13x - 15y, subject to the constraints: **[3]**
$x + y \leq 7, 2x - 3y + 6 \geq 0, x \geq 0, y \geq 0$.

OR

Solve the Linear Programming Problem graphically:
Minimize Z = 5x + 3y Subject to
2x + y $\geq$ 10
x + 3y $\geq$ 15
x $\leq$ 10
y $\leq$ 8
x, y $\geq$ 0

30. Two dice are thrown. Defined are the following two events A and B: **[3]**
$A = \{(x, y) : x + y = 9\}, B = \{(x, y) : x \neq 3\}$, where (x, y) denote a point in the sample space.
Check if events A and B are independent or mutually exclusive.

31. If y = a sin x + b cos x, then prove that $y^2 + \left(\frac{dy}{dx}\right)^2 = a^2 + b^2$. **[3]**

Section D

32. Find Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the lines x + y = 2. **[5]**

33. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a – b| is divisible by 2} is an equivalence relation. Write all the equivalence classes of R. **[5]**

OR

Check whether the relation S in the set of real numbers R defined by $S = \{(a, b)$: where $a - b + \sqrt{2}$ is an irrational number$\}$ is reflexive, symmetric or transitive.

34. If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-7}$ are perpendicular to each other, find the value of k and hence write the vector equation of a line perpendicular to these two lines and passing through the point $(3, -4, 7)$. **[5]**

OR

Show that the lines $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5}$ and $\frac{x-2}{2} = \frac{y-1}{3} = \frac{z+1}{-2}$ intersect and find their point of intersection.

35. By using determinants, solve the following system of equations: **[5]**
x + y + z = 1
x + 2y + 3z = 4
x + 3y + 5z = 7

Section E

36. A departmental store sends bills to charge its customers once a month. Past experience shows that 70% of its customers pay their first month bill in time. The store also found that the customer who pays the bill in time has the probability of 0.8 of paying in time next month and the customer who doesn't pay in time has the probability of 0.4 of paying in time the next month. **[4]**
Based on the above information, answer the following questions:

i. Let E_1 and E_2 respectively denote the event of customer paying or not paying the first month bill in time.

Find $P(E_1), P(E_2)$. **(1)**

ii. Let A denotes the event of customer paying second month's bill in time, then find $P(A \mid E_1)$ and $P(A \mid E_2)$. **(1)**

iii. a. Find the probability of customer paying second month's bill in time. **(2)**

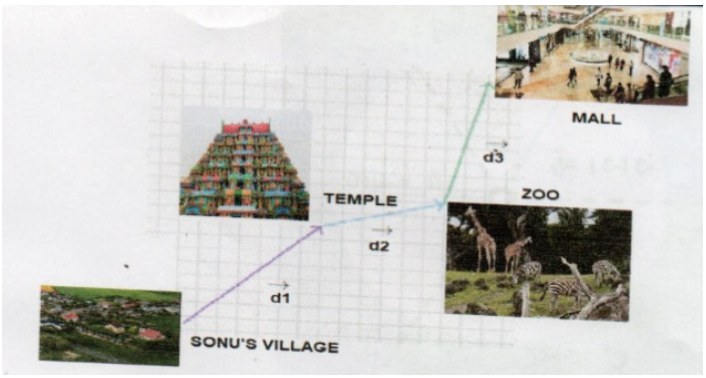
OR

b. Find the probability of customer paying first month's bill in time if it is found that customer has paid the second month's bill in time. **(2)**

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Sonu left from his village on weekend. First, he travelled d_1 displacement up to a temple. After this, he left for the zoo and travelled d_2 displacement. After this he left for shopping in a mall - Total driving time of Deepal from village to Mall was 1.5 hr.

If $d_1 = (6, 8)$ $d_2 = (3, 4)$ and $d_3 = (7, 12)$ km



i. What is the total displacement from village to Mall? (1)

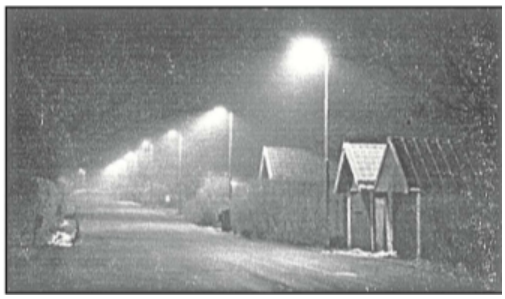
ii. What is the speed of Sonu from Village to Mall? (1)

iii. What is the Displacement from Village to Zoo? (2)

OR

What is the displacement from temple to Mall? (2)

38. **[4]**



A small town is analyzing the pattern of a new street light installation. The lights are set up in such a way that the intensity of light at any point x metres from the start of the street can be modelled by $f(x) = e^x \sin x$, where x is in metres.

Based on the above, answer the following:

i. Find the intervals on which the $f(x)$ is increasing or decreasing, $x \in [0, \pi]$. **(2)**

ii. Verify, whether each critical point when $x \in [0, \pi]$ is a point of local maximum or local minimum or a point of inflexion. **(2)**

Solution

Section A

1.

(b) x = 2

Explanation:

Given, $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$

We know that, if A is a skew-symmetric matrix, then

$A = -A^T$ ...(i)

From Eq. (i) We, get

$$\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 & x \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -x \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$$

On comparing the corresponding element, we get

$-2 = -x \Rightarrow x = 2$

2.

(c) 35 sq units

Explanation:

$$\Delta = \frac{1}{2}\begin{vmatrix} -2 & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -2 & 4 & 1 \\ 4 & -10 & 0 \\ 7 & 0 & 0 \end{vmatrix}$$ = 35 sq units

3. **(a)** -6

Explanation:

Apply $C_1 \rightarrow (C_1 - C_3)$ and take (x - 7) common from C_1.

4.

(b) -1

Explanation:

For continuity left hand limit must be equal to right hand limit and value at the point.

Continuous at x = 2..

L.H.L = $\lim_{x \to 2^-}(kx + 5)$

$\Rightarrow \lim_{h \to 0}(k(2 - h) + 5)$

$\Rightarrow$ k(2 - 0) + 5 = 2k + 5

R.H.L = $\lim_{x \to 2^+}(x + 1)$

$\Rightarrow \lim_{h \to 0}(2 + h + 1)$

$\Rightarrow$ 2 + 0 + 1

= 3

As f(x) is continuous,we get

$\because$ 2k + 5 = 3

k = -1.

5. **(a)** 1

Explanation:

Z-axis makes an angle 90° with X-axis, 90° with Y-axis and 0° with Z-axis

$\therefore$ Direction cosines of Z-axis: cos 90°, cos 90°, cos 0° i.e., 0, 0, 1

Therefore, sum of the direction cosines = 0 + 0 + 1 = 1

6. **(b)** Both (i) and (ii)

Explanation:

Both (i) and (ii)

7. **(a)** 12

Explanation:

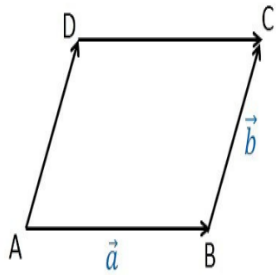
The feasible region as shown in the figure, has objective function F = 3x - 4y.

Corner points	**Corresponding value of F = 3x - 4y**
(0, 0)	0
(12, 6)	12 $\leftarrow$ Maximum
(0, 4)	-16

Hence, the maximum value of F is 12.

8. **(c)** $\sqrt{42}$

Explanation:



Given that,

$a = 3\hat{i} + 1\hat{j} + 4\hat{k}$

$b = 1\hat{i} - 1\hat{j} + 1\hat{k}$

Area of Parallelogram ABCD $= |\vec{a} \times \vec{b}|$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix}$$

$= \hat{i}(1 \times 1 - (-1) \times 4) - \hat{j}(3 \times 1 - 1 \times 4) + \hat{k}(3 \times -1 - 1 \times 1)$

$= 5\hat{i} + \hat{j} - 4\hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{25 + 1 + 16} = \sqrt{42}$

9. **(b)** $-e^{\cos^2 x} + C$

Explanation:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c; \int e^x dx = e^x + c$

Therefore ,

Put $\cos^2$ x=t $\Rightarrow$ 2 cos x(-sin x) dx=dt $\Rightarrow$ -sin 2xdx=dt

$= -\int e^t dt$

= -e^t + c

= $-e^{\cos^2 x} + c$

10.

(b) 4

Explanation:

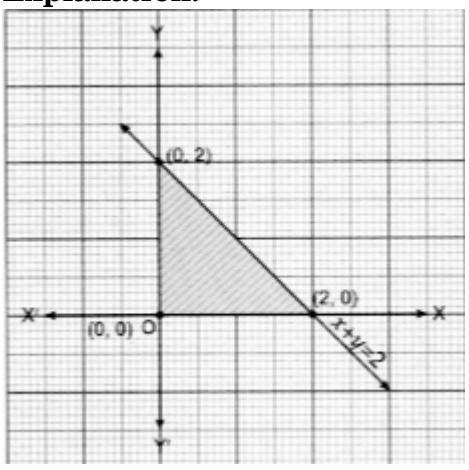
Here, A = $\begin{bmatrix} 5 & 8 & 11 \\ 4 & 5 & 13 \\ 7 & 5 & 5 \end{bmatrix}$

Thus, number of elements more than 5, is 4.

11.

(b) at infinite number of points

Explanation:



Feasible region is shaded region with corner points (0, 0), (2, 0) and (0, 2)

Z(0, 0) = 0

Z(2, 0) = 2⟵ maximise

Z(0, 2) = 2 ⟵ maximise

Z_{max} = 2 obtained at (2, 0) and (0, 2) so is obtained at any point on line segment joining (2, 0) and (0, 2).

12.

(b) two

Explanation:

Given that, $\vec{a} = 2\hat{1} + \hat{j} + 2\hat{k}$, $\vec{b} = \hat{j} + \hat{k}$

Now, a vector which is perpendicular to both $\vec{a}$ and $\vec{b}$ is given by

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{1} & \hat{j} & \hat{k} \\ 2 & 1 & 2 \\ 0 & 1 & 1 \end{vmatrix} = \vec{i}(1 - 2) - \vec{j}(2 - 0) + \vec{j} = k(2 - 0) = -\hat{i} - 2\hat{j} + 2\hat{k}$

Now, $\vec{a} \times \vec{b} = \sqrt{(-1)^2 + (-2)^2 + (2)^2} = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$

$\therefore$ the required unit vector

$= \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} = \frac{-i-2\hat{j}+2\hat{k}}{3} = \frac{-1}{3}\,\hat{i} - \frac{2}{3}\,\hat{j} + \frac{2}{3}k$

There are two perpendicular directions to any plane. Thus, another unit vector perpendicular to $\vec{a}$ and $\vec{b}$ is given by $-\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} = \frac{\vec{b}\times\vec{a}}{|\vec{b}\times\vec{a}|}$

$\Rightarrow \frac{\vec{b}\times\vec{a}}{|\vec{b}\times\vec{a}|} = \frac{1}{3}\,\hat{i} + \frac{2}{3}\,\hat{j} - \frac{2}{3}k$

Hence, there are two unit length vectors perpendicular to the $\vec{a}$ and $\vec{b}$.

13.

(d) $\frac{\pi}{4}$

Explanation:

$\text{A} = \begin{vmatrix} 0 & \sin\theta \\ \cos\theta & 1 \end{vmatrix}$ and $\text{A}^{\text{T}} = \text{A} \Rightarrow \begin{vmatrix} 0 & \cos\theta \\ \sin\theta & 1 \end{vmatrix} = \begin{vmatrix} 0 & \sin\theta \\ \cos\theta & 1 \end{vmatrix}$

$\Rightarrow \sin\theta = \cos\theta \Rightarrow \tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$

14.

(b) 0.6

Explanation:

0.6

15.

(b) $\log(1+y) = x - \frac{x^2}{2} + C$

Explanation:

Here, $\frac{dy}{dx} = 1 - x + y - xy$

$\frac{dy}{dx} = 1 - x + y(1-x)$

$\frac{dy}{dx} = (1+y)(1-x)$

$\frac{\text{dy}}{1+\text{y}} = (1-\text{x})\text{dx}$

On integrating on both sides, we obtain

$\log(1+y) = x - \frac{x^2}{2} + c$

16.

(c) $\cos^{-1}\frac{2}{\sqrt{3}}$

Explanation:

Given that, $|\vec{a}| = 2$ and $|\vec{b}| = \sqrt{3}$

Also, $\overrightarrow{a}.\vec{b} = 4$

The angle between two vectors is given by

$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\cdot|\vec{b}|}$

$\therefore \cos\theta = \frac{4}{2\cdot\sqrt{3}} = \frac{2}{\sqrt{3}}$

$\therefore \theta = \cos^{-1}\frac{2}{\sqrt{3}}.$

17.

(d) $a - b$

Explanation:

$a - b$

18. **(a)** $k = \frac{1}{\sqrt{3}}$ or $k = -\frac{1}{\sqrt{3}}$

Explanation:

We know that if l, m, n are direction cosines of a lines then $l^2 + m^2 + n^2 = 1$

so, $k^2 + k^2 + k^2 = 1$

$3k^2 = 1$

$k^2 = \frac{1}{3}$

taking squareroot on both sides , we have $k = \frac{1}{\sqrt{3}}$ or $k = -\frac{1}{\sqrt{3}}$

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

20.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

Assertion Consider $x^2 + 1 = 17$

$\Rightarrow x^2 = 16$

$\Rightarrow x = \pm 4$

Hence, pre-images of 17 are ± 4.

Hence, both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.

Section B

21. Let $x = \cos\theta$

$\therefore \theta = \cos^{-1} x$

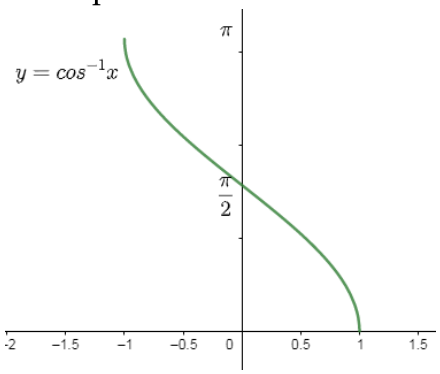
$\sec^{-1}\left(\frac{1}{2x^2-1}\right) = \sec^{-1}\left(\frac{1}{2\cos^2\theta - 1}\right) = \sec^{-1}\left(\frac{1}{\cos 2\theta}\right) = \sec^{-1}(\sec 2\theta)$

$= 2\theta$

$= 2\cos^{-1} x$

OR

Principal value branch of $\cos^{-1} x$ is $[0, \pi]$ and its graph is shown here,



22. Given:- $f(x) = (x - 1)e^x + 1$

$\Rightarrow f'(x) = \frac{d}{dx}((x-1)e^x + 1)$

$= f'(x) = e^x + (x - 1)e^x$

$= f'(x) = e^x(1 + x - 1)$

$= f'(x) = xe^x$

as given

$x > 0$

$= e^x > 0$

$= xe^x > 0$

$= f'(x) > 0$

Hence, the condition for f(x) to be increasing

Thus, f(x) is increasing for all $x > 0$

23. Let r be the radius and V be the volume of the sphere at any t then,

$V = \frac{4}{3}\pi r^3$

Differentiating both sides with respect to t,

$\Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\Rightarrow \frac{dr}{dt} = \frac{1}{4\pi r^2}\frac{dV}{dt}$

$\Rightarrow \frac{dr}{dt} = \frac{25}{4\pi(5)^2}$

$\Rightarrow \frac{dr}{dt} = \frac{1}{4\pi}\text{cm/sec}$

Now, let S be the surface area of the sphere area at any t then

$S = 4\pi r^2$

Differentiating both sides with respect to t,

$\Rightarrow \frac{dS}{dt} = 8\pi r\frac{dr}{dt}$

$\Rightarrow \frac{dS}{dt} = 8\pi(5) \times \frac{1}{4\pi}$

$\Rightarrow \frac{dS}{dt} = 10\text{cm}^2/\text{sec}$

OR

Let x denote the area of the circle of variable radius r.

$\because$Area of circle $(x) = \pi r^2$

$\therefore$ Rate of change of area x w.r.t. r

$\frac{dx}{dr} = \pi(2r) = 2\pi r$

a. When r = 3 cm, then

$\frac{dx}{dr} = 2\pi(3) = 6\pi cm^2/\sec$

b. When r = 4 cm, then

$\frac{dx}{dr} = 2\pi(4) = 8\pi cm^2/\sec$

24. We have

$I = \int \frac{x^3+x}{x^4-9}dx = \int \frac{x^3}{x^4-9}dx + \frac{xdx}{x^4-9} = I_1 + I_2$

Now $I_1 = \int \frac{x^3}{x^4-9}$

Put $t = x^4 - 9$ so that $4x^3\,dx = dt$. Therefore

$I_1 = \frac{1}{4}\int \frac{dt}{t} = \frac{1}{4}\log|t| + C_1 = \frac{1}{4}\log|x^4 - 9| + C_1$

Again, $I_2 = \int \frac{xdx}{x^4-9}$

Put $x^2 = u$ so that $2x\,dx = du$. Then

$I_2 = \frac{1}{2}\int \frac{du}{u^2-(3)^2} = \frac{1}{2\times 6}\log\left|\frac{u-3}{u+3}\right| + C_2$

$= \frac{1}{12}\log\left|\frac{x^2-3}{x^2+3}\right| + C_2$

Thus $I = I_1 + I_2$

$= \frac{1}{4}\log|x^4 - 9| + \frac{1}{12}\log\left|\frac{x^2-3}{x^2+3}\right| + C$

25. Let x cm be the radius and y be the circumference of the circle at any time t.

Rate of increase of radius of circle = 0.7 cm/sec

$\Rightarrow \frac{dx}{dt}$ is positive and = 0.7 cm/sec

$\Rightarrow \frac{dx}{dt}$ = 0.7 cm / sec ...(i)

$\Rightarrow y = 2\pi x$

$\therefore$ Rate of change of circumference of circle $= \frac{dy}{dt}$

$= 2\pi\frac{d}{dt}x = 2\pi\,(0.7)$ (from (i))

= 1.4πcm/sec

Section C

26. Let $I = \int (x+2)\sqrt{3x+5}\,dx$

Let x + 2 = λ (3x + 5) + μ

On equating the coefficients of like powers of x on both sides, we get

3λ = 1 and 5λ + μ = 2

$\Rightarrow \quad \lambda = \frac{1}{3}$ and $5 \times \frac{1}{3} + \mu$ = 2

$\Rightarrow \quad \lambda = \frac{1}{3}$ and $\mu = \frac{1}{3}$

$\therefore I = \int \{\lambda(3x+5) + \mu\}\sqrt{3x+5}\,dx$

$= \lambda \int (3x+5)\sqrt{3x+5}\,dx + \mu \int \sqrt{3x+5}\,dx$

$= \lambda \int (3x+5)^{\frac{3}{2}}\,dx + \mu \int (3x+5)^{\frac{1}{2}}\,dx$

$= \lambda \times \frac{(3x+5)^{\frac{5}{2}}}{\frac{5}{2}\times 3} + \mu \frac{(3x+5)^{\frac{3}{2}}}{3\times\frac{3}{2}}$ + c

$= \frac{1}{3} \times \frac{2}{15} \times (3x+5)^{\frac{5}{2}} + \frac{1}{3} \times \frac{2}{9}(3x+5)^{\frac{3}{2}} + c$

$= \frac{2}{45} \times (3x+5)^{\frac{5}{2}} + \frac{2}{27} \times (3x+5)^{\frac{3}{2}} + c$

$= \frac{2}{9} \times (3x+5)^{\frac{3}{2}} \left[\frac{1}{5} \times (3x+5)^1 + \frac{1}{3}\right] + c$

$= \frac{2}{9} \times (3x+5)^{\frac{3}{2}} \left[\frac{3(3x+5)+5}{15}\right] + c$

$= \frac{2}{9} \times (3x+5)^{\frac{3}{2}} \frac{(9x+15+5)}{15} + c$

$= \frac{2}{135} \times (3x+5)^{\frac{3}{2}}$ (9x + 20) + c

OR

We have,

$\int_0^{\frac{\pi}{2}} \sqrt{1+\sin x}\,dx = \int_0^{\frac{\pi}{2}} \sqrt{1 + \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}\,dx$ ...(We use sin x = $\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$)

$\Rightarrow \quad \int_0^{\frac{\pi}{2}} \sqrt{1+\sin x}\,dx = \int_0^{\frac{\pi}{2}} \sqrt{\frac{\left(1+\tan\frac{x}{2}\right)^2}{1+\tan^2\frac{x}{2}}}\,dx$

$= \int_0^{\frac{\pi}{2}} \sqrt{\frac{\left(1+\tan\frac{x}{2}\right)^2}{\sec^2\frac{x}{2}}}\,dx$

$= \int_0^{\frac{\pi}{2}} \left(\frac{1+\tan\frac{x}{2}}{\sec\frac{x}{2}}\right) dx$

$= \int_0^{\frac{\pi}{2}} \left(\cos\frac{x}{2} + \sin\frac{x}{2}\right) dx$

$= \left[2\sin\frac{x}{2} - 2\cos\frac{x}{2}\right]_0^{\frac{\pi}{2}}$

$= 2\left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - 0 + 1\right]$

$\therefore \int_0^{\frac{x}{2}} \sqrt{1+\sin x}\,dx = 2$

27. We have,

$I = \int_2^8 |x-5|dx$

The function |x - 5| can be written as,

$|x-5| = \begin{cases} x-5, & \text{if } x \in (5,8) \\ -(x-5), & \text{if } x \in (2,5) \end{cases}$

Hence,

$I = \int_2^5 -(x-5)dx + \int_5^8 (x-5)dx$

$= -\left[\frac{x^2}{2} - 5x\right]_2^5 + \left[\frac{x^2}{2} - 5x\right]_5^8$

$= -\left[\left(\frac{25}{2} - 25\right) - \left(\frac{4}{2} - 10\right)\right] + \left[\left(\frac{64}{2} - 40\right) - \left(\frac{25}{2} - 25\right)\right]$

$= -\left[-\frac{25}{2} + 8\right] + \left[(-8) + \left(\frac{25}{2}\right)\right]$

$= \frac{25}{2} - 8 - 8 + \frac{25}{2}$

$= 25 - 16 = 9$

$\therefore \int_2^8 |x-5|dx = 9$

28. $x^2dy + y(x+y)dx = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x} - \left(\frac{y}{x}\right)^2$

Putting $\frac{y}{x} = v \Rightarrow y = vx, \frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = -v - v^2,$

separating the variable and integrating

$\int \frac{1}{v^2+2v}dv = -\int \frac{1}{x}dx$

$\Rightarrow \int \frac{1}{(v+1)^2-1}dv = -\int \frac{1}{x}dx$

$\Rightarrow \frac{1}{2}\log\left|\frac{v}{v+2}\right| = \log\left|\frac{C}{x}\right|$

The solution of the differential equation is, $\left\|\frac{y}{y+2x}\right\| = \frac{C^2}{x^2}$ or $x^2y = k(y+2x)$

OR

Given the differential equation is -

$(1 + x^2)\, dy + 2xy\, dx = \cot x\, dx$

$\Rightarrow (1 + x^2)dy = (\cot x - 2xy)dx$

On dividing both sides by $(1 + x^2)$, we get

$dy = \frac{\cot x - 2xy}{1+x^2}dx$

$\Rightarrow \quad \frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{\cot x}{1+x^2}$

which is a linear differential equation of 1st order and is of the form

$\frac{dy}{dx} + Py = Q$

Here, $P = \frac{2x}{1+x^2}$ and $Q = \frac{\cot x}{1+x^2}$

$\therefore \quad \text{IF} = e^{\int Pdx} = e^{\int \frac{2x}{1+x^2}} = e^{\log|1+x^2|} = 1 + x^2$

$\left[\begin{array}{l} \text{put } 1 + x^2 = t \Rightarrow 2xdx = dt, \text{ then} \\ \int \frac{2x}{1+x^2}dx = \int \frac{dt}{t} = \log|t| = \log|1 + x^2| \end{array}\right]$

The solution of linear differential equation is given by

$y \times \text{IF} = \int (Q \times \text{IF})dx + C$

$\therefore \quad y\left(1 + x^2\right) = \int \frac{\cot x}{1+x^2} \times \left(1 + x^2\right) dx + C$

$\Rightarrow \quad y\left(1 + x^2\right) = \int \cot x dx + C$

$\Rightarrow \quad y\left(1 + x^2\right) = \log|\sin x| + C \quad [\because \int \cot x dx = \log|\sin x|]$

On dividing both sides by $(1 + x^2)$, we get

$y = \frac{\log|\sin x|}{1+x^2} + \frac{C}{1+x^2}$

which is the required solution.

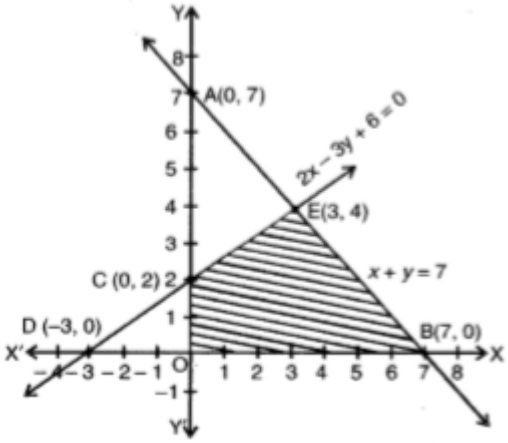
29. Consider$x + y = 7$

When $x = 0,\ then\ y = 7$ and

when $y = 0,\ then\ x = 7$

So, A(0, 7) and B(7, 0) are the points on line

$x + y = 7$



Consider $2x - 3y + 6 = 0$

When x = 0, then y = 2 and when y = 0, then x = - 3, So C(0, 2) and D(-3, 0) are the points on line

$2x - 3y + 6 = 0$

Also, we have x > 0 and v > 0.

The feasible region OBEC is bounded, so, minimum value will obtain at a comer point of this feasible region.

Corner points are O(0, 0), B(7, 0), E(3, 4) and C(0, 2)

$Z = 13x - 15y$

At $O(0, 0), Z = 0$

At $B(7, 0), Z = 13(7) - 15(0) = 91$

At $E(3, 4), Z = 13(3) - 15(4) = -21$

At $C(0, 2), Z = 13(0) - 15(2)$

= -30 (minimum)

Hence , the minimum value is -30 at the point (0, 2).

OR

First, we will convert the given inequations into equations, we obtain the following equations:

2x + y = 10, x + 3 y = 15, x = 10, y = 8

Region represented by 2x + y $\geq$ 10 :

The line 2x + y = 10 meets the coordinate axes at A(5,0) and B(0,10) respectively. By joining these points we obtain the line 2 x + y = 10. Clearly (0, 0) does not satisfies the inequation 2x + y $\geq$ 10 . So, the region in x-y plane which does not contain the origin represents the solution set of the inequation 2 x + y $\geq$ 10

Region represented by x + 3 y $\geq$ 15:

The line x + 3 y = 15 meets the coordinate axes at C(15,0) and D(0,5) respectively. By joining these points we obtain the line x + 3 y = 15 .

Clearly (0,0) satisfies the inequation x + 3y $\geq$ 15. (0,0) the region in x y plane which does not contain the origin represents the solution set of the inequation x +

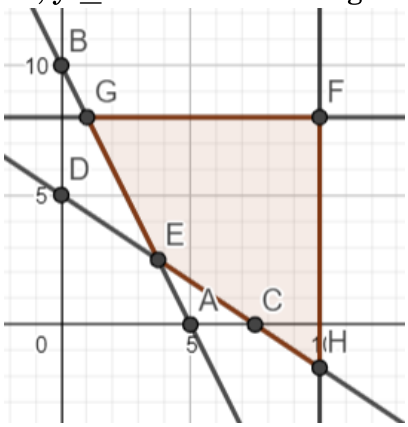
3 y $\geq$ 15

The line x = 10 is the line that passes through the point (10,0) and is parallel to Y-axis x $\leq$ 10 is the region to the left of the line x = 10.

The line y = 8 is the line that passes through the point (0,8) and is parallel to X-axis. y $\leq$ 8 is the region below the line y = 8.

The region represented by x $\geq$ 0 and y $\geq$ 0 since every point in the first quadrant satisfies these

inequations. So, the first quadrant is the region represented by the inequations $x \geq 0$ and $y \geq 0$.
The feasible region determined by subject to the constraints are $2x + y \geq 10$, $x + 3y \geq 15$, $x \leq 10$, $y \leq 8$ and the non-negative restrictions $x \geq 0$ and $y \geq 0$ are as follows.

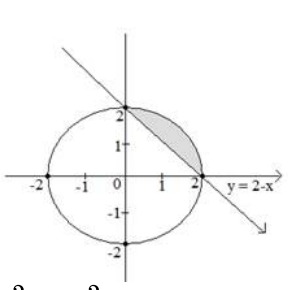


The corner points of the feasible region are E(3, 8) H (10, $\frac{5}{3}$) F(10, 8) and G(1, 8)
The value of the objective function $z = 5x + 3y$ at the corner points are as follows:
E(3, 4) $5 \times 3 + 3 \times 4 = 27$
$H\left(10, \frac{5}{3}\right) = 5 \times 10 + 3 \times \frac{5}{3} = 55$
F(10, 8) = $5 \times 10 + 3 \times 8 = 74$
G(1, 8) = $5 \times 1 + 3 \times 8 = 29$

30. A = {(3, 6), (4, 5), (5, 4), (6, 3)
$P(A) = \frac{4}{36} = \frac{1}{9}, P(B) = \frac{30}{36} = \frac{5}{6}$
$P(A \cap B) = \frac{3}{36} = \frac{1}{12}$
$P(A) \times P(B) = \frac{5}{54} \neq P(A \cap B)$
Therefore, A and B are not independent.
A and B are not mutually exclusive as $A \cap B \neq \varnothing$

31. Given, $y^2 + \left(\frac{dy}{dx}\right)^2 = a^2 + b^2$.....(i)
We shall first find $\frac{dy}{dx}$ and then use it in L.H.S of (i).
Now, y = a sin x + b cos x.....(ii)
Therefore,on differentiating both sides of Eq.(ii) w.r.t x, we get,
$\frac{dy}{dx}$ = a cos x - b sin x
Now, let us take LHS of Eq.(i),
Here, LHS = $y^2 + \left(\frac{dy}{dx}\right)^2$
Therefore,on putting the value of y and dy/dx, we get
LHS = $(a \sin x + b \cos x)^2 + (a \cos x - b \sin x)^2$
= $a^2 \sin^2 x + b^2 \cos^2 x + 2ab \sin x \cos x + a^2 \cos^2 x - b^2 \sin^2 x + 2ab \sin x \cos x$
= $a^2 \sin^2 x + b^2 \cos^2 x + a^2 \cos^2 x - b^2 \sin^2 x$
= $a^2 (\sin^2 x + \cos^2 x) + b^2 (\sin^2 x + \cos^2 x)$
= $a^2 + b^2$
= RHS

Section D

32. 

$x^2 + y^2 = 4$ ...(1)

x + y = 2 ...(2)

From (2), y = 2 - x

Put this value of y in (1), we get,

$x^2 + (2 - x)^2 = 4$

$\Rightarrow 2x^2 - 4x = 0$

$\Rightarrow 2x(x - 2) = 0$

$\Rightarrow x = 0, 2$

When x = 0, y = 2 - 0 = 2

When x = 2, y = 2 - 2 = 0

$\therefore$ points of intersection are (0, 2) and (2, 0)

Required area = area of quadrant in first quadrant - area of triangle.

$= \int_0^2 \sqrt{4 - x^2}\, dx - \int_0^2 (2 - x) dx$

$= \int_0^2 \sqrt{(2)^2 - x^2}\, dx - \int_0^2 (2 - x) dx$

$= \left[\frac{x}{2}\sqrt{(2)^2 - x^2} + \frac{(2)^2}{2}\sin^{-1}\frac{x}{2}\right]_0^2 - \left[2x - \frac{x^2}{2}\right]_0^2$

$= \left[\frac{2}{2}\sqrt{4 - 4} + \frac{4}{2}\sin^{-1}1\right] - \left[0 + \frac{4}{2}\sin^{-1}0\right] - \left[\left(4 - \frac{4}{2}\right) - (0 - 0)\right]$

$= \left(0 + \frac{4}{2} \times \frac{\pi}{2}\right) - (0 + 2 \times 0) - 2 + 0 = \pi - 2$

33. R = {(a,b) = |a.b| is divisible by 2.

where $a, b \in A = \{1, 2, 3, 4, 5\}$

reflexivty

For any a$\in$A, |a–a|=0 Which is divisible by 2.

$\therefore$ (a, a) $\in$ r for all a $\in$ A

So ,R is Reflexive

Symmetric :

Let $(a, b) \in R$ for all a, b $\in$ R

|a–b| is divisible by 2

|b–a| is divisible by 2

$(a, b) \in r \Rightarrow (b, a) \in R$

So, R is symmetirc .

Transitive :

Let $(a, b) \in R$ and $(b, c) \in R$ then

$(a, b) \in R$ and $(b, c) \in R$

|a–b| is divisible by 2

|b–c| is divisible by 2

Two cases :

Case 1:

When b is even

$(a, b) \in R$ and $(b, c) \in R$

|a–c| is divisible by 2

|b–c| is divisible by 2

|a–c| is divisible by 2

$\therefore$ (a, c) $\in$ R

Case 2:

When b is odd

$(a, b) \in R$ and $(b, c) \in R$

|a–c| is divisible by 2

|b–c| is divisible by 2

|a–c| is divisible by 2

Thus, $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$

So R is transitive.

Hence , R is an equivalence relation

OR

Reflexive: For $a \in S$

$\Rightarrow a - a + \sqrt{2}$ is irrational number

$\Rightarrow \sqrt{2}$ is irrational number

$\Rightarrow (a, a) \in S$

Thus, S is Reflexive Relation.

Symmetric: Let $(a, b) \in S \Rightarrow a - b + \sqrt{2}$ is irrational number but $b - a + \sqrt{2}$ may not be irrational number

For example, $(\sqrt{2}, 1) \in S \Rightarrow \sqrt{2} - 1 + \sqrt{2} = 2\sqrt{2} - 1$ is irrational number}

$(1, \sqrt{2}) \notin S$ as $1 - \sqrt{2} + \sqrt{2} = 1$ is not irrational number

$\therefore (b, a) \notin S$, So S is NOT Symmetric Relation.

Transitive: Let $(a, b) \in S \Rightarrow a - b + \sqrt{2}$ is irrational number $\&(b, c) \in S \Rightarrow b - c + \sqrt{2}$ is irrational number but $a - c + \sqrt{2}$ may not be irrational number

For example, $(1, \sqrt{3}) \in S \Rightarrow 1 - \sqrt{3} + \sqrt{2}$ is irrational number

$(\sqrt{3}, \sqrt{2}) \in S \Rightarrow \sqrt{3} - \sqrt{2} + \sqrt{2} = \sqrt{3}$ is irrational number

But $(1, \sqrt{2}) \notin S$ as $1 - \sqrt{2} + \sqrt{2} = 1$ is not irrational number

$\therefore (a, c) \notin S, So\ S$ is NOT Transitive Relation.

Thus, S is Reflexive But Neither Symmetric nor Transitive Relation.

34. $L_1 : \frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2} \Rightarrow$ direction ratio's of $L_1 = \langle -3, 2k, 2 \rangle$

$L_2 : \frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-7} \Rightarrow$ direction ratio's of $L_2 = \langle 3k, 1, -7 \rangle$

Since $L_1 \perp L_2$,

$-9k + 2k - 14 = 0 \Rightarrow k = -2$

Thus, d.r.'s of $L_1 =< -3, -4, 2 >$, d.r.'s of $L_2 =< -6, 1, -7 >$

Now the vector perpendicular to both $L_1 \& L_2$ is given by

$$\vec{b} = \begin{Vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & -4 & 2 \\ -6 & 1 & -7 \end{Vmatrix} = 26\hat{i} - 33\hat{j} - 27\hat{k}$$

Thus, Equation of the required line is

$\vec{r} = (3\hat{i} - 4\hat{j} + 7\hat{k}) + \lambda(26\hat{i} - 33\hat{j} - 27\hat{k})$

OR

Given Cartesian equations of lines

$\mathrm{L_1} = \frac{\mathrm{x-1}}{3} = \frac{\mathrm{y+1}}{2} = \frac{\mathrm{z-1}}{5}$

Line L1 is passing through point (1, -1, 1) and has direction ratios (3, 2, 5)

Thus, vector equation of line L1 is

$\vec{\mathrm{r}} = (\hat{\mathrm{i}} - \hat{\mathrm{j}} + \hat{\mathrm{k}}) + \lambda(3\hat{\mathrm{i}} + 2\hat{\mathrm{j}} + 5\hat{\mathrm{k}})$

And

$L_2 : \frac{x-2}{2} = \frac{y-1}{3} = \frac{z+1}{-2}$

Line L_2 is passing through point (2, -1, 1) and has direction ratios (2, 3, -2)

Thus, vector equation of line L2 is

$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(2\hat{i} + 3\hat{j} - 2\hat{k})$

Now, to calculate distance between the lines,

$\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 5\hat{k})$

$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(2\hat{i} + 3\hat{j} - 2\hat{k})$

Here,we have

$\overrightarrow{a_1} = 1 - j + \hat{k}$

$\overrightarrow{b_1} = 3\hat{i} + 2\hat{j} + 5\hat{k}$

$\overrightarrow{a_2} = 2\hat{i} + \hat{j} - \hat{k}$

$\overrightarrow{b_2} = 2\hat{i} + 3\hat{j} - 2\hat{k}$

Thus,

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 5 \\ 2 & 3 & -2 \end{vmatrix}$$

$= \hat{i}(-4 - 15) - \hat{j}(-6 - 10) + \hat{k}(9 - 4)$

$\Rightarrow \overrightarrow{b_1} \times \overrightarrow{b_2} = -19\hat{i} + 16\hat{j} + 5\hat{k}$

$\Rightarrow |\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{(-19)^2 + 16^2 + 5^2}$

$= \sqrt{361 + 256 + 25}$

$= \sqrt{642}$

$\overrightarrow{a_2} - \overrightarrow{a_1} = (2 - 1)\hat{i} + (1 + 1)\hat{j} + (-1 - 1)\hat{k}$

$\therefore \overrightarrow{a_2} - \overrightarrow{a_1} = 1 + 2\hat{j} - 2\hat{k}$

Now,

$(\overrightarrow{b_1} \times \overrightarrow{b_2}) \cdot (\overrightarrow{a_2} - \overrightarrow{a_1}) = (-19\hat{1} + 16\hat{j} + 5\hat{k}) \cdot (\hat{i} + 2\hat{j} - 2\hat{k})$

$= ((-19) \times 1) + (16 \times 2) + (5 \times (-2))$

= -19 + 32 - 10

= 3

Thus, the shortest distance between the given lines is

$$d = \left| \frac{(\overrightarrow{b_1} \times \overrightarrow{b_2}) \cdot (\overrightarrow{a_2} - \overrightarrow{a_1})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|} \right|$$

$\Rightarrow d = \left| \frac{3}{\sqrt{642}} \right|$

$\therefore d = \frac{3}{\sqrt{642}}$ units

As $d \neq 0$

Hence, given lines do not intersect each other.

35. For the given system of equations, we have

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{vmatrix} = 1 \times (10 - 9) - 1 \times (5 - 3) + 1 \times (3 - 2) = 0,$$

$$D_1 = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & 3 \\ 7 & 3 & 5 \end{vmatrix} = 1 \times (10 - 9) - 1 \times (20 - 21) + 1 \times (12 - 14) = 0,$$

$D_2 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 4 & 3 \\ 1 & 7 & 5 \end{vmatrix}$ = 1 × (20 - 21) - 1 × (5 - 3) + 1 × (7 - 4) = 0,

and, $D_3 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 7 \end{vmatrix}$ = 1 × (14 -1 2) - 1 × (7 - 4) + 1 × (3 - 2) = 0.

Thus, we have $D = D_1 = D_2 = D_3 = 0$.

So, either the system is consistent with infinitely many solutions or it is inconsistent. Consider the first two equations, these equations can be written as

x + y = 1 - z

x + 2y = 4 - 3z

In order to solve these equations let us use Cramer's rule.

Here, $D = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$ = 2 - 1 = 1, $D_1 = \begin{vmatrix} 1-z & 1 \\ 4-3z & 2 \end{vmatrix}$ = 2 - 2z - 4 + 3z = z - 2

and, $D_2 = \begin{vmatrix} 1 & 1-z \\ 1 & 4-3z \end{vmatrix}$ = 4 - 3z -1 + z = 3 - 2z.

$\therefore x = \frac{D_1}{D}$ and $y = \frac{D_2}{D}$

$\Rightarrow$ x = z - 2, y = 3 - 2z.

Let z = k, where k is any real number. Then, we get

x = k - 2, y = 3 - 2k and z = k

Substituting these values in the LHS of third equation x + 3y + 5z = 7, we get

k - 2 + 3(3 - 2k) + 5k = k - 2 + 9 - 6k + 5k = 7 = RHS

Hence, these values satisfy the third equation.

Hence, x = k - 2, y = 3 - 2k , z = k is a solution of the given system of equation for every value of k.

Section E

36. i. $P(E_1) = \frac{7}{10} = 0.7, P(E_2) = \frac{3}{10} = 0.3$

ii. $P(A \mid E_1) = 0.8, P(A \mid E_2) = 0.4$

iii. a. $P(A) = P(E_1) \cdot P(A \mid E_1) + P(E_2) \cdot P(A \mid E_2)$ = 0.7 $\times 0.8 + 0.3 \times 0.4 = 0.68$ or $\frac{17}{25}$

OR

b. $P(A) = \frac{P(E_1) \cdot P(A|E_1)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2)} = \frac{14}{17}$

37. i. Total displacement = $|\overrightarrow{d_1}| + |\overrightarrow{d_2}| + |\overrightarrow{d_3}|$

$|\overrightarrow{d_1}| = \sqrt{6^2 + 8^2}$

= $\sqrt{36 + 64}$

= $\sqrt{100}$

= 10 km

$|\overrightarrow{d_2}| = \sqrt{3^2 + 4^2}$

= $\sqrt{9 + 16}$

= $\sqrt{25}$

= 5 km

$|\overrightarrow{d_3}| = \sqrt{7^2 + 12^2}$

= $\sqrt{49 + 144}$

= 13.89

Total displacement = 10 + 5 + 13.89

= 28.89

$\approx$ 29 km

ii. Speed = = $\frac{\text{Total distance}}{\text{Total time}}$

= $\frac{28.89}{1.5}$

= 19.26 km/hr

iii. Displacement form village to zoo = $d_1 + d_2$

= 10 + 5

= 15 km

OR

Displacement from temple to mall = $d_2 + d_3$

= 5 + 13.89

= 18.89

$\approx$ 19 km

38. i. $f'(x) = e^x(\cos x + \sin x)$

For critical points, $f'(x) = 0$

$\Rightarrow \cos x + \sin x = 0$

$\Rightarrow \cos x = -\sin x$

For x to be a critical point $x \in (0, \pi)$, hence, $x = \frac{3\pi}{4}$

For all $x \in \left[0, \frac{3\pi}{4}\right], f'(x) \geq 0$

Hence, f is increasing in $\left[0, \frac{3\pi}{4}\right]$

$\left(0, \frac{3\pi}{4}\right)$ or $\left[0, \frac{3\pi}{4}\right)$ or $\left(0, \frac{3\pi}{4}\right]$

For all $x \in \left[\frac{3\pi}{4}, \pi\right], f'(x) \leq 0$

Hence, f is decreasing in $\left[\frac{3\pi}{4}, \pi\right]$

$\left(\frac{3\pi}{4}, \pi\right)$ or $\left(\frac{3\pi}{4}, \pi\right]$ or $\left[\frac{3\pi}{4}, \pi\right)$

ii. $x = \frac{3\pi}{4}$ is a critical point

$f''(x) = e^x(\cos x - \sin x) + e^x(\cos x + \sin x)$

$= 2e^x \cos x$

$f''\left(\frac{3\pi}{4}\right) = -ve$

Hence, $\frac{3\pi}{4}$ is a point of local maximum.