Class XII Session 2026-27
Subject - Mathematics
Sample Question Paper - 5

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. 1This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix}$, then (A - B)' is equal to **[1]**

   a) A'B' b) A' - B'

   c) A' + B' d) B'A'

2. If A is a square matrix of order 3, IA'I = - 3, then $|AA'|$ = **[1]**

   a) 9 b) -3

   c) -9 d) 3

3. The differential equation $x\frac{dy}{dx} - y = x^2$, has the general solution **[1]**

   a) $y + x^2 = 2cx$ b) $y - x^3 = 2cx$

   c) $2y - x^3 = cx$ d) $2y + x^2 = 2cx$

4. If $f(x) = \begin{cases} \frac{\sin^2 ax}{x^2}, & x \neq 0 \\ 1, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of a is: **[1]**

   a) ± 1 b) -1

   c) 1 d) 0

5. The angle between the lines $\frac{x-2}{2} = \frac{y-1}{7} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{y-4}{2} = \frac{z-5}{4}$ is **[1]**

   a) $\frac{\pi}{2}$ b) $\frac{\pi}{6}$

   c) $\cos^{-1}\left(\frac{3}{8}\right)$ d) $\frac{\pi}{3}$

6. General solution of $\frac{dy}{dx} + 2y = \sin x$ is **[1]**

a) $y = \frac{1}{5}(2\sin x + \cos x) - Ce^{-2x}$ b) $y = \frac{1}{5}(2\sin x + \cos x) + Ce^{-2x}$

c) $y = \frac{1}{5}(2\sin x - \cos x) - Ce^{-2x}$ d) $y = \frac{1}{5}(2\sin x - \cos x) + Ce^{-2x}$

7. The restrictions imposed on decision variables involved in an objective function of a linear programming problem are called: **[1]**

a) constraints b) optimal solutions

c) feasible solutions d) infeasible solutions

8. $\sin\left[\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to **[1]**

a) $\frac{1}{4}$ b) 1

c) $\frac{1}{3}$ d) $\frac{1}{2}$

9. $\int \tan^2 \frac{x}{2} dx = ?$ **[1]**

a) $\tan \frac{x}{2} + x + C$ b) $\tan \frac{x}{2} - x + C$

c) $2\tan \frac{x}{2} - x + C$ d) $2\tan \frac{x}{2} + x + c$

10. If f(x) = x^2 + 4x - 5 and A = $\begin{vmatrix} 1 & 2 \\ 4 & -3 \end{vmatrix}$, then f(A) is equal to **[1]**

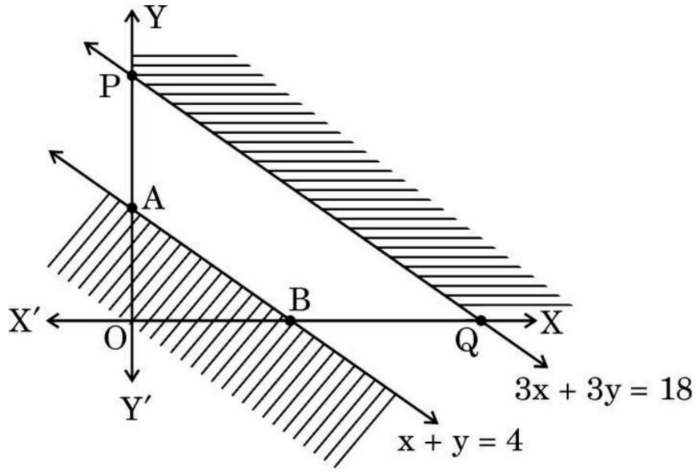
a) $\begin{vmatrix} 8 & 4 \\ 8 & 0 \end{vmatrix}$ b) $\begin{vmatrix} 0 & -4 \\ 8 & 8 \end{vmatrix}$

c) $\begin{vmatrix} 2 & 1 \\ 2 & 0 \end{vmatrix}$ d) $\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}$

11. In a Linear Programming Problem (LPP), the objective function $Z = 2x + 5y$ is to be maximised under the following constraints: **[1]**

$x + y \leq 4, 3x + 3y \geq 18, x, y \geq 0$

Study the graph and select the correct option.



The solution of the given LPP:

a) lies in the shaded unbounded region.

b) lies in the combined region of $\triangle AOB$ and unbounded shaded region.

c) does not exist.

d) lies in $\triangle AOB$.

12. The greatest integer function defined by $f(x) = [x], 1 < x < 3$ is not differentiable at $x =$ **[1]**

a) 1 b) $\frac{3}{2}$

c) 0 d) 2

13. If the direction ratios of a line are proportional to 1, - 3, 2, then its direction cosines are **[1]**

a) $-\frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}$ b) $\frac{1}{\sqrt{14}}, -\frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}$

c) $-\frac{1}{\sqrt{14}}, -\frac{2}{\sqrt{14}}, -\frac{3}{\sqrt{14}}$ d) $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$

14. Let A and B be two independent events. The probability that both A and B occur is $\frac{1}{12}$ and the probability that neither A nor B occurs is $\frac{1}{2}$. The respective probabilities of A and B are **[1]**

a) none of thse b) $\frac{1}{3}$ and $\frac{1}{4}$ or $\frac{1}{4}$ and $\frac{1}{3}$

c) $\frac{1}{6}$ and $\frac{1}{2}$ d) $\frac{1}{2}$ and $\frac{1}{6}$

15. The integrating factor of the differential equation $x\frac{dy}{dx} - y = x^4 - 3x$ is: **[1]**

a) $\log\left(x^{-1}\right)$ b) -x

c) x d) x^{-1}

16. The function f(x) = $\frac{\lambda \sin x + 2\cos x}{\sin x + \cos x}$ is increasing, if, **[1]**

a) $\lambda > 2$ b) $\lambda < 2$

c) $\lambda > 1$ d) $\lambda < 1$

17. If $y = \tan^{-1}\frac{\cos x}{1+\sin x}$ then $\frac{dy}{dx} = ?$ **[1]**

a) $\frac{1}{2}$ b) $\frac{-1}{2}$

c) 1 d) 0

18. If the direction ratios of a line are 2, 3 and -6, then direction cosines of the line making obtuse angle with Y-axis are **[1]**

a) $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$ b) $\frac{-2}{7}, \frac{3}{7}, \frac{-6}{7}$

c) $\frac{-2}{7}, \frac{-3}{7}, \frac{-6}{7}$ d) $\frac{-2}{7}, \frac{-3}{7}, \frac{6}{7}$

19. **Assertion (A):** The lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ are perpendicular, when $\vec{b_1} \cdot \vec{b_2} = 0$. **[1]**

Reason (R): The angle θ between the lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is given by $\cos\theta = \frac{\vec{b_1} \cdot \vec{b_2}}{\left|\vec{b_1}\right|\left|\vec{b_2}\right|}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** $\int \frac{dx}{x^2+2x+3} = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right) + c$ **[1]**

Reason (R): $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Evaluate $\Delta = \begin{vmatrix} 2 & 3 & -2 \\ 1 & 2 & 3 \\ -2 & 1 & -3 \end{vmatrix}$ by expanding it along the second row. **[2]**

OR

If $A = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$, prove that $A^{-1} = A^T$

22. Evaluate: $\int \frac{1-\sin 2x}{x+\cos^2 x}dx$ **[2]**

23. The total revenue in Rupees received from the sale of x units of the product is given by R (x) = $13x^2 + 26x + 15$. Find Marginal Revenue when 17 units are produced. **[2]**

OR

Find the maximum and minimum values of f(x) = $2x^3$ - 24x + 107 in the interval [1, 3].

24. If E_1 and E_2 are independent events such that P(E_1) = 0.3 and P(E_2) = 0.4, find $P\left(\bar{E}_1 \cap \bar{E}_2\right)$. **[2]**

25. Find the general solution for differential equation: $\cos x(1+\cos y)dx - \sin y(1+\sin x)dy = 0$ **[2]**

Section C

26. Evaluate: $\int_0^1 \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)dx$ **[3]**

27. Show that the function f(x) defined by f(x) = $\begin{cases} \frac{\sin x}{x} + \cos x, & x > 0 \\ 2, & x = 0 \\ \frac{4(1-\sqrt{1-x})}{x}, & x < 0 \end{cases}$ is continuous at x = 0. **[3]**

28. Evaluate $\int \frac{\sin(x-a)}{\sin(x+a)}dx$. **[3]**

OR

Evaluate: $\int_0^{\pi} \frac{x}{9\sin^2 x + 16\cos^2 x}dx$

29. A population grows at a rate of 5% per year. How long does it take for the population to double? **[3]**

OR

Solve the differential equation: ($2x^2$ y + y^3) dx + (xy^2 - $3x^3$) dy = 0

30. Find the foot of the perpendicular from (0, 2, 7) on the line $\frac{x+2}{-1} = \frac{y-1}{3} = \frac{z-3}{-2}$. **[3]**

OR

If $\vec{a} + \vec{b} + \vec{c} = \overrightarrow{0}$, prove that $(\vec{a} \times \vec{b}) = (\vec{b} \times \vec{c}) = (\vec{c} \times \vec{a})$.

31. Determine the maximum value of Z = 11x + 7y subject to the constraints: **[3]**

$2x + y \leqslant 6, x \leqslant 2, x \geqslant 0, y \geqslant 0$.

Section D

32. Sketch the graph of $y = x|x|$ and hence find the area bounded by this curve, X-axis and the ordinates $x = -2$ and $x = 2$, using integration. **[5]**

33. If $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$ find (i) Magnitude of $\vec{a} \times \vec{b}$ (ii) A unit vector which is $\perp to\ both\ \vec{a}$ and $\vec{b}$ (iii) The cosine and sine of the angle b/w the vectors $\vec{a}$ and $\vec{b}$. **[5]**

OR

If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, then show that $\vec{a} - \vec{d}$ is parallel to $\vec{b} - \vec{c}$, where $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$.

34. A furniture workshop produces three types of furniture - chairs, tables and beds each day. On a particular day the total number of furniture pieces produced is 45. It was also found that production of beds exceeds that of chairs by 8, while the total production of beds and chairs together is twice the production of tables. Determine the units produced of each type of furniture, using matrix method. **[5]**

35. The relation between the height of the plant ($y\ cm$) with respect to exposure to sunlight is governed by the equation $y = 4x - \frac{1}{2}x^2$, where x is the number of days exposed to sunlight. **[5]**

i. Find the rate of growth of the plant with respect to sunlight. **(2)**

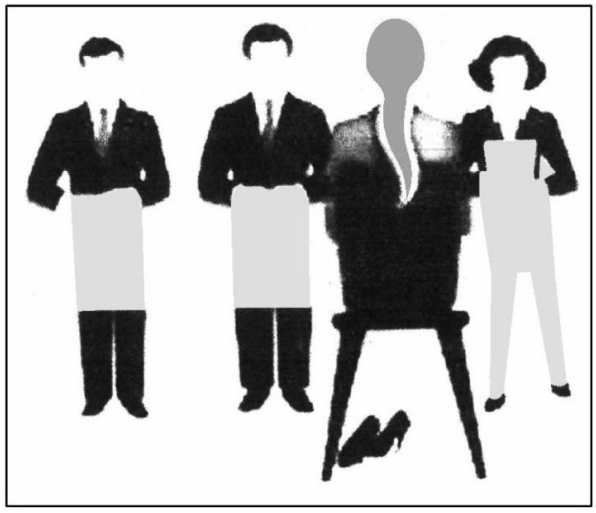
ii. In how many days will the plant attain its maximum height? What is the maximum height? **(3)**

OR

Given the perimeter of a rectangle, show that its diagonal is minimum when it is a square.

Section E

36. Rohit, Jaspreet and Alia appeared for an interview for three vacancies in the same post. The probability of Rohit's selection is $\frac{1}{5}$, Jaspreet's selection is $\frac{1}{3}$ and Alia's selection is $\frac{1}{4}$. The event of selection is independent of each other. **[4]**



Based on the above information, answer the following questions:

i. What is the probability that at least one of them is selected? **(1)**

ii. Find $P(G \mid \overline{H})$ where G is the event of Jaspreet's selection and $\overline{H}$ denotes the event that Rohit is not selected. **(1)**

iii. a. Find the probability that exactly one of them is selected. **(2)**

OR

b. Find the probability that exactly two of them are selected. **(2)**

37. A class-room teacher is keen to assess the learning of her students the concept of "relations" taught to them. She writes the following five relations each defined on the set $A = \{1, 2, 3\}$: **[4]**

$R_1 = \{(2, 3), (3, 2)\}$

$R_2 = \{(1, 2), (1, 3), (3, 2)\}$

$R_3 = \{(1, 2), (2, 1), (1, 1)\}$

$R_4 = \{(1, 1), (1, 2), (3, 3), (2, 2)\}$

$R_5 = \{(1, 1), (1, 2), (3, 3), (2, 2), (2, 1), (2, 3), (3, 2)\}$

The students are asked to answer the following questions about the above relations:

i. Identify the relation which is reflexive, transitive but not symmetric. (1)

ii. Identify the relation which is reflexive and symmetric but not transitive. (1)

iii. a. Identify the relations which are symmetric but neither reflexive nor transitive. (1+1)

OR

b. What pairs should be added to the relation R_2 to make it an equivalence relation? (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Mansi started to read the notes on the topic 'differentiability' which she has prepared in the class of mathematics. She wanted to solve the questions based on this topic, which teacher gave as home work. She has written following matter in her notes:

Let f(x) be a real valued function, then its Left Hand Derivative (LHD) is:

$\mathrm{L}f'(a) = \lim_{h \to 0} \frac{f(a-h)-f(a)}{-h}$

Right Hand Derivative (RHD) is:

$\mathrm{R}f'(a) = \lim\limits_{h \to 0} \frac{f(a+h)-f(a)}{h}$

Also, a function f(x) is said to be differentiable at x = a if its LHD and RHD at x = a exist and are equal.

For the function, $f(x) = \begin{cases} |x-3|, & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$

i. Find the value f'(-1). (1)

ii. Find the value of f'(2). (1)

iii. Check the differentiability of function at x = 1. (2)

OR

Check the differentiability of the given function at x = 1. (2)

Solution

Section A

1.

(b) A' - B'

Explanation:

(A - B) $= \begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 5 & 6 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -2 & -3 \\ -2 & 3 \end{bmatrix}$ and (A - B)' $= \begin{bmatrix} 0 & -2 & -2 \\ 0 & -3 & 3 \end{bmatrix}$

Also, A' - B' $= \begin{bmatrix} 1 & 4 & 5 \\ 2 & 1 & 6 \end{bmatrix} - \begin{bmatrix} 1 & 6 & 7 \\ 2 & 4 & 3 \end{bmatrix}$

$= \begin{bmatrix} 0 & -2 & -2 \\ 0 & -3 & 3 \end{bmatrix}$

= (A - B)'

2. **(a)** 9

Explanation:

IAA'| = |A| |A'| = (-3)(-3) = 9

3.

(c) 2y - x^3 = cx

Explanation:

We have,

$x\frac{dy}{dx} - y = x^2$

$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = x$

Comparing with $\frac{dy}{dx}$ - Py = Q

$\Rightarrow P = \frac{-1}{x}, Q = x$

I.F. = $e^{\int Pdx} = e^{\int \frac{-1}{x}dx} = e^{-\log x} = \frac{1}{x}$

Multiplying $\frac{1}{x}$ on both sides,

$\frac{1}{x}\frac{dy}{dx} - \frac{y}{x^2} = 1$

$\frac{d}{dx}\frac{y}{x} = 1$

$\int \frac{d}{dx}\frac{y}{x} = \int xdx$

$\frac{y}{x} = \frac{x^2}{2} + c$

2y = x^3 + cx

2y - x^3 - cx=0

4. **(a)** ± 1

Explanation:

± 1

5. **(a)** $\frac{\pi}{2}$

Explanation:

Let's consider the first parallel vector to be $\vec{a} = 2\hat{\imath} + 7\hat{\jmath} - 3\hat{k}$ and second parallel vector be $\overrightarrow{\text{b}} = -\hat{\imath} + 2\hat{\jmath} + 4\hat{\text{k}}$

For the angle, we can use the formula $\cos\alpha = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\times|\vec{b}|}$

For that, we need to find the magnitude of these vectors

$|\vec{a}| = \sqrt{3^2+2^2+(7)^2}$

$= \sqrt{62}$

$|\overrightarrow{b}| = \sqrt{1+2^2+4^2}$

$= \sqrt{21}$

$\Rightarrow \cos\alpha = \frac{(2\hat{i}+7\hat{j}-3\hat{k})\cdot(-\hat{i}+2\hat{j}+4\hat{k})}{\sqrt{21}\times\sqrt{62}}$

$\Rightarrow \cos\alpha = \frac{-2+14-12}{\sqrt{21}\times\sqrt{62}}$

$\Rightarrow \cos\alpha = \frac{0}{\sqrt{21}\times\sqrt{62}}$

$\Rightarrow \alpha = \cos^{-1} 0$

The negative sign does not affect anything in cosine as cosine is positive in the fourth quadrant

$\therefore \alpha = \frac{\pi}{2}$

6.

(d) $y = \frac{1}{5}(2\sin x - \cos x) + Ce^{-2x}$

Explanation:

$\frac{dy}{dx} + 2y = \sin x \Rightarrow P = 2, Q = \sin x$

$\Rightarrow I.F. = e^{\int 2dx} = e^{2x}$

$\Rightarrow ye^{2x} = \int \sin x . e^{2x} dx \Rightarrow ye^{2x}$

$= e^{2x}\frac{1}{5}(2\sin x - \cos x) + C$

$\Rightarrow y = \frac{1}{5}(2\sin x - \cos x) + Ce^{-2x}$

7. **(a)** constraints

Explanation:

constraints

8.

(b) 1

Explanation:

$\sin\left[\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right]$

= $\sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)$ = $\sin\left(\frac{\pi}{2}\right)$ = 1

9.

(c) $2\tan\frac{x}{2} - x + C$

Explanation:

Given integral is $\int \tan^2\frac{x}{2}$

Let, $\frac{x}{2} = z$

$\Rightarrow$ dx = 2dz

So,

$\int \tan^2\frac{x}{2} dx$

$= 2\int \tan^2 z dz$

$= 2\int \frac{\sin^2 z}{\cos^2 z} dz$

$= 2\int \frac{1-\cos^2 z}{\cos^2 z} dz$ where c is the integrating constant.

$= 2\int \left(\sec^2 z - 1\right) dz$

= 2 [tan z - z] + c

$= 2\left[\tan\frac{x}{2} - \frac{x}{2}\right] + c$

Hence, $\int \tan^2\frac{x}{2} = 2\left[\tan\frac{x}{2} - \frac{x}{2}\right] + c$

10. **(a)** $\begin{vmatrix} 8 & 4 \\ 8 & 0 \end{vmatrix}$

Explanation:

$A = \begin{vmatrix} 1 & 2 \\ 4 & -3 \end{vmatrix}, A^2 = \begin{vmatrix} 1 & 2 \\ 4 & -3 \end{vmatrix}\begin{vmatrix} 1 & 2 \\ 4 & -3 \end{vmatrix} = \begin{vmatrix} 9 & -4 \\ -8 & 17 \end{vmatrix}$

$f(x) = x^2 + 4x - 5$

$\because f(A) = A^2 + 4A - 5I = \begin{vmatrix} 9 & -4 \\ -8 & 17 \end{vmatrix} + \begin{vmatrix} 4 & 8 \\ 16 & -12 \end{vmatrix} + \begin{vmatrix} -5 & 0 \\ 0 & -5 \end{vmatrix} = \begin{vmatrix} 8 & 4 \\ 8 & 0 \end{vmatrix}$

11. **(c)** does not exist.

Explanation:

does not exist.

12. **(d)** 2

Explanation:

2

13. **(b)** $\frac{1}{\sqrt{14}}, -\frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}$

Explanation:

$\frac{1}{\sqrt{14}}, -\frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}$

The direction ratios of the line are proportional to 1, -3, 2

$\therefore$ The direction cosines of the line are

$\frac{1}{\sqrt{1^2+(-3)^2+2^2}}, \frac{-3}{\sqrt{1^2+(-3)^2+2^2}}, \frac{2}{\sqrt{1^2+(-3)^2+2^2}}$

$= \frac{1}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{2}{\sqrt{14}}$

14. **(b)** $\frac{1}{3}$ and $\frac{1}{4}$ or $\frac{1}{4}$ and $\frac{1}{3}$

Explanation:

Since A & B are independents and therefore $\bar{A}$ & $\bar{B}$ are independent

$\therefore P(A \cap B) = P(A)P(B)$ and $P(\overline{A} \cap \overline{B}) = P(\overline{A})\,.\,P(\overline{B})$

$\Rightarrow P(\bar{A} \cdot P(\bar{B}) = \frac{1}{12}$ .....(1)

$P(\bar{A}) \cdot P(\bar{B}) = \frac{1}{2}$

$\Rightarrow [1 - P(A)][1 - P(B)] = \frac{1}{2} \Rightarrow 1 - P(B) - P(A) + P(A) \cdot P(B) = \frac{1}{2}$

$\Rightarrow 1 - P(B) - P(A) + \frac{1}{12} = \frac{1}{2}$ ...(By (1))

$\Rightarrow P(B) = \frac{7}{12} - P(A)$ ...(2)

$\therefore$ From (1), $P(A)\left[\frac{7}{12} - P(A)\right] = \frac{1}{12}$

$\Rightarrow 12P(A)^2 - 7P(A) + 1 = 0$

$\Rightarrow (4(P(A) - 1)\,(3P(A) - 1) = 0$

$\Rightarrow$ P(A) = $\frac{1}{4}$ or P(A) = $\frac{1}{3}$

When $P(A) = \frac{1}{4}(1)$ gives $P(B) = \frac{1}{3}$ and when $P(A) = \frac{1}{3}, (1)$ gives $P(B) = \frac{1}{4}$

15.

(d) x^{-1}

Explanation:

$x\frac{dy}{dx} - y = x^4 - 3x$

$\frac{dy}{dx} - \frac{y}{x} = \frac{x\left(x^3-3\right)}{x}$

$\frac{dy}{dx} - \frac{y}{x} = x^3 - 3$

Compare with $\frac{dy}{dx} + Py = Q$.

$P = -\frac{1}{x}$

$IF = e^{\int Pdx}$

$= e^{\int -\frac{1}{x}dx}$

$= e^{-\log x}$

$= e^{\log -1}$

$= x^{-1}$

$= \frac{1}{x}$

16. **(a)** $\lambda > 2$

Explanation:

$\lambda > 2$

17.

(b) $\frac{-1}{2}$

Explanation:

Given that $y = \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$

Using $\cos x = \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}$, $\sin x = 2\sin \frac{x}{2}\cos \frac{x}{2}$ and $\cos^2 \theta + \sin^2 \theta = 1$

Therefore,

$y = \tan^{-1}\left(\frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2} + 2\sin \frac{x}{2}\cos \frac{x}{2}}\right) = \tan^{-1}\left(\frac{\left(\cos \frac{x}{2} - \sin \frac{x}{2}\right)\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)}{\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)^2}\right)$

$\Rightarrow y = \tan^{-1} \frac{\cos \frac{x}{2} - \sin \frac{x}{2}}{\cos \frac{x}{2} + \sin \frac{x}{2}}$

Dividing by $\cos \frac{x}{2}$ in numerator and denominator, we get

$y = \tan^{-1} \frac{1-\tan \frac{x}{2}}{1+\tan \frac{x}{2}}$

Using $\tan\left(\frac{\pi}{4} - x\right) = \frac{1-\tan x}{1+\tan x}$, we obtain

$y = \tan^{-1} \tan\left(\frac{\pi}{4} - \frac{x}{2}\right)$

$= \frac{\pi}{4} - \frac{x}{2}$

Differentiating with respect to x, we

$\frac{dy}{dx} = -\frac{1}{2}$

18.

(d) $\frac{-2}{7}, \frac{-3}{7}, \frac{6}{7}$

Explanation:

Direction cosines of the line, whose direction ratios are 2, 3, -6 are:

$\frac{2}{\sqrt{2^2+3^2+(-6)^2}}, \frac{3}{\sqrt{2^2+3^2+(-6)^2}}, \frac{-6}{\sqrt{2^2+3^2+(-6)^2}}$

or $\frac{2}{7}, \frac{3}{7}, \frac{-6}{7}$

Since, line makes obtuse angle with Y-axis, then $\cos\beta < 0$

Therefore, direction cosines are $\frac{-2}{7}, \frac{-3}{7}, \frac{6}{7}$.

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$

This is a standard integral and hence R is true.

$\int \frac{dx}{x^2+2x+3} = \int \frac{dx}{(x+1)^2+(\sqrt{2})^2}$

$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right) + c$

Hence A is true and R is the correct explanation for A.

Section B

21. By using the definition, expanding given determinant with respect to R_2

$$\Delta = \begin{vmatrix} 2 & 3 & -2 \\ 1 & 2 & 3 \\ -2 & 1 & -3 \end{vmatrix}$$

$$\Rightarrow \Delta = (-1)^{2+1}(1)\begin{vmatrix} 3 & -2 \\ 1 & -3 \end{vmatrix} + (-1)^{2+2}(2)\begin{vmatrix} 2 & -2 \\ -2 & -3 \end{vmatrix} + (-1)^{2+3}(3)\begin{vmatrix} 2 & 3 \\ -2 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = -\begin{vmatrix} 3 & -2 \\ 1 & -3 \end{vmatrix} + 2\begin{vmatrix} 2 & -2 \\ -2 & -3 \end{vmatrix} - 3\begin{vmatrix} 2 & 3 \\ -2 & 1 \end{vmatrix}$$

$\Rightarrow$ A = - (- 9 + 2) + 2(- 6 - 4) - 3(2 + 6) = 7 - 20 - 24 = - 37

OR

Given:

$$A = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}, \quad A^{\top} = \frac{1}{9}\begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix}$$

$|A| = \frac{1}{9}\left[-8(16+56) - 1(16-7) + 4(-32-4)\right] = -81$

Cofactors of A are:

$C_{11} = 72\ C_{21} = -36\ C_{31} = -9$

$C_{12} = -9\ C_{22} = -36\ C_{32} = 72$

$C_{13} = -36\ C_{23} = -63\ C_{33} = -36$

$$\text{adj A} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^{T}$$

$$= \begin{bmatrix} 72 & -9 & -36 \\ -36 & -36 & -63 \\ -9 & 72 & -36 \end{bmatrix}^{T}$$

So, adj A = $\begin{bmatrix} 72 & -36 & -9 \\ -9 & -36 & 72 \\ -36 & -63 & -36 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{-81}\begin{bmatrix} 72 & -36 & -9 \\ -9 & -36 & 72 \\ -36 & -63 & -36 \end{bmatrix}$

Hence, $A^{-1} = \frac{1}{9}\begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix}$

$\Rightarrow A^{-1} = A^T$

22. Let $I = \int \frac{1-\sin 2x}{x+\cos^2 x}dx$ ...(i)

Also let $x + \cos^2 x = t$ then, we have

$d(x + \cos^2 x) = dt$

$(1 - 2\cos x \sin x)\, dx = dt$

$\Rightarrow \quad dx = \frac{dt}{1-2\cos x \sin x}$

Putting $x + \cos^2 x = t$ and $dx = \frac{dt}{1-2\cos x \sin x}$ in equation (i), we get

$I = \int \frac{1-\sin 2x}{t} \times \frac{dt}{1-2\cos x \sin x}$

$= \int \frac{1-\sin 2x}{t} \times \frac{dt}{1-\sin 2x}$

$= \int \frac{dt}{t}$

$= \log |t| + c$

$= \log |x + \cos^2 x| + c$

$\therefore I = \log |x + \cos^2 x| + c$

23. $MR = \frac{d}{dx}(R(x)) = 26x + 26$

$MR]_{x=17} = 25 \times 17 + 26$

$= 425 + 26$

$= 451$

OR

We have, $f(x) = 2x^3 - 24x + 107$

$\Rightarrow f'(x) = 6x^2 - 24$

Now, $f'(x) = 0 \Rightarrow 6x^2 - 24 = 0 \Rightarrow x = \pm 2$

But, $x = -2 \notin [1, 3]$. So $x = 2$ is the only stationary point.

Let us now compute the values of f(x) at x = 1, 2, 3.

$f(1) = 2 - 24 + 107 = 85$, $f(2) = 2(2)^3 - 24(2) + 107 = 75$

and, $f(3) = 2(3)^3 - 24 \times 3 + 107 = 89$

Clearly, largest of these values is 89 and the least is 75.

Hence, the maximum value of f(x) is 89 which it attains at x = 3 and the minimum value is 75 which is attained at x = 2.

24. Given: E_1 and E_2 are two independent events such that $P(E_1) = 0.3$ and $P(E_2) = 0.4$

$P(\overline{E_1} \cap \overline{E_2}) = P(\overline{E_1}) \times P(\overline{E_2})$

since , $P(E_1) = 0.3$ and $P(E_2) = 0.4$

$\Rightarrow P(\overline{E_1}) = 1 - P(E_1) = 0.7$ and $P(\overline{E_2}) = 1 - P(E_2) = 0.6$

Since, E_1 and E_2 are two independent events.

Therefore,we have,

$P(\overline{E_1} \cap \overline{E_2})$ =$P(\bar{E}_1) \times P(\bar{E}_2)$ = 0.7 × 0.6 = 0.42

This is the required probability.

25. Rearranging the terms we get:

$\frac{\cos x dx}{(1+\sin x)} = \frac{\sin y dy}{(1+\cos y)}$

Integrating both the sides we get:

$\Rightarrow \int \frac{\cos x dx}{(1+\sin x)} = \int \frac{\sin y dy}{(1+\cos y)} + c$

$\Rightarrow$log | 1 + sin x| = -log | 1 + cos y| + log c

$\Rightarrow$log | 1 + sin x| + log | 1 + cos y| = log c

$\Rightarrow$(1 + sin x)(1 + cos y) = c

(1 + sin x)(1 + cos y)=c.

This is the required solution.

Section C

26. Clearly, we can write the given function as

$\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) = \begin{cases} 3\tan^{-1} x, & \text{if } -1/\sqrt{3} < x < 1\sqrt{3} \\ -\pi + 3\tan^{-1} x, & \text{if } x > 1/\sqrt{3} \\ \pi + 3\tan^{-1} x, & \text{if } x < -1/\sqrt{3} \end{cases}$

$\therefore \quad I = \int_0^1 \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) dx$

$\Rightarrow \quad I = \int_0^{1/\sqrt{3}} \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) dx + \int_{1/\sqrt{3}}^1 \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ [Using additive property]

$\Rightarrow \quad I = \int_0^{1/\sqrt{3}} 3\tan^{-1} x dx + \int_{1/\sqrt{3}}^1 \left(-\pi + 3\tan^{-1} x\right) dx$

$I = \left\{ \int_0^{1/\sqrt{3}} 3\tan^{-1} x dx + \int_{1/\sqrt{3}}^1 3\tan^{-1} x dx \right\} + \int_{1/\sqrt{3}}^1 -\pi dx$

$\Rightarrow \quad I = \int_0^1 3\tan^{-1} x - \pi \int_{1/\sqrt{3}}^1 dx$ [Using additive property]

$\Rightarrow \quad I = 3\left[x\tan^{-1} x - \frac{1}{2}\log\left(1+x^2\right)\right]_0^1 - \pi[x]_{1/\sqrt{3}}^1$ ...

$\left[\because \int \tan^{-1} x dx = x\tan^{-1} x - \frac{1}{2}\log\left(1+x^2\right)\right]$

$\Rightarrow \quad I = 3\left[\left(\frac{\pi}{4} - \frac{1}{2}\log 2\right) - 0\right] - \pi\left[1 - \frac{1}{\sqrt{3}}\right] = \pi\left(\frac{1}{\sqrt{3}} - \frac{1}{4}\right) - \frac{3}{2}\log 2$

27. To show that the given function is continuous at x = 0, we show that

$(LHL)_{x=0} = (RHL)_{x=0}$ = f(0) ....(i)

Here, we have f(x) = $\begin{cases} \frac{\sin x}{x} + \cos x, & x > 0 \\ 2, & x = 0 \\ \frac{4(1-\sqrt{1-x})}{x}, & x < 0 \end{cases}$

Now, LHL = $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{4(1-\sqrt{1-x})}{x}$

$= \lim_{h\to 0} \frac{4[1-\sqrt{1-(0-h)}]}{0-h}$

$= \lim_{h\to 0} \frac{4[1-\sqrt{1+h}]}{-h}$

$= \lim_{h\to 0} \frac{4[1-\sqrt{1+h}]}{-h} \times \frac{1+\sqrt{1+h}}{1+\sqrt{1+h}}$

$= \lim_{h\to 0} \frac{4\left[(1)^2-(\sqrt{1+h})^2\right]}{-h[1+\sqrt{1+h}]}$

$= \lim_{h\to 0} \frac{4[1-(1+h)]}{-h[1+\sqrt{1+h}]}$

$= \lim_{h\to 0} \frac{-h\times 4}{-h[1+\sqrt{1+h}]}$

$= \lim_{h\to 0} \frac{4}{1+\sqrt{1+h}}$

$= \frac{4}{1+\sqrt{1}} = \frac{4}{2} = 2$

and RHL = $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \left(\frac{\sin x}{x} + \cos x\right)$

$\Rightarrow \quad \text{RHL} = \lim_{h\to 0} \left(\frac{\sin h}{h} + \cos h\right)$

$= \lim_{h\to 0} \frac{\sin h}{h} + \lim_{h\to 0} \cos h$

= 1 + cos 0

= 1 + 1

= 2

Also , given that x = 0, f(x) = 2 $\Rightarrow$f(0) = 2

Since, $(\text{LHL})_{x=0} = (\text{RHL})_{x=0} = f(0) = 2$

Therefore, f(x) is continuous at x = 0.

28. According to the question, $I = \int \frac{\sin(x-a)}{\sin(x+a)} dx$

Put $x + a = t \Rightarrow$ dx = dt

$\therefore I = \int \frac{\sin(t-a-a)}{\sin t} dt = \int \frac{\sin(t-2a)}{\sin t} dt$

$= \int \frac{\sin t \cos 2a - \cos t \sin 2a}{\sin t} dt$

[$\because sin(A-B) = sinAcosB - cosAsinB$]

$= \int \cos 2a dt - \int \sin 2a \cdot \cot t dt$

$= cos2a[t] - sin2a[log|sint|] + C_1$

$= (x+a)cos2a - sin2alog|sin(x+a)| + C_1$

[put t = x + a)

$= xcos2a - sin2alog|sin(x+a)| + C1$

OR

$I = \int_0^\pi \frac{xdx}{16\cos^2 x + 9\sin^2 x}$ ...(i)

$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$I = \int_0^\pi \frac{(\pi-x)dx}{16\cos^2(\pi-x) + 9\sin^2(\pi-x)}$

$I = \int_0^\pi \frac{(\pi-x)dx}{16\cos^2 x + 9\sin^2 x}$ ...(ii)

Add eqn (i) and (ii)

$2I = \int_0^\pi \frac{\pi-x+x}{16\cos^2 x + 9\sin^2 x} dx$

$I = \frac{\pi}{2\times 9} \int_0^\pi \frac{1}{\frac{16}{9}\cos^2 x + \sin^2 x} dx$

Dividing by $\cos^2$x both sides

$I = \frac{\pi}{18} \int_0^\pi \frac{sec^2 x}{\left(\frac{4}{3}\right)^2 + \tan^2 x} dx$

Put tan x = t, $\sec^2$x dx = dt

$I = \frac{\pi}{18} \int_0^\infty \frac{dt}{\left(\frac{4}{3}\right)^2 + t^2}$

$I = \frac{\pi}{18} \left[\tan^{-1}\left(\frac{t}{\frac{4}{3}}\right)\right]_0^\infty$

$I = \frac{\pi}{18} \left[\tan^{-1}\infty - \tan^{-1} 0\right]$

$I = \frac{\pi}{18} \left[\frac{\pi}{2} - 0\right]$

$I = \frac{\pi^2}{36}$

29. Let P_0 be the initial population and let the population after t years be P. We have,

$\frac{dP}{dt} = \frac{5P}{100}$

$\Rightarrow \frac{dP}{dt} = \frac{P}{20}$

$\Rightarrow \frac{dP}{P} = \frac{1}{20}dt$

$\Rightarrow \frac{dP}{P} = \frac{1}{20}dt$

$\Rightarrow \int \frac{1}{P}dP = \frac{1}{20}\int 1.\, dt$

$\Rightarrow \log P = \frac{1}{20}t + C$ ...(i)

Initially i.e. at t = 0, we have P = P_0 . Putting t = 0 and P = P_0 in (i), we have,

$\log P_0 = \frac{1\times 0}{20} + C \Rightarrow C = \log P_0$

Substituting C = log P_0 in (i), we get

$\log P = \frac{1}{20}t + \log P_0 \Rightarrow \log \frac{P}{P_0} = \frac{1}{20}t \Rightarrow t = \frac{20}{1}\log\left(\frac{P}{P_0}\right)$

When P = $2P_0$, we have,

$t = \frac{20}{1}\log\left(\frac{P_0}{P_0}\right) = \frac{20}{1}\log 2$

Thus, the population is doubled in 20 log 2 years.

OR

The given differential equation is,

$(2x^2\, y + y^3)\, dx + (xy^2 - 3x^3)\, dy = 0$

$\frac{dy}{dx} = -\frac{\left(2x^2y+y^3\right)}{\left(xy^2-3x^3\right)}$

It is a homogeneous equation

Put y = vx and $x\frac{\mathrm{dv}}{\mathrm{dx}} + \mathrm{v} = \frac{\mathrm{dy}}{\mathrm{dx}}$

So, $\mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}} + \mathrm{v} = \frac{\left(2\mathrm{x}^2\mathrm{vx}+\mathrm{v}^3\mathrm{x}^3\right)}{\left(-\mathrm{xv}^2\mathrm{x}^2+3\mathrm{x}^3\right)}$

$\mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}} = \frac{\left(2\mathrm{v}+\mathrm{v}^3\right)}{\left(-\mathrm{v}^2+3\right)} - \mathrm{v}$

$\mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}} = \frac{\left(2\mathrm{v}^3-\mathrm{v}\right)}{\left(-\mathrm{v}^2+3\right)}$

Integrating Both Sides We get,

$\int \frac{\left(-v^2+3\right)}{\left(2v^3-v\right)}dv = \int \frac{1}{x}dx$

$\frac{\left(-v^2+3\right)}{v\left(2v^2-1\right)} = \frac{A}{v} + \frac{Bv+C}{\left(2v^2-1\right)}$

$3 - \mathrm{v}^2 = \mathrm{A}(2\mathrm{v}^2 - 1) + (\mathrm{Bv} + \mathrm{C})\, \mathrm{v}$

$3 - \mathrm{v}^2 = (2\mathrm{A} + \mathrm{B})\, \mathrm{v}^2 + \mathrm{Cv} - \mathrm{A}$

Comparing the coefficient of like power of v

A = -3

C = 0

and 2A + B = -1

$\Rightarrow$ B = 5

So, $\int \frac{-3}{\mathrm{v}}\mathrm{dv} + \int \frac{5\mathrm{v}}{2\mathrm{v}^2-1}\mathrm{dv} = \int \frac{1}{\mathrm{x}}\mathrm{dx}$

$-3\int \frac{1}{\mathrm{v}}\mathrm{dv} + \frac{5}{4}\int \frac{4\mathrm{v}}{2\mathrm{v}^2-1}\mathrm{dv} = \int \frac{1}{\mathrm{x}}\mathrm{dx}$

$-3 \log \mathrm{v} + \frac{5}{4} \log (2\mathrm{v}^2 - 1) = \log \mathrm{x} + \log \mathrm{c}$

$-12 \log \mathrm{v} + 5 \log (2\mathrm{v}^2 - 1) = 4 \log \mathrm{x} + 4 \log \mathrm{c}$

$\frac{\left(2v^2-1\right)^5}{v^{12}} = \mathrm{x}^4\, \mathrm{c}^4$

$\mathrm{x}^2\mathrm{c}^4\mathrm{y}^{12} = \left(2y^2 - x^2\right)^5$

30. Suppose L be the foot of the perpendicular drawn from the point P(0, 2, 7) to the given line.

The coordinates of a general point on the line $\frac{x+2}{-1} = \frac{y-1}{3} = \frac{z-3}{-2}$ are given by

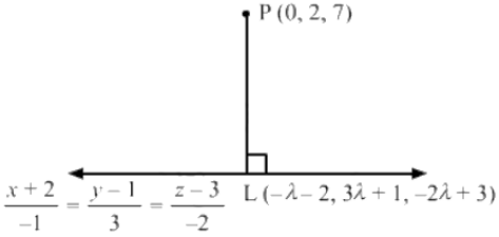
$\frac{x+2}{-1} = \frac{y-1}{3} = \frac{z-3}{-2} = \lambda$

$\Rightarrow x = -\lambda - 2$

$y = 3\lambda + 1$

$z = -2\lambda + 3$

Suppose the coordinates of L be $(-\lambda - 2, 3\lambda + 1, -2\lambda + 3)$



The direction ratios of PL are proportional to $-\lambda - 2 - 0, 3\lambda + 1 - 2, -2\lambda + 3 - 7$, i.e., $-\lambda - 2, 3\lambda - 1, -2\lambda - 4$

The direction ratios of the given line are proportional to -1, 3, -2, but PL is perpendicular to the given line.

$\therefore -1(-\lambda - 2) + 3(3\lambda - 1) - 2(-2\lambda - 4) = 0$

$\Rightarrow \lambda = \frac{1}{2}$

put $\lambda = -\frac{1}{2}$ in $(-\lambda - 2, 3\lambda + 1, -2\lambda + 3)$

We obtain the coordinates of L as $\left(-\frac{3}{2}, -\frac{1}{2}, 4\right)$

OR

Given,

$\vec{a} + \vec{b} + \vec{c} = \vec{0} \Rightarrow \vec{a} + \vec{b} = -\vec{c}$

$\Rightarrow (\vec{a} + \vec{b}) \times \vec{b} = (-\vec{c}) \times \vec{b}$

$\Rightarrow (\vec{a} \times \vec{b}) + (\vec{b} \times \vec{b}) = (-\vec{c}) \times \vec{b}$ [by the distributive law]

$\Rightarrow (\vec{a} \times \vec{b}) + \vec{0} = (\vec{b} \times \vec{c})$ [$\because \quad \vec{b} \times \vec{b} = \vec{0}$ and $(-\vec{c}) \times \vec{b} = \vec{b} \times \vec{c}$]

$\Rightarrow \vec{a} \times \vec{b} = \vec{b} \times \vec{c}$ ...(i)

Also, $\vec{a} + \vec{b} + \vec{c} = \vec{0} \Rightarrow \vec{b} + \vec{c} = -\vec{a}$

$\Rightarrow (\vec{b} + \vec{c}) \times \vec{c} = (-\vec{a}) \times \vec{c}$

$\Rightarrow (\vec{b} \times \vec{c}) + (\vec{c} \times \vec{c}) = (-\vec{a}) \times \vec{c}$ [by the distributive law]

$\Rightarrow (\vec{b} \times \vec{c}) + \vec{0} = \vec{c} \times \vec{a}$ [$\because \vec{c} \times \vec{c} = \vec{0}$ and $(-\vec{a}) \times \vec{c} = \vec{c} \times \vec{a}$]

$\Rightarrow \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$ ...(ii)

From (i) and (ii), we get

$\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

31. We have, maximise Z = 11x + 7y ..... (i)

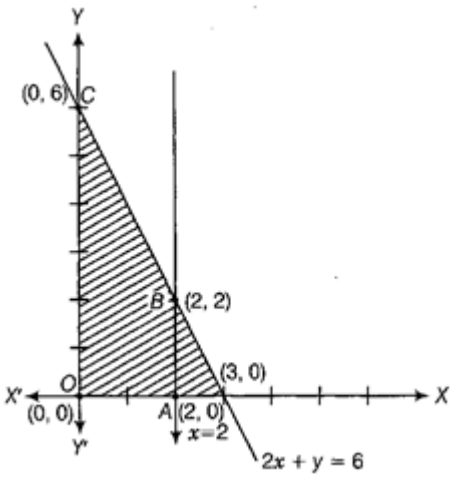
Subject to the constraints

$2x + y \leqslant 6$ ..... (ii)

$x \leqslant 2$ .... (iii)

$x \geqslant 0, y \geqslant 0$ ..... (iv)

We see that, the feasible region as shaded determined by the system of constraint (ii) to (iv) is OABC and is bounded. So, now we shall use corner point method to determine the maximum value of Z.

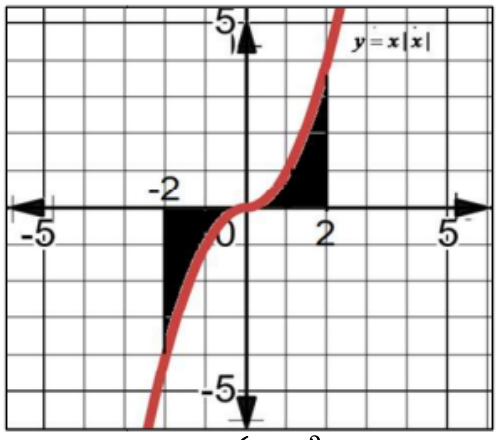


Corner Points	Corresponding value of Z
(0, 0)	0
(2, 0)	22
(2, 2)	36
(0, 6)	42 (Maximum)

Hence, the maximum value of Z is 42 at (0, 6).

Section D

32.



As, $y = x|x| = \begin{cases} -x^2, x < 0 \\ x^2, x \geq 0 \end{cases}$

Area of the shaded region $= \int_{-2}^{2} y dx = 2\int_{0}^{2} y dx = 2\int_{0}^{2} x^2 dx$

$= 2\left(\frac{x^3}{3}\right)_0^2$

$= 2\left(\frac{8}{3}\right) = \frac{16}{3}$

33. i. $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 2 & -2 & 4 \end{vmatrix}$

$= 8\hat{i} - 8\hat{j} + 8\hat{k}$

$\left|\vec{a} \times \vec{b}\right| = \sqrt{8^2 + 8^2 + 8^2} = \sqrt{64 + 64 + 64} = 8\sqrt{3}$

ii. A unit vector which is perpendicular to both $\vec{a}$ and $\vec{b} = \frac{\vec{a} \times \vec{b}}{\left|\vec{a} \times \vec{b}\right|} = \frac{8\hat{i} - 8\hat{j} - 8\hat{k}}{8\sqrt{3}}$

iii. $\vec{a}.\vec{b} = (3\hat{i} + \hat{j} + 2\hat{k})(2\hat{i} - 2\hat{j} + 4\hat{k}) = 12$

$|\vec{a}| = \sqrt{9 + 1 + 4} = \sqrt{14}$

$\left|\vec{b}\right| = \sqrt{4 + 4 + 16} = \sqrt{24}$

$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$

$\frac{12}{(\sqrt{14})(\sqrt{24})} = \frac{\sqrt{3}}{\sqrt{7}}$

$\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$

$= \frac{8\sqrt{3}}{\sqrt{14}\sqrt{24}}$

$= \frac{2}{\sqrt{7}}$

OR

According to the question, $\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$ ...(i)

and $\vec{a}\times\vec{c} = \vec{b}\times\vec{d}$ ...(ii)

Subtracting Eq. (ii) from Eq. (i),

$(\vec{a}\times\vec{b}) - (\vec{a}\times\vec{c}) = (\vec{c}\times\vec{d}) - (\vec{b}\times\vec{d})$

$\Rightarrow (\vec{a}\times\vec{b}) - (\vec{a}\times\vec{c}) + (\vec{b}\times\vec{d}) - (\vec{c}\times\vec{d}) = \vec{0}$

$\Rightarrow \quad \vec{a}\times(\vec{b}-\vec{c}) + (\vec{b}-\vec{c})\times\vec{d} = \vec{0}$

$\Rightarrow \quad \vec{a}\times(\vec{b}-\vec{c}) - \vec{d}\times(\vec{b}-\vec{c}) = \vec{0}$

$[\because \vec{a}\times\vec{b} = -\vec{b}\times\vec{a}]$

$\therefore \quad (\vec{a}-\vec{d})\times(\vec{b}-\vec{c}) = \vec{0}$

$[\because \vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$, given]

The cross-product of vectors $\vec{a}-\vec{d}$ and $\vec{b}-\vec{c}$ is a zero vector, so $\vec{a}-\vec{d}$ is parallel $\vec{b}-\vec{c}$.

34. Let the numbers of chairs, tables and beds produced be x, y and z respectively.

$\therefore x+y+z = 45; -x \ +0.y+z = 8; x-2y+z = 0$

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix}$

$|A| = 1(0+2) - 1(-1-1) \ +1(2-0) = 6 \neq 0$

$\therefore A^{-1}$ exists

$AX = B \Rightarrow X = A^{-1}B$

$\text{adj}(A) = \begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{6}\begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix}$

$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix}\begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix} = \begin{bmatrix} 11 \\ 15 \\ 19 \end{bmatrix}$

So, $x = 11, y = 15, z = 19$

Hence the numbers of chairs, tables and beds produced are 11, 15 and 19 respectively.

35. i. Rate of Growth of the Plant: The rate of growth of the plant with respect to sunlight is the derivative of the height function y with respect to time x. The height function is given by:

$y = 4x - \frac{1}{2}x^2$

Differentiating with respect to x, we get:

$\frac{dy}{dx} = 4 - x.$

Thus, the rate of growth of the plant with respect to sunlight is:

$\frac{dy}{dx} = 4 - x.$

ii. Maximum Height: To find when the plant reaches its maximum height, we set the rate of growth $\frac{dy}{dx}$ to zero:

$4 - x - 0 \rightarrow x - 4.$

Thus, the plant reaches its maximum height in 4 days. Now, substitute $x = 4$ into the height equation to find the maximum height:

$y - 4(4) - \frac{1}{2}(4)^2 - 16 - 8 - 8\ cm$

Hence, the maximum height of the plant is 8 cm.

OR

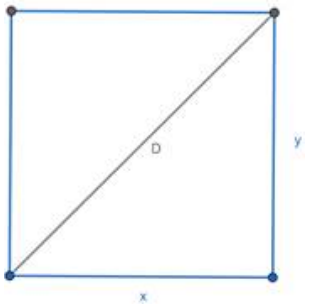
Given, Rectangle with a given perimeter.

Let us consider,

'p' as the fixed perimeter of the rectangle.

'x' and 'y' be the sides of the given rectangle.

Diagonal of the rectangle, $D = \sqrt{x^2 + y^2}$. (using the hypotenuse formula)



Now as consider the perimeter of the rectangle,

p = 2(x +y)

p = 2x + 2y

$y = \frac{p-2x}{2}$ ... (i)

Consider the diagonal of the rectangle,

$D = \sqrt{x^2 + y^2}$

Substituting (i) in the diagonal of the rectangle,

$D = \sqrt{x^2 + \left(\frac{p-2x}{2}\right)^2}$

[squaring both sides]

$Z = D^2 = x^2 + \left(\frac{p-2x}{2}\right)^2$ ... (ii)

For finding the maximum/ minimum of a given function, we can find it by differentiating it with x and then equating it to zero. This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.

Differentiating both sides with respect to x:

$\frac{dZ}{dx} = \frac{d}{dx}\left[x^2 + \left(\frac{p-2x}{2}\right)^2\right]$

$\frac{dZ}{dx} = \frac{d}{dx}\left(x^2\right) + \frac{1}{4}\frac{d}{dx}\left[(p-2x)^2\right]$

$\frac{dZ}{dx} = 2x + \frac{1}{4}[2(p-2x)(-2)]$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

= 2x - p + 2x

$\frac{dz}{dx} = 4x - p$ ... (iii)

To find the critical point, we need to equate equation (iii) to zero.

$\frac{dZ}{dx} = 4x - p = 0$

4x –p = 0

4x = p

$x = \frac{p}{4}$

Now to check if this critical point will determine the minimum diagonal, we need to check with the second differential which needs to be positive.

Consider differentiating both sides equation (iii) with x:

$\frac{d^2Z}{dx^2} = \frac{d}{dx}[4x - p]$

$\frac{d^2Z}{dx^2} = \frac{d}{dx}(4x) - \frac{d}{dx}(p)$

= 4 + 0

[Since $\frac{d}{dx}(\text{ constant }) = 0$ and $\frac{d}{dx}(x^n) = nx^{n-1}$]

$\frac{d^2z}{dx^2} = 4$ ... (iv)

Now, consider the value of $\left(\frac{d^2z}{dx^2}\right)_{x=\frac{p}{4}}$

$\frac{d^2Z}{dx^2} = 4 > 0$

As $\left(\frac{d^2z}{dx^2}\right)_{x=\frac{p}{4}} = 4 > 0$, so the function Z is minimum at $x = \frac{p}{4}$.

Now substituting $x = \frac{p}{4}$ in equation (i):

$y = \frac{p-2\left(\frac{p}{4}\right)}{2}$

$y = \frac{p-\frac{p}{2}}{2} = \frac{p}{4}$

$\therefore y = \frac{p}{4}$

As x = y = $\frac{p}{4}$ the sides of the taken rectangle are equal, we can clearly say that a rectangle with minimum diagonal which has a given perimeter is a square.

Section E

36. Given $P(\text{Rohit}) = \frac{1}{5}, P(\text{Jaspreet}) = \frac{1}{3}, P(\text{Alia}) = \frac{1}{4}$

i. P (atleast one of them is selected) $= 1 - P$ (no one is selected) $= 1 - \left(\frac{4}{5} \times \frac{2}{3} \times \frac{3}{4}\right) = \frac{3}{5}$

ii. $P(G \mid \overline{H}) = \frac{P(G \cap \bar{H})}{P(\bar{H})} = \frac{1}{3}$

iii. a. P (exactly one of them selected)

$= P(R) \times P(\overline{J}) \times P(\overline{A}) + P(\overline{R}) \times P(J) \times P(\overline{A}) + P(\overline{R}) \times P(\overline{J}) \times P(A)$

$= \frac{6+12+8}{60} = \frac{13}{30}$

OR

b. P (exactly two of them selected)

$= P(R) \times P(J) \times P(\overline{A}) + P(R) \times P(\overline{J}) \times P(A) + P(\overline{R}) \times P(J) \times P(A)$

$= \frac{3+2+4}{60} = \frac{3}{20}$

37. i. R_4

ii. R_5

iii. a. $\boldsymbol{R}_1$ and $\boldsymbol{R}_3$

OR

b. Required pairs to be added to make the relation R_2 as an equivalence relation are:
$(1,1), (2,2), (3,3), (2,1), (3,1)$ and $(2,3)$

38. i. f(x) = $\frac{x}{2} - \frac{3}{2}$, x < 1

$\therefore$ f(-1) = $\frac{-1}{2} - \frac{3}{2}$ = -2

ii. f(x) = -1, $1 \leq x \leq 3$

$\therefore$ f(2) = - 1

iii. We have, $f(x) = \begin{cases} x-3, & x \geq 3 \\ 3-x, & 1 \leq x < 3 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$

LHD at x = 1 = $\lim_{h \to 0} \frac{f(1-h)-f(1)}{-h}$

$= \lim_{h \to 0} \frac{-1}{h} \left[\frac{(1-h)^2}{4} - \frac{3(1-h)}{2} + \frac{13}{4} - 2 \right]$

$= \lim_{h \to 0} \left(\frac{1+h^2-2h-6+6h+13-8}{-4h} \right)$

$= \lim_{h \to 0} \left(\frac{h^2+4h}{-4h} \right)$ = 1

RHD at x = 1 = $\lim_{h \to 0} \frac{f(1+h)-f(1)}{h}$

= $\lim_{h \to 0} \frac{3-(1+h)-2}{h}$

= $\lim_{h \to 0} \frac{3-(1+h)-2}{h}$

= $\lim_{h \to 0} -\frac{h}{h}$ = -1

Since, LHD = RHD

Thus, f(x) is differentiable at x = 1.

OR

We have

f(x) = $\begin{cases} x^3 - 1, & 1 < x < \infty \\ x - 1, & -\infty < x \leq 1 \end{cases}$

LHD at x = 1

f'(1) = $\lim_{h \to 0} \frac{f(1-h)-f(1)}{-h}$

= $\lim_{h \to 0} \frac{(1-h)-1-0}{-h}$

RHD at x = 1

f'(1) = $\lim_{h \to 0} \frac{f(1+h)-f(1)}{h}$

= $\lim_{h \to 0} \frac{(1+h)^3-1-0}{h}$

= $\lim_{h \to 0}$ h^2 + 3 + 3h = 3

Clearly, LHD $\neq$ RHD

Thus, f(x) is not differentiable at x = 1.