Class XII Session 2026-27
Subject - Mathematics
Sample Question Paper - 4

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculators is not allowed.

Section A

1. For two matrices A and B, given that $A^{-1} = \frac{1}{4}\ B$, then inverse of (4A) is: **[1]**

   a) $\frac{1}{16}\ B$ b) B

   c) $\frac{1}{4}\ B$ d) $4\ B$

2. If A is a square matrix of order 3 such that the value of $|adj \cdot A| = 8$, then the value of $\left|A^T\right|$ is : **[1]**

   a) 8 b) $2\sqrt{2}$

   c) $\sqrt{2}$ d) $-\sqrt{2}$

3. If A is a square matrix of order 3, IA'I = - 3, then |AA'| = **[1]**

   a) 9 b) -3

   c) -9 d) 3

[1]

4. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then value of x is

a) 3 b) 6

c) ± 3 d) ± 6

5. Find the angle between the following pairs of lines: $\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k}$ + $\lambda\left(3\hat{i} + 2\hat{j} + 6\hat{k}.\right)$ and $\vec{r} = 7\hat{i} - 6\hat{k} + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}.\right)$,$\lambda, \mu \in R$ **[1]**

a) $\theta = \cos^{-1}(\frac{19}{21})$ b) $\theta = \sin^{-1}\left(\frac{19}{21}\right)$

c) $\theta = \cot^{-1}\left(\frac{19}{21}\right)$ d) $\theta = \tan^{-1}\left(\frac{19}{21}\right)$

6. What is the product of the order and degree of the differential equation $\frac{d^2y}{dx^2}\sin y + \left(\frac{dy}{dx}\right)^3 \cos y = \sqrt{y}$? **[1]**

a) 3 b) 6

c) not defined d) 2

7. The number of corner points of the feasible region determined by the constraints x - y $\geq$ 0, 2y $\leq$ x + 2, x $\geq$ 0, y $\geq$ 0 is: **[1]**

a) 3 b) 5

c) 4 d) 2

8. If $\begin{bmatrix} 3x & 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ -15 \end{bmatrix} = 0$, then the value of x is: **[1]**

a) ± 5 b) 5

c) $2\sqrt{5}$ d) zero

9. $\int \frac{(\cos 2x - \cos 2\alpha)}{(\cos x - \cos \alpha)} dx$ is equal to: **[1]**

a) 2sin x + x cos α + C b) 2 sin x + 2x cos α + C

c) sin x + x cos α + C d) 2sin x - x cos α + C

10. Find the matrix A^2, where $A = [a_{ij}]$ is a 2×2 matrix whose elements are given by a_{ij} = maximum (i, j) - minimum (i, j) : **[1]**

a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

11. The position of origin (0, 0) w.r.t. feasible region represented by x + y $\geq$ 1 is **[1]**

a) on the line x - y = 0 b) on the line x + y = 0

c) not in the region d) in the region

12. The vector equation of XY-plane is **[1]**

a) $\vec{r} \cdot \hat{i} = 0$ b) $\vec{r} \cdot \hat{k} = 0$

c) $\vec{r} \cdot \vec{n} = 1$ d) $\vec{r} \cdot \hat{j} = 0$

13. If A is a non singular matrix of order 3×3 and $|\text{adj A}| = |\text{A}|^{\text{k}}$, then write the value of k. **[1]**

a) -2 b) 1

c) 4 d) 2

14. Chances that three persons A, B, and C go to the market are $30\%, 60\%$ and 50% respectively. The probability that at least one will go to the market is: **[1]**

a) $\frac{43}{50}$ b) $\frac{14}{10}$

c) $\frac{7}{50}$ d) $\frac{9}{100}$

15. Solution of $\frac{dy}{dx} = \sec y$ is **[1]**

a) $x = \sin 2y + C$ b) $x = \sin y + C$

c) $x = cos\, y + C$ d) $x = cos\, 2y + C$

16. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$ and $|\vec{a} \times \vec{b}| = 3\sqrt{3}$, then the value of $\vec{a} \cdot \vec{b}$ is **[1]**

a) 9 b) $\frac{1}{3}$

c) $\frac{1}{9}$ d) 3

17. What is the rate of change of $\sqrt{x^2 + 16}$ w.r.t. x^2 at x = 3? **[1]**

a) $\frac{1}{15}$ b) $\frac{1}{20}$

c) $\frac{1}{5}$ d) $\frac{1}{10}$

18. The angle which the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{0}$ makes with the positive direction of Y-axis is : **[1]**

a) $\frac{7\pi}{4}$ b) $\frac{5\pi}{4}$

c) $\frac{3\pi}{4}$ d) $\frac{5\pi}{6}$

19. **Assertion (A):** The rate of change of area of a circle with respect to its radius r when r = 6 cm is 12π cm^2/cm. **[1]**

Reason (R): Rate of change of area of a circle with respect to its radius r is $\frac{dA}{dr}$, where A is the area of the circle.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The modulus function f : R $\rightarrow$ R given by f(x) = |x| is neither one-one nor onto. **[1]**

Reason (R): The signum function f : R $\rightarrow$ R given by f(x) = $\begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$ is bijective.

a) Both A and R are true and R is the

b) Both A and R are true but R is not

correct explanation of A. the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

Section B

21. Find the value of $\tan^{-1}\left(\tan\frac{3\pi}{5}\right)+\cos^{-1}\left(\cos\frac{13\pi}{6}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$ **[2]**

OR

Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$.

22. A ladder 13 m long is leaning against the wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of 2 m/s. How fast is the height on the wall decreasing when the foot of the ladder is 12 m away from the wall? **[2]**

23. Determine the values of x for which $f(x)=\frac{x-2}{x+1}, x\neq -1$ is increasing or decreasing. **[2]**

OR

For the curve $y=5x-2x^3$, if x increases at the rate of 2 units/s, then how fast is the slope of the curve changing when $x=2$?

24. Find $\int e^{\cot^{-1}x}\left(\frac{1-x+x^2}{1+x^2}\right)dx$. **[2]**

25. The radius of a cylinder is decreasing at a rate of 2 cm/s and the altitude is increasing at the rate of 3 cm/s. Find the rate of change of volume of this cylinder when its radius is 4 cm and altitude is 6 cm. **[2]**

Section C

26. Find $\int \frac{\cos\theta}{(4+\sin^2\theta)(5-4\cos^2\theta)}d\theta$. **[3]**

OR

Evaluate: $\int_0^5(|x-1|+|x-2|+|x-5|)dx$

27. Find $\int \frac{(2x-5)e^{2x}}{(2x-3)^3}dx$. **[3]**

28. Solve the following differential equation $(\cot^{-1}y+x)dy=(1+y^2)dx$. **[3]**

OR

Find the particular solution of the differential equation

$\left(xe^{\frac{y}{x}}+y\right)dx=xdy$, given that $y=1$ when $x=1$.

29. Minimize Z=400x + 200y subject to **[3]**

$5x+2y\geq 30$

$2x+y\geq 15$

$x\leq y, x\geq 0, y\geq 0$

OR

Solve the following linear programming problem graphically:

Maximise $Z=20x+30y$

Subject to the constraints:

$x+y\leq 80$

$2x+3y\geq 100$

$x \geq 14$

$y \geq 14$

30. The probability that a student buys a colouring book is 0.7 and that she buys a box of colours is 0.2. The probability that she buys a colouring book, given that she buys a box of colours, is $0 \cdot 3$. Find the probability that the student: **[3]**

i. Buys both the colouring book and the box of colours.

ii. Buys a box of colours given that she buys the colouring book.

31. Find $\frac{dy}{dx}$, if $y = (\cos x)^x + \cos^{-1}\sqrt{x}$. **[3]**

Section D

32. Find the area bounded by the circle $x^2 + y^2 = 16$ and the line $\sqrt{3}y = x$ in the first quadrant, using integration. **[5]**

33. Let A = R - {3}, B = R - {1}. If $f : A \to B$ be defined by $f(x) = \frac{x-2}{x-3}\ \forall x \in A$. Then, show that f is bijective. **[5]**

OR

Let n be a positive integer. Prove that the relation R on the set Z of all integers numbers defined by (x, y) ∈ R ⇔ x - y is divisible by n, is an equivalence relation on Z.

34. Find the foot of the perpendicular drawn from the point $(1, 1, 4)$ on the line $\frac{x+2}{5} = \frac{y+1}{2} = \frac{-z+4}{-3}$. **[5]**

OR

Find the point of intersection of the lines

$\vec{r} = \hat{i} - \hat{j} + 6\hat{k} + \lambda(3\hat{i} - \hat{k})$, and

$\vec{r} = -3\hat{j} + 3\hat{k} + \mu(\hat{i} + 2\hat{j} - \hat{k})$

Also, find the vector equation of the line passing through the point of intersection of the given lines and perpendicular to both the lines.

35. Using matrices, solve the following system of linear equations: **[5]**

x + 2y -3z = -4

2x + 3y + 2z = 2

3x - 3y - 3z = 11

Section E

36. A biased die is tossed and respective probabilities for various faces to turn up are the following: **[4]**

Face	1	2	3	4	5	6
Probability	0·1	0·24	0·19	0·18	0·15	K



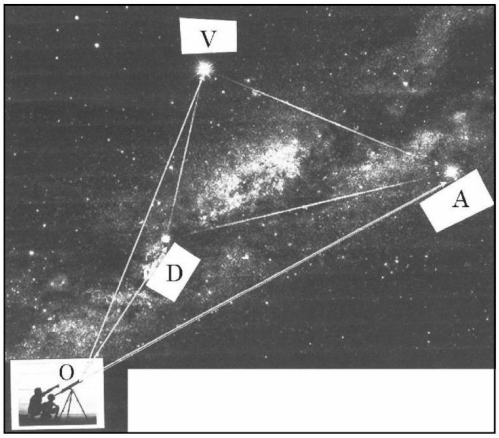
Based on the above information, answer the following questions:

i. What is the value of K? (2)

ii. If a face showing an even number has turned up, then what is the probability that it is the face with 2 or 4? (2)

37. An instructor at the astronomical centre shows three among the brightest stars in a **[4]**

particular constellation. Assume that the telescope is located at $O(0,0,0)$ and the three stars have their locations at the points D, A and V having position vectors $2\hat{i}+3\hat{j}+4\hat{k}, 7\hat{i}+5\hat{j}+8\hat{k}$ and $-3\hat{i}+7\hat{j}+11\hat{k}$ respectively.



Based on the above information, answer the following questions:

i. How far is the star V from star A? **(1)**

ii. Find a unit vector in the direction of $\overrightarrow{DA}$. **(1)**

iii. a. Find the measure of $\angle VDA$. **(2)**

OR

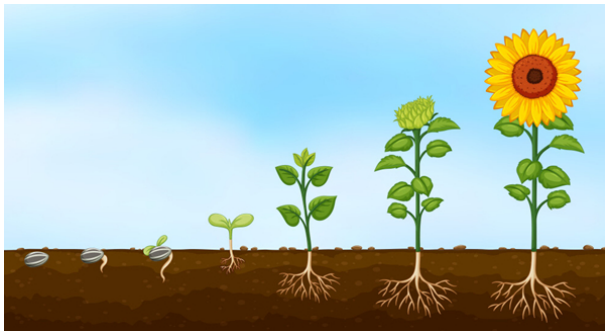
b. What is the projection of vector $\overrightarrow{DV}$ on vector $\overrightarrow{DA}$? **(2)**

38. In an agricultural institute, scientists do experiments with varieties of seeds to grow them in different environments to produce healthy plants and get more yield. **[4]**

A scientist observed that a particular seed grew very fast after germination. He had recorded growth of plant since germination and he said that its growth can be defined by the function

$f(x) = \frac{1}{3}x^3 - 4x^2 + 15x + 2, 0 \leq x \leq 10$

where x is the number of days the plant is exposed to sunlight.



On the basis of the above information, answer the following questions:

i. What are the critical points of the function f(x)? **(2)**

ii. Using second derivative test, find the minimum value of the function. **(2)**

Solution

Section A

1. **(a)** $\frac{1}{16}\ B$

   Explanation:

   $\frac{1}{16}\ B$

2. **(b)** $2\sqrt{2}$

   Explanation:

   $2\sqrt{2}$

3. **(a)** 9

   Explanation:

   IAA'| = |A| |A'| = (-3)(-3) = 9

4. **(d)** ± 6

   Explanation:

   We have $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$

   $\Rightarrow$ $2x^2$ - 40 = 18 + 14

   $\Rightarrow$ $2x^2$ = 72

   $\Rightarrow$ x^2 = 36

   $\Rightarrow$ x = ± 6

5. **(a)** $\theta = \cos^{-1}(\frac{19}{21})$

   Explanation:

   If θ is the acute angle between $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$, then cosine of the angle between these two lines is given by : $\cos\theta = \left|\frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{\left|\overrightarrow{b_1}\right|\left|\overrightarrow{b_2}\right|}\right|$

   Here, $\overrightarrow{b_1} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, $\overrightarrow{b_2} = \hat{i} + 2\hat{j} + 2\hat{k}$

   Then,

   $\cos\theta = \left|\frac{(3\hat{i}+2\hat{j}+6\hat{k}).(\hat{i}+2\hat{j}+2\hat{k})}{\left|(3\hat{i}+2\hat{j}+6\hat{k})\right|\left|(\hat{i}+2\hat{j}+2\hat{k})\right|}\right|$

   $\cos\theta = \left|\frac{19}{\sqrt{49}\sqrt{9}}\right| = \left|\frac{19}{21}\right| \Rightarrow \theta = \cos^{-1}\left(\left|\frac{19}{21}\right|\right)$

6. **(d)** 2

   Explanation:

   2

7. **(a)** 3

   Explanation:

   3

   Since it contains 3 equation in which one is y axis so only one 3 corner points are there

8. **(a)** ± 5
Explanation:
± 5

9. **(b)** 2 sin x + 2x cos α + C
Explanation:
Given
$\int \frac{(\cos 2x - \cos 2\alpha)}{(\cos x - \cos \alpha)} dx$
$= \int \frac{-2\sin\left(\frac{2x+2\alpha}{2}\right)\sin\left(\frac{2x-2\alpha}{2}\right)}{-2\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$
$= \int \frac{\sin(x+\alpha)\sin(x-\alpha)}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$
$= \int \frac{2\sin\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x+\alpha}{2}\right) \times 2\sin\left(\frac{x-\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$
$= 2\int 2\cos\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)$
$= 2\int \cos\left(\frac{x+\alpha}{2} + \frac{x-\alpha}{2}\right) + \cos\left(\frac{x+\alpha}{2} - \frac{x-\alpha}{2}\right)$
$= 2\int (\cos x + \cos \alpha) dx$
$= 2[\sin x + x\cos \alpha] + c$
where c is the integrating constant.

10. **(d)** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Explanation:
$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

11. **(c)** not in the region
Explanation:
Since (0, 0) does not satisfy x + y $\geq$ 1
i.e., 0 + 0 $\neq$ 1
$\Rightarrow$ (0, 0) not lie in feasible region represented by x + y $\geq$ 1.

12. **(b)** $\vec{r} \cdot \hat{k} = 0$
Explanation:
$\vec{r} \cdot \hat{k} = 0$
eq qxy-plane is
$rk = 0.$
or $z = 0$ is Cartesian form.

13. **(d)** 2
Explanation:

$|\text{adj A}| = |\text{A}|^{\text{k}}$ (Given)

But $|\text{adj A}| = |\text{A}|^2$

$\Rightarrow \text{k} = 2$

14. **(a)** $\frac{43}{50}$

Explanation:

$\frac{43}{50}$

15. **(b)** $x = \sin y + C$

Explanation:

$cosydy = dx$

$\int cosydy = \int dx$

$siny + c = x$

16. **(d)** 3

Explanation:

3

17. **(d)** $\frac{1}{10}$

Explanation:

Let u = $\sqrt{x^2 + 16}$ and v = x^2

Now, $\frac{du}{dx} = \frac{1}{2\sqrt{x^2+16}} \times 2\text{x} = \frac{x}{\sqrt{x^2+16}}$ and $\frac{dv}{dx} = 2\text{x}$

Now, rate of change of u w.r.t. v is

$\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{x}{\sqrt{x^2+16}} \times \frac{1}{2x}$

$\frac{du}{dv} = \frac{1}{2\sqrt{x^2+16}}$

$\Rightarrow \frac{du}{dv}$ at(x = 3) $= \frac{1}{2\sqrt{9+16}}$

$= \frac{1}{2\sqrt{25}}$

$= \frac{1}{2\times5} = \frac{1}{10}$

$\therefore \frac{d\sqrt{(x^2+16)}}{d(x^2)} = \frac{1}{10}$ at x = 3

18. **(c)** $\frac{3\pi}{4}$

Explanation:

$\frac{3\pi}{4}$

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

20. **(c)** A is true but R is false.

Explanation:

Assertion: Here, f : R $\rightarrow$ R is given by

$$f(x) = |x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

It is seen that f(-1) = |-1| = 1, f(1) = |1| = 1

Therefore, f(-1) = f(1) but -1 $\neq$ 1

Therefore, f is not one-one.

Now, consider -1 $\in$ R

It is known that f(x) = |x| is always non-negative

Thus, there does not exist any element x in domain R such that f (x) = |x| = -1.

Therefore, f is not onto.

Hence, the modulus function is neither one-one nor onto.

Reason: f : R $\rightarrow$ R, $f(x) = \begin{cases} 1, & \text{if} \quad x > 0 \\ 0, & \text{if} \quad x = 0 \\ -1, & \text{if} \quad x < 0 \end{cases}$

It is seen that f(1) = f(2) = 1 but 1 $\neq$ 2.

Therefore, f is not one-one

Now, as f(x) takes only three values (1, 0 or -1), therefore for the element -2 in codomain R, there does not exist any x in domain R such that f(x) = -2

Therefore, f is not onto. Hence, the Signum function is neither one-one nor onto.

Section B

21. $\tan^{-1}\left(\tan\frac{3\pi}{5}\right)+\cos^{-1}\left(\cos\frac{13\pi}{6}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$

$= -\frac{2\pi}{5}+\frac{\pi}{6}-\frac{\pi}{6}$

$= -\frac{2\pi}{5}$

OR

We have, $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$.

$= \tan^{-1}\left(\tan\frac{5\pi}{6}\right)+\cot^{-1}\left(\cot\frac{\pi}{3}\right)+\tan^{-1}(-1)$.

$= \tan^{-1}\left[\tan\left(\pi-\frac{\pi}{6}\right)\right]+\cot^{-1}\left[\cot\left(\frac{\pi}{3}\right)\right]+tan^{-1}\left[\tan\left(\pi-\frac{\pi}{4}\right)\right]$

$= \tan^{-1}\left(-\tan\frac{\pi}{6}\right)+\cot^{-1}\left(\cot\frac{\pi}{3}\right)+\tan^{-1}\left(-\tan\frac{\pi}{4}\right)$

$\left[\begin{array}{c} \because \tan^{-1}(\tan x) = x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \\ \cot^{-1}(\cot x) = x, x \in (0,\ \pi) \\ and\ \tan^{-1}(-x) = -\tan^{-1}x \end{array}\right]$

$= -\frac{\pi}{6}+\frac{\pi}{3}-\frac{\pi}{4} = \frac{-2\pi+4\pi-3\pi}{12}$

$= \frac{-5\pi+4\pi}{12} = \frac{-\pi}{12}$

22. $x^2+y^2 = 169$

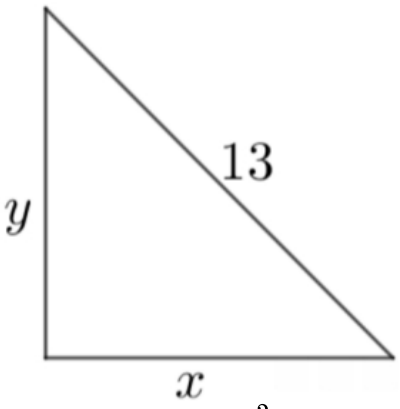
Differentiate both sides w.r.t. t

$2x\frac{dx}{dt}+2y\frac{dy}{dt} = 0$

$\Rightarrow 12(2)+5\left(\frac{dy}{dt}\right) = 0[\because \text{ when } x = 12m, y = 5m]$

$\Rightarrow \frac{dy}{dt} = -\frac{24}{5}$

Hence, the height decreases at the rate of $\frac{24}{5}\ m/s$



23. Given: $f(x) = \frac{x-2}{x+1}, x \neq -1$

$\Rightarrow \quad f'(x) = \frac{(x+1)\times 1-(x-2)\times 1}{(x+1)^2} = \frac{3}{(x+1)^2}, x \neq -1$

$f'(x) = \frac{3}{(x+1)^2} > 0$ for all $x \in R - |-1|$

So, f(x) is increasing on $R - \{-1\}$

Now, we have to find the intervals in which $f(x) = \frac{x}{2} + \frac{2}{x}, x \neq 0$ is increasing or decreasing

$f(x) = \frac{x}{2} + \frac{2}{x}$

$f'(x) = \frac{1}{2} - \frac{2}{x^2} = \frac{x^2-4}{2x^2}$

$\frac{x^2-4}{2x^2} > 0$

$\Rightarrow \quad x^2 - 4 > 0$

$\Rightarrow \quad (x-2)(x+2) > 0$

$\Rightarrow \quad x < -2 \text{ or } , x > 2$

$\Rightarrow \quad x \in (-\infty, -2) \cup (2, \infty)$

So, f(x) is increasing on $(-\infty, -2) \cup (2, \infty)$

$\frac{x^2-4}{2x^2} < 0$

$x^2 - 4 < 0$

$\Rightarrow \quad (x-2)(x+2) < 0$

$\Rightarrow \quad x \in (-2, 2)$

But, domain (f) = $R - \{0\}$. So, $f(x)$ is decreasing on $(-2, 0) \cup (0, 2)$

OR

$y = 5x - 2x^3$

Given $\frac{dx}{dt} = 2$ units $/s$

slope of the curve $= \frac{dy}{dx} = 5 - 6x^2 = m$

$\frac{dm}{dt} = -12x\frac{dx}{dt} = -12x(2) = -24x$

at $x = 2, \frac{dm}{dt} = -24(2) = -48$

Hence, slope of curve is decreasing at the rate of 48

24. Put $\cot^{-1} x = t \therefore x = \cot t$ and $\frac{1}{1+x^2}dx = -dt$

$\therefore \int e^{\cot^{-1} x}\left(\frac{1-x+x^2}{1+x^2}\right)dx = -\int e^t(1 - \cot t + \cot^2 t)dt = \int e^t(\cot t - \operatorname{cosec}^2 t)dt$

$= e^t \cot t + c$

$= xe^{\cot^{-1} x} + c$

///////

25. $\frac{dr}{dt} = -2\ cm/s, \frac{dh}{dt} = 3\ cm/s, \left(\frac{dV}{dt}\right)_{r=4,\ h=6} = ?$

$V = \pi r^2\ h \Rightarrow \frac{dV}{dt} = 2\pi r \cdot \frac{dr}{dt} \cdot\ h + \pi r^2 \frac{dh}{dt}$

When $r = 4\ cm$ and $h = 6\ cm$,

$\frac{dV}{dt} = 2\pi(4)(-2)(6) + \pi(4)^2(3) = -48\pi\ cm^3/s$

Volume is decreasing at the rate of $48\pi\ cm^3/s$

Section C

26. According to the question, $I = \int \frac{\cos\theta}{(4+\sin^2\theta)(5-4\cos^2\theta)]} d\theta$

$= \int \frac{\cos\theta}{(4+\sin^2\theta)[5-4(1-\sin^2\theta)]} d\theta \quad [\because cos^2\theta = 1 - sin^2\theta]$

$= \int \frac{\cos\theta}{(4+\sin^2\theta)(5-4+4\sin^2\theta)} d\theta$

$= \int \frac{\cos\theta}{(4+\sin^2\theta)(1+4\sin^2\theta)} d\theta$

Let $\sin\theta = t \Rightarrow \cos\theta d\theta = dt$

Then, $I = \int \frac{dt}{(4+t^2)(1+4t^2)}$

let , $\frac{1}{(4+t^2)(1+4t^2)} = \frac{A}{4+t^2} + \frac{B}{1+4t^2}$

using partial fractions

At t = 0, $\frac{A}{4} + \frac{B}{1} = \frac{1}{4\times1} \Rightarrow \quad A + 4B = 1$ ...(i)

At $t = 1, \frac{A}{5} + \frac{B}{5} = \frac{1}{5\times5} \Rightarrow 5A + 5B = 1$ ...(ii)

On solving Equations (i) and (ii), we get

$A = \frac{-1}{15} \quad and \quad B = \frac{4}{15}$

$\frac{1}{(4+t^2)(1+4t^2)} = \frac{-\frac{1}{15}}{4+t^2} + \frac{\frac{4}{15}}{1+4t^2}$

$\Rightarrow \quad \frac{1}{(4+t^2)(+4t^2)} = \frac{-1}{15(4+t^2)} + \frac{4}{15(1+4t^2)}$

Integrating both sides w.r.t. t,

$\Rightarrow \int \frac{1}{(4+t^2)(1+4t^2)} dt = \frac{-1}{15}\int \frac{1}{4+t^2} dt + \frac{4}{15}\int \frac{1}{1+4t^2} dt$

$= \frac{-1}{15}\int \frac{1}{2^2+t^2} + \frac{4}{15\times4}\int \frac{1}{\left(\frac{1}{2}\right)^2+t^2} dt$

$= \frac{-1}{15}\cdot\frac{1}{2}\tan^{-1}\frac{t}{2} + \frac{1}{15}\cdot\frac{1}{1/2}\tan^{-1}\frac{t}{1/2} + C \quad \left[\because \int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c\right]$

put $t = \sin\theta$

$= \frac{-1}{30}\tan^{-1}\frac{\sin\theta}{2} + \frac{2}{15}\tan^{-1} 2\sin\theta + C$

OR

$|x-1| + |x-2| + |x-5| = (1-x) + (2-x) + (5-x) = 8 - 3x.$

For $1 \le x \le 2$:

$(x-1) + (2-x) + (5-x) = 6 - x$

For $2 \le x \le 5$:

$(x-1) + (x-2) + (5-x) = x + 2$

Now integrate on each interval:

$\int_0^1 (8-3x)dx = \left[8x - \frac{3}{2}x^2\right]_0^1 = 8 - \frac{3}{2} = \frac{13}{2}$

$\int_1^2 (6-x)dx = \left[6x - \frac{1}{2}x^2\right]_1^2 = (12-2) - \left(6 - \frac{1}{2}\right) = 10 - 5.5 = \frac{9}{2}$

$\int_2^5 (x+2)dx = \left[\frac{1}{2}x^2 + 2x\right]_2^5 = \left(\frac{25}{2} + 10\right) - (2+4) = \frac{33}{2}$

Sum:

$\frac{13}{2} + \frac{9}{2} + \frac{33}{2} = \frac{55}{2}$

27. According to the question, $I = \int \frac{(2x-5)e^{2x}}{(2x-3)^3}dx$

$= \int \frac{(2x-3-2)e^{2x}}{(2x-3)^3}dx$

$= \int \frac{(2x-3)e^{2x}}{(2x-3)^3}dx - 2\int \frac{e^{2x}}{(2x-3)^3}dx$

$= \int \frac{e^{2x}}{(2x-3)^2}dx - 2\int \frac{e^{2x}}{(2x-3)^3}dx$

$= \int_{II} e^{2x} \underset{I}{(2x-3)^{-2}}dx - 2\int e^{2x}(2x-3)^{-3}dx \left[\because \frac{1}{a^n} = a^{-n}\right]$

Using integration by parts for first part ,

$=(2x-3)^{-2}\int e^{2x}dx - \int\left\{\frac{d}{dx}(2x-3)^{-2}.\int e^{2x}dx\right\}dx - 2\int e^{2x}(2x-3)^{-3}dx$

$= (2x-3)^{-2}\ \frac{e^{2x}}{2} - \int -2(2x-3)^{-3}.2\times\frac{e^{2x}}{2}dx\ - 2\int e^{2x}(2x-3)^{-3}dx$

$= \frac{e^{2x}(2x-3)^{-2}}{2} + C + 2\int e^{2x}(2x-3)^{-3}dx\ - 2\int e^{2x}(2x-3)^{-3}dx$

$= \frac{e^{2x}(2x-3)^{-2}}{2} + C$

28. According to the question ,$(cot^{-1}y + x)dy = (1+y^2)dx$

$\Rightarrow \quad \frac{dx}{dy} = \frac{\cot^{-1}y + x}{1+y^2}$

$\Rightarrow \quad \frac{dx}{dy} + \left(-\frac{1}{1+y^2}\right)x = \frac{\cot^{-1}y}{1+y^2}$

This is a linear differential equation of the form $\frac{dx}{dy} + Px = Q$,

Where , $P = \frac{-1}{1+y^2}$ and $Q = \frac{\cot^{-1}y}{1+y^2}$

We know that , $\text{IF} = e^{-\int\frac{1}{1+y^2}dy} = e^{\cot^{-1}y}$

Now, the solution of linear differential equation is given by $x.IF = \int(Q\times\text{IF})dy + C$

$\therefore \quad xe^{\cot^{-1}y} = \int\frac{\cot^{-1}y}{(1+y^2)}e^{\cot^{-1}y}dy + C$

put $\cot^{-1}y = t$

$\Rightarrow \frac{1}{1+y^2}dy = -dt$

$xe^{\cot^{-1}y} = -\int te^t dt + C$

$= -e^t(t-1) + C$

$\Rightarrow \quad xe^{\cot^{-1}y} = e^{\cot^{-1}y}\left(1 - \cot^{-1}y\right) + C \ \left[\because t = \cot^{-1}y\right]$

which is the required solution.

OR

$\frac{dy}{dx} = e^{\frac{y}{x}} + \frac{y}{x} = f\left(\frac{y}{x}\right)$ so, its a homogeneous differential equation

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Now, $v + x\frac{dv}{dx} = e^v + v$

$\Rightarrow \int e^{-v}dv = \int\frac{1}{x}dx$

$\Rightarrow -e^{-v} = \log|x| + c \Rightarrow -e^{\frac{-y}{x}} = \log|x| + c \ldots (1)$

Now, $x = 1, y = 1$, gives $c = -e^{-1}$

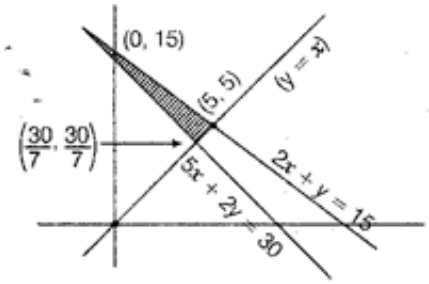
Thus, $\log|x| + e^{\frac{-y}{x}} = e^{-1}$

29. we have minimise Z=400x + 200y subject to 5x + 2y$\geqslant$30.

$2x + y \geqslant 15, x \leqslant y, x \geqslant 0, y \geqslant 0$

On solving x-y=0 and 5x+2y=30, we get

$y = \frac{30}{7}, x = \frac{30}{7}$



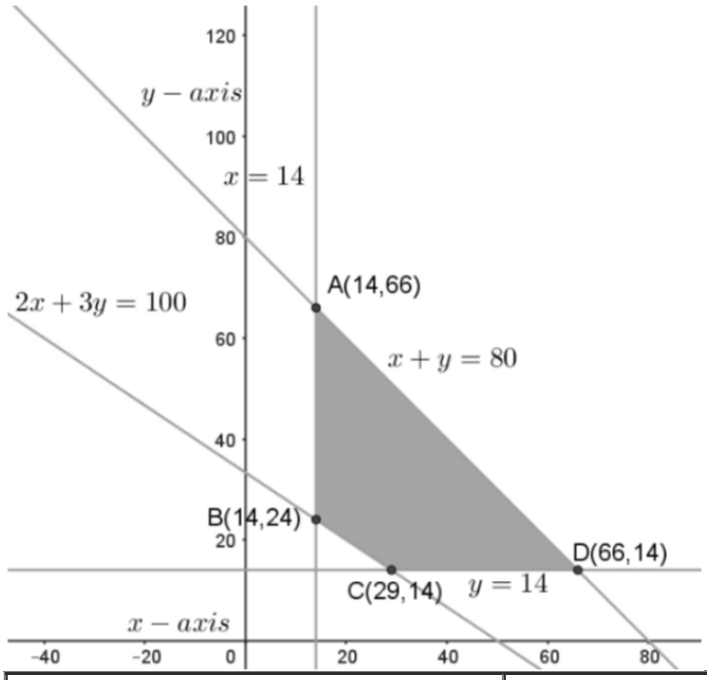
On solving x - y = 0 and 2x + y = 15 we get x = 5, y = 5

So, from the shaded feasible region it is clear that coordinates of corner points are (0,15) , (5,5) and $\left(\frac{30}{7}, \frac{30}{7}\right)$.

Corner Points	**Corresponding value of X = 400x + 200y**
(0, 15)	3000
(5, 5)	3000
$\left(\frac{30}{7}, \frac{30}{7}\right)$	$400 \times \frac{30}{7} + 200 \times \frac{30}{7} = \frac{18000}{7}$
	= 2571.43 (minimum)

Hence, the minimum is Rs 2571.43.

OR



Corner Point	**Value of $Z = 20x + 30y$**
$A(14, 66)$	**2260**
$B(14, 24)$	**1000**
$C(66, 14)$	**1740**
$D(29, 14)$	**1000**

Max(Z) = 2260

30. Let A be the event of buying colouring book and B be the event of buying coloured box.

$P(A) = 0.7, P(B) = 0.2, P(A/B) = 0.3$

i. $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} \Rightarrow 0.3 = \frac{P(A \cap B)}{0.2}$

$\Rightarrow P(A \cap B) = 0.06$ or $\frac{3}{50}$

ii. $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

$= \frac{0.06}{0.7} = \frac{3}{35}$ or 0.086

31. Let $y = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$u = (\cos x)^x \Rightarrow \log u = x \log \cos x$

$\Rightarrow \frac{du}{dx} = (\cos x)^x [\log \cos x - x \tan x]$

$v = \cos^{-1} \sqrt{x} \Rightarrow \frac{dv}{dx} = -\frac{1}{2\sqrt{x}\sqrt{1-x}}$

$\therefore \frac{dy}{dx} = (\cos x)^x [\log \cos x - x \tan x] - \frac{1}{2\sqrt{x}\sqrt{1-x}}$

Section D

32. According to the question ,

Given equation of circle is $x^2 + y^2 = 16$ ...(i)

Equation of line given is ,

$\sqrt{3}y = x$ ...(ii)

$\Rightarrow y = \frac{1}{\sqrt{3}}x$ represents a line passing through the origin.

To find the point of intersection of circle and line ,

substitute eq. (ii) in eq.(i) , we get

$x^2 + \frac{x^2}{3} = 16$

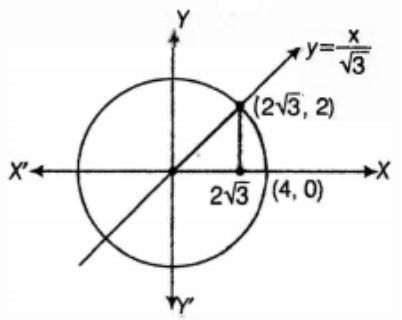
$\frac{3x^2 + x^2}{3} = 16$

$\Rightarrow 4x^2 = 48$

$\Rightarrow x^2 = 12$

$\Rightarrow x = \pm 2\sqrt{3}$

When $x = 2\sqrt{3}$, then $y = \frac{2\sqrt{3}}{\sqrt{3}} = 2$



Required area (In first quadrant) = (Area under the line $y = \frac{1}{\sqrt{3}}x$ from x = 0 to $2\sqrt{3}$) + (Area under the circle from $x = 2\sqrt{3}$ to x=4)

$= \int_0^{2\sqrt{3}} \frac{1}{\sqrt{3}} x dx + \int_{2\sqrt{3}}^4 \sqrt{16 - x^2}\, dx$

$= \frac{1}{\sqrt{3}} \left[\frac{x^2}{2}\right]_0^{2\sqrt{3}} + \left[\frac{x}{2}\sqrt{16 - x^2} + \frac{(4)^2}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_{2\sqrt{3}}^4$

$= \frac{1}{2\sqrt{3}}\left[(2\sqrt{3})^2 - 0\right] + \left[0 + 8\sin^{-1}(1) - \frac{2\sqrt{3}}{2}\sqrt{16 - 12} - 8\sin^{-1}\left(\frac{2\sqrt{3}}{4}\right)\right]$

$= 2\sqrt{3} + 8\left(\frac{\pi}{2}\right) - \frac{2\sqrt{3}}{2} \times 2 - 8\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= 2\sqrt{3} + 4\pi - 2\sqrt{3} - 8\left(\frac{\pi}{3}\right)$

$= 4\pi - \frac{8\pi}{3}$

$= \frac{12\pi - 8\pi}{3}$

$= \frac{4\pi}{3}$ sq units.

33. Given that, A = R - {3}, B = R - {1}.

$f : A \to B$ is defined by $f(x) = \frac{x-2}{x-3}\ \forall x \in A$

For injectivity

Let $f(x_1) = f(x_2) \Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$

$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6$

$\Rightarrow -3x_1 - 2x_2 = -3x_2 - 2x_1$

$\Rightarrow -x_1 = -x_2 \Rightarrow x_1 = x_2$

So, f(x) is an injective function

For surjectivity

Let $y = \frac{x-2}{x-3} \Rightarrow$ x - 2 = xy - 3y

$\Rightarrow$ x(1 - y) = 2 - 3y $\Rightarrow x = \frac{2-3y}{1-y}$

$\Rightarrow x = \frac{3y-2}{y-1} \in A,\ \forall y \in B$ [codomain]

So, f(x) is surjective function.

Hence, f(x) is a bijective function.

OR

We observe the following properties of relation R.

Reflexivity: For any a $\in$N

a - a = 0 = 0 $\times$ n

$\Rightarrow$ a - a is divisible by n

$\Rightarrow$ (a, a) $\in$ R

Thus, (a, a) $\in$ for all a $\in$ Z. So, R is reflexive on Z

Symmetry: Let (a, b) $\in$R. Then,

(a, b) $\in$ R

$\Rightarrow$ (a - b) is divisible by n

$\Rightarrow$ (a - b) = np for some p $\in$ Z

$\Rightarrow$ b - a = n (-p)

$\Rightarrow$ b - a is divisible by n $\left[\because p \in Z \Rightarrow -p \in Z\right]$

$\Rightarrow$ (b, a) $\in$ R

Thus, (a, b) $\in$ R $\Rightarrow$ (b, a) $\in$R for all a, b $\in$ Z.

So, R is symmetric on Z.

Transitivity: Let a, b, c $\in$ Z such that (a, b) $\in$R and (b, c) $\in$ R. Then,

(a, b) $\in$ R

$\Rightarrow$ (a - b) is divisible by n

$\Rightarrow$ a - b = np for some p $\in$ Z

and, (b, c) $\in$ R

$\Rightarrow$ (b - c) is divisible by n

$\Rightarrow$ b - c = nq for some q $\in$ Z

$\therefore$ (a, b) $\in$ R and (b, c) $\in$ R

$\Rightarrow$ a - b = np and b - c = nq

$\Rightarrow$ (a - b) + (b - c) = np + nq

$\Rightarrow$ a - c = n (p + q)

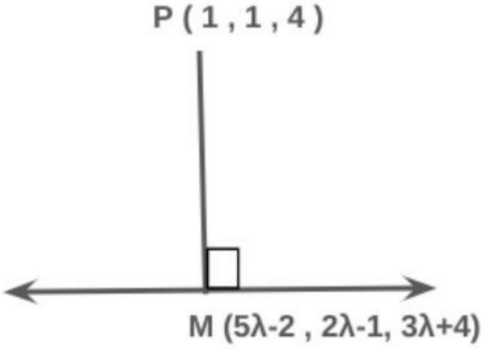
$\Rightarrow$ a - c is divisible by n $[\because p, q \in Z \Rightarrow p+q \in Z]$

$\Rightarrow$ (a, c) $\in$ R

Thus, (a, b) $\in$ R and (b, c) $\in$ R $\Rightarrow$(a, c) $\in$ R for all a, b, c $\in$ Z.

34. Let $\frac{x+2}{5} = \frac{y+1}{2} = \frac{z-4}{3} = \lambda$

Coordinate of general point on the given line are $M(5\lambda - 2, 2\lambda - 1, 3\lambda + 4)$



Direction Ratios of PM vector are $< 5\lambda - 3, 2\lambda - 2, 3\lambda >$

Since, $PM \perp \boldsymbol{l}$

$\Rightarrow 5(5\lambda - 3) + 2(2\lambda - 2) + 3(3\lambda) = 0$

$\Rightarrow \lambda = \frac{1}{2}$

Hence, coordinates of M are $\left(\frac{1}{2}, 0, \frac{11}{2}\right)$

OR

If the given lines intersect then,

$\hat{i} - \hat{j} + 6\hat{k} + \lambda(3\hat{i} - \hat{k}) = -3\hat{j} + 3\hat{k} + \mu(\hat{i} + 2\hat{j} - \hat{k})$

Solving the equations, $1 + 3\lambda = \mu, -1 = 2\mu - 3$, we get $\lambda = 0, \mu = 1$

which do not satisfy the equation, $6 - \lambda = 3 - \mu$.

$\therefore$ The lines do not intersect, hence no point of intersection

35. Given system of linear equations are

x + 2y -3z = -4

2x + 3y + 2z = 2

3x - 3y - 3z = 11

It can be written as $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$

or AX = B where A = $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -3 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -3 \end{vmatrix}$ = 1(-9 + 6) -2(-6 - 6) -3(-6 - 9) = -3 +24 +45 = 66

$|A| = 66 \neq 0$

$\therefore$ X = A^{-1} B

A_{11}= -3 , A_{12}= 12 , A_{13} = -15 , A_{21}= 15 , A_{22}= 6, A_{23}= 9, A_{31} = 13, A_{32} = -8, A_{33} = -1

adj A = $\begin{bmatrix} -3 & 15 & 13 \\ 12 & 6 & -8 \\ -15 & 9 & -1 \end{bmatrix}$

$A^{-1} = \frac{1}{66}\begin{bmatrix} -3 & 15 & 13 \\ 12 & 6 & -8 \\ -15 & 9 & -1 \end{bmatrix}$

So $X = \frac{1}{66}\begin{bmatrix} -3 & 15 & 13 \\ 12 & 6 & -8 \\ -15 & 9 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$

$= \frac{1}{66}\begin{bmatrix} 185 \\ -124 \\ 67 \end{bmatrix}$

$= \begin{bmatrix} \frac{185}{66} \\ \frac{-62}{33} \\ \frac{67}{66} \end{bmatrix}$

$x = \frac{185}{66}$, $y = -\frac{62}{33}$, $z = \frac{67}{66}$

Section E

36. i. P(1) + P(2) + P(3) + P(4) + P(5) + P(6)

$\Rightarrow$ 0.1 + 0.24 + 0.19 + 0.18 + 0.15 + K = 1

$\Rightarrow$ K = 0.14

ii. A: face shows 2 or 4

B: even face have turned up

Required probability = P (A/B) = $\frac{P(A \cap B)}{P(B)}$

here P (A$\cap$B) = P(2) + P(4)

= 0.24 + 0.18 = 0.42

P(B) = P(2) + P(4) + P(6) = 0.56

$\therefore$ P(A/B) = $\frac{0.42}{0.56} = \frac{3}{4}$

37. i. $\overrightarrow{AV}$ = Position Vector of V - Position Vector of A

$= -3\hat{i} + 7\hat{j} + 11\hat{k} - 7\hat{i} - 5\hat{j} - 8\hat{k} \quad = -10\hat{i} + 2\hat{j} + 3\hat{k}$

Thus, $|\overrightarrow{AV}| = \sqrt{100 + 4 + 9} = \sqrt{113}$ units

ii. $\overrightarrow{DA}$ = Position Vector of A $-$ Position Vector of D

$= 7\hat{i} + 5\hat{j} + 8\hat{k} - 2\hat{i} - 3\hat{j} - 4\hat{k} = 5\hat{i} + 2\hat{j} + 4\hat{k}$

Unit vector in the direction of $\overrightarrow{DA} = \frac{5\hat{i}+2\hat{j}+4\hat{k}}{3\sqrt{5}}$

iii. a. $\overrightarrow{DV} = -5\hat{i} + 4\hat{j} + 7\hat{k}$

$\angle VDA = \cos^{-1}\left(\frac{\overrightarrow{DV} \cdot \overrightarrow{DA}}{|\overrightarrow{DV}||\overrightarrow{DA}|}\right) = \cos^{-1}\left(\frac{11\sqrt{2}}{90}\right)$

OR

b. $\overrightarrow{DV} = -5\hat{i} + 4\hat{j} + 7\hat{k}$

Projection of $\overrightarrow{DV}$ on $\overrightarrow{DA} = \left(\frac{\overrightarrow{DV} \cdot \overrightarrow{DA}}{|\overrightarrow{DA}|}\right) = \frac{11\sqrt{5}}{15}$

38. i. $f^{I}(x) = x^2 - 8x + 15 = (x - 3)(x - 5)$

$f^{I}(x) = 0 \Rightarrow x = 3, 5$ are the critical points.

ii. Now $f^{II}(x) = 2x - 8$

$f^{II}(3) < 0$ and $f^{II}(5) > 0$

so, mimimum value of f(x) is at x = 5.

min. value = $f(5) = \frac{5^3}{3} - 4(5)^2 + 15(5) + 2 = \frac{56}{3}$