Class XII Session 2026-27
Subject - Mathematics
Sample Question Paper - 3

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculators is not allowed.

Section A

1. If A is a 2×3 matrix such that AB and AB' both are defined, then order of the matrix B is **[1]**

   a) 3×2 b) 2×2

   c) 2×1 d) 3×3

2. If $P = \begin{vmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{vmatrix}$ is the adjoint of a 3×3 matrix A and $|A| = 4$, then α is equal to **[1]**

   a) 11 b) 4

   c) 5 d) 13

3. If $A = \begin{bmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{bmatrix}$ is singular then k =? **[1]**

a) $\frac{16}{3}$ b) $\frac{33}{2}$

c) $\frac{34}{5}$ d) $\frac{33}{4}$

4. Let f(x) = $\begin{cases} x\sin\frac{1}{x}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$. Then **[1]**

a) f(x) is discontinuous at x = 0 b) f(x) is continuous at x = 0

c) f(x) is not defined at x = 0 d) $\lim_{x\to 0} f(x)$ does not exist

5. The lines $\frac{1-x}{2} = \frac{y-1}{3} = \frac{z}{1}$ and $\frac{2x-3}{2p} = \frac{y}{-1} = \frac{z-4}{7}$ are perpendicular to each other for p equal to: **[1]**

a) 3 b) 2

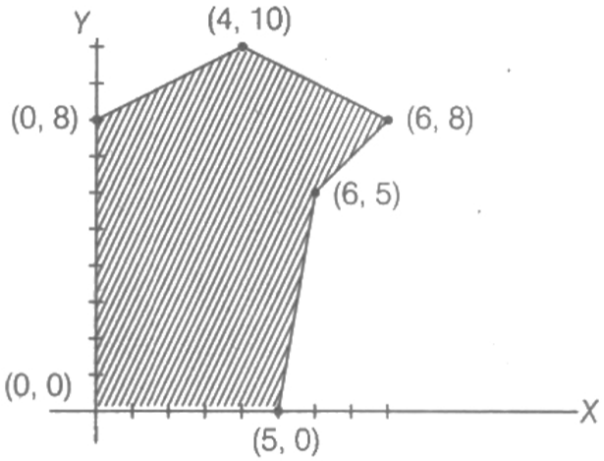
c) $-\frac{1}{2}$ d) $\frac{1}{2}$

6. The general solution of the DE $x^2\frac{dy}{dx} = x^2 + xy + y^2$ is **[1]**

a) $\tan^{-1}\frac{y}{x} = \log x + C$ b) $\tan^{-2}\frac{y}{x} = \log x + C$

c) $\tan^{1}\frac{y}{x} = \log y + C$ d) $\tan^{-1}\frac{x}{y} = \log x - C$

7. The feasible solution for a LPP is shown in following figure. Let Z = 3x - 4y be the objective function. Minimum of Z occurs at **[1]**



a) (4, 10) b) (0, 8)

c) (5, 0) d) (0, 0)

8. The unit vector perpendicular to the vectors $\hat{i} - \hat{j}$ and $\hat{i} + \hat{j}$ is **[1]**

a) $\frac{\hat{i}-\hat{j}}{\sqrt{2}}$ b) $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

c) $\hat{k}$ d) $-\hat{k} + \hat{j}$

9. $\int e^{x}\left(\frac{x\log x+1}{x}\right)dx$ is equal to **[1]**

a) $\frac{e^{x}}{x} + c$ b) log (e^x log x) + c

c) e^x log x + c d) x log x + e^x + c

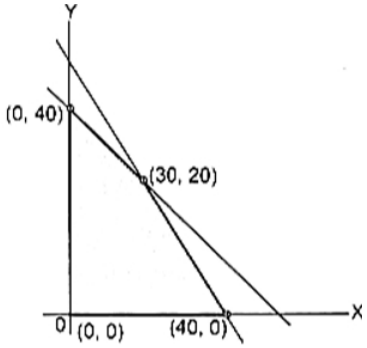
[1]

10. In the matrix equation $\begin{bmatrix} x+y+z \\ x+z \\ y+2z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$, the value of z is:

a) 2 b) -2

c) 1 d) -1

11. The maximum value of Z = 0.7x + y for feasible region given below is **[1]**



a) 50 b) 40

c) 45 d) 41

12. If $|\vec{a}| = \sqrt{2}$, $|\vec{b}| = \sqrt{3}$ and $|a+b| = \sqrt{6}$, then what is $|\vec{a}+\vec{b}|$ equal to? **[1]**

a) 2 b) 3

c) 4 d) 1

13. The value of det A where A= $\begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$ lies in the interval **[1]**

a) [0 ,2] b) [2,4]

c) (1,2) d) (2,1)

14. X and Y are independent events such that $P(X \cap \bar{Y}) = \frac{2}{5}$ and $P(X) = \frac{3}{5}$. Then P(Y) is equal to: **[1]**

a) $\frac{1}{5}$ b) $\frac{1}{3}$

c) $\frac{2}{5}$ d) $\frac{2}{3}$

15. The differential equation $\frac{dy}{dx} = F(x, y)$ will not be a homogeneous differential equation, if F(x, y) is: **[1]**

a) $\frac{y}{x}$ b) $\cos x - \sin\left(\frac{y}{x}\right)$

c) $\frac{x^2+y^2}{xy}$ d) $\cos^2\left(\frac{x}{y}\right)$

16. If θ is the angle between the vectors $2\hat{i} - 2\hat{j} + 4\hat{k}$ and $3\hat{i} + \hat{j} + 2\hat{k}$, then sin θ = **[1]**

a) $\frac{2}{3}$ b) $\sqrt{\frac{2}{7}}$

c) $\frac{2}{\sqrt{7}}$ d) $\frac{\sqrt{2}}{7}$

17. The relationship between a and b, so that the function f defined by **[1]**

f(x) = $\begin{cases} ax+1, & \text{if } x \leq 3 \\ bx+3, & \text{if } x > 3 \end{cases}$ is continuous at x = 3, is

a) a - b = $\frac{3}{2}$

b) a + b = 2

c) a = b + $\frac{2}{3}$

d) a + b = $\frac{2}{3}$

18. Find the shortest distance between the lines $\vec{r} = \hat{i} + 2\hat{j} + \hat{k}$ + $\lambda\left(\hat{i} - \hat{j} + \hat{k}.\right)$ $and\, \vec{r} = 2\hat{i} - \hat{j} - \hat{k}$ $+\mu\left(2\hat{i} + \hat{j} + \widehat{2k.}\right)$, $\lambda, \mu \in R$ **[1]**

a) $\frac{5\sqrt{2}}{2}$

b) $\frac{4\sqrt{2}}{2}$

c) $\frac{3\sqrt{2}}{5}$

d) $\frac{3\sqrt{2}}{2}$

19. **Assertion (A):** f(x) = $2x^3$ - $9x^2$ + 12x - 3 is increasing outside the interval (1, 2). **[1]**
Reason (R): f'(x) < 0 for x ∈ (1, 2).

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Let a relation R defined from set B to B such that B = {1, 2, 3, 4} and R = {1, 1), (2, 2), (3, 3), (1, 3), (3, 1)}, then R is transitive. **[1]**
Reason (R): A relation R in set A is called transitive, if (a, b) ∈ R and (b, c) ∈ R ⇒ (a, c) ∈ R, ∀ a, b, c ∈ A.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Write the interval for the principal value of function and draw its graph: $\cot^{-1}$ x. **[2]**

OR

Find the domain of $f(x) = \sin^{-1}\left(-x^2\right)$.

22. Find the maximum and minimum values, if any, of the following function given by f(x) = |sin 4x + 3| on R. **[2]**

23. A man is walking at the rate of 6.5 km/hr towards the foot of a tower 120 m high. At what rate is he approaching the top of the tower when he is 50 m away from the tower **[2]**

OR

Find the interval in which the function $f(x) = x^3 - 6x^2 + 9x + 15$ is increasing or decreasing.

24. Evaluate the Integral: $\int \frac{(2x-1)}{\sqrt{x^2-x-1}} dx$ **[2]**

25. The volume of a cube is increasing at the rate of 6 cm^3/s. How fast is the surface area of cube increasing, when the length of an edge is 8 cm? **[2]**

Section C

26. Evaluate: $\int_0^{\pi} x \sin^3 x dx$ **[3]**

OR

Evaluate: $\int_0^{\pi/2} \frac{\sin^2 x}{1+\sin x \cos x} dx$

27. Find $\int \frac{\sin^6 x}{\cos^8 x} dx$. **[3]**

28. Solve the following differential equation: **[3]**

xdy - (y - x^3)dx = 0

OR

Find the particular solution of the differential equation

$\frac{dy}{dx} - 2xy = 3x^2 e^{x^2}; y(0) = 5$

29. Show that the minimum of Z occurs at more than two points. **[3]**

Minimise and Maximise Z = x + 2y subject to

$x + 2y \geq 100, 2x - y \leq 0, 2x + y \leq 200$; $x,\ y \geq 0$.

OR

Solve the Linear Programming Problem graphically:

Maximize Z = 2x + 3y Subject to

x + y $\geq$ 1

10x + y $\geq$ 5

x + 10y $\geq$ 1

x, y $\geq$ 0

30. There are 3 bags, each containing 5 white balls and 3 black balls. Also there are 2 bags, each containing 2 white balls and 4 black balls. A white ball is drawn at random. Find the probability that this white ball is from a bag of the first group. **[3]**

31. Find all points of discontinuity of f where f is defined as follows,f(x) = **[3]**

$$\begin{cases} |x| + 3, x \leq -3 \\ -2x, -3 < x < 3 \\ 6x + 2, x \geq 3 \end{cases}$$

Section D

32. Find the area enclosed by the parabola 4y = $3x^2$ and the line 2y = 3x + 12 **[5]**

33. Show that the function f : R - {3} $\rightarrow$ R - {1} given by $f(x) = \frac{x-2}{x-3}$ is a bijection. **[5]**

OR

Let A = R – {3} and B = R – {1}. Consider the function f: A $\Rightarrow$B defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

34. Find the co-ordinates of the foot of the perpendicular drawn from the point $(2, 3, -8)$ to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$. **[5]**

Also, find the perpendicular distance of the given point from the line.

OR

Find the shortest distance between the following lines: $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$.

35. Show that the matrix, $A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$ satisfies the equation, $A^3 - A^2 - 3A - I_3 = O$. Hence, find A^{-1} **[5]**

Section E

36. Some students are having a misconception while comparing decimals. For example, a student may mention that $78.56 > 78.9$ as $7856 > 789$. In order to assess this concept, a decimal comparison test was administered to the students of class VI through the following question : In the recently held Sports Day in the school, 5 students participated in a javelin throw competition. The distances to which they have thrown the javelin are shown below in the table : **[4]**

Name of student	Distance of javelin (in meters)
Ajay	47.7
Bijoy	47.07
Kartik	43.09
Dinesh	43. 9
Devesh	45.2

The students were asked to identify who has thrown the javelin the farthest.

Based on the test attempted by the students, the teacher concludes that 40% of the students have the misconception in the concept of decimal comparison and the rest do not have the misconception. 80% of the students having misconception answered Bijoy as the correct answer in the paper. 90% of the students who are identified with not having misconception, did not answer Bijoy as their answer.

On the basis of the above information, answer the following questions :

i. What is the probability of a student not having misconception but still answers Bijoy in the test? **(1)**

ii. What is the probability that a randomly selected student answers Bijoy as his answer in the test? **(1)**

iii. a. What is the probability that a student who answered as Bijoy is having misconception? **(2)**

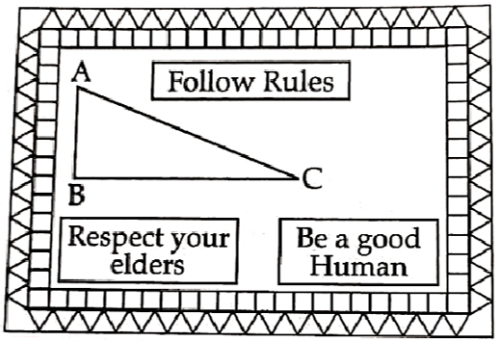
OR

b. What is the probability that a student who answered as Bijoy is amongst students who do not have the misconception? **(2)**

37. **Read the following text carefully and answer the questions that follow:** **[4]**

The slogans on chart papers are to be placed on a school bulletin board at the points A, B and C displaying A (follow Rules), B (Respect your elders) and C (Be a good human). The coordinates of these points are (1, 4, 2), (3, -3, -2) and (-2, 2, 6),

respectively.



i. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ be the position vectors of points A, B, C, respectively, then find $|\vec{a}+\vec{b}+\vec{c}|$. (1)

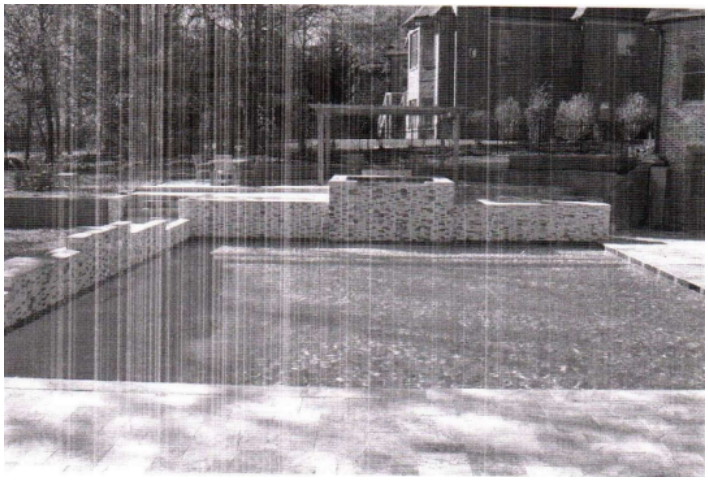
ii. If $\vec{a} = 4\hat{i} + 6\hat{j} + 12\hat{k}$, then find the unit vector in direction of $\vec{a}$. (1)

iii. Find area of $\triangle$ABC. (2)

OR

Write the triangle law of addition for $\triangle$ABC. Suppose, if the given slogans are to be placed on a straight line, then the value of $|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$. (2)

38. A housing society wants to commission a swimming pool for its residents. For this, they have to purchase a square piece of land and dig this to such a depth that its capacity is 250 cubic metres. Cost of land is ₹ 500 per square metre. The cost of digging increases with the depth and cost for the whole pool is ₹ 4000 $(\text{depth})^2$. **[4]**



Suppose the side of the square plot is x metres and depth is h metres.

On the basis of the above information, answer the following questions:

i. Write cost C(h) as a function in terms of h. **(1)**

ii. Find critical point. **(1)**

iii. Use second derivative test to find the value of h for which cost of constructing the pool is minimum. What is the minimum cost of construction of the pool? **(2)**

OR

iii. Use first derivative test to find the depth of the pool so that cost of construction is minimum. Also, find relation between x and h for minimum cost. **(2)**

Solution

Section A

1.

(d) 3×3

Explanation:

3×3

2. **(a)** 11

Explanation:

11

3.

(b) $\frac{33}{2}$

Explanation:

When a given matrix is singular then the given matrix determinant is 0.

|A| = 0

Given,

$$A = \begin{pmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{pmatrix}$$

|A| = 0

1(-4k + 6) -k(-12 + 4) + 3(9 - 2k) = 0

-4k + 6 + 12k - 4k + 27 - 6k = 0

-2k + 33 = 0

k = $\frac{33}{2}$.

Which is the required solution.

4.

(b) f(x) is continuous at x = 0

Explanation:

$$Given, f(x) = \begin{cases} x \sin \frac{1}{x}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$$

At x = 0, f(0) = 0

LHL = $\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h)$

= $\lim_{h \to 0} f(-h) = \lim_{h \to 0} (-h) \sin\left(\frac{1}{-h}\right)$

= $\lim_{h \to 0} h \sin \frac{1}{h}$ |$\because$ sin -$(-\theta)$ = - sin θ|

=0 $\times$(An oscillating value lies between - 1 and 1) = 0

RHL = $\lim_{x \to 0^+} f(x) = \lim_{h \to 0}$ f(0 + h)

= $\lim_{h \to 0} f(h) = \lim_{h \to 0} h \sin\left(\frac{1}{h}\right)$

= 0 $\times$ An oscillating value lies between -1 and 1) = 0

Here, LHL= RHL = 0

Hence, f(x) is continuous at x = 0.

5.

(b) 2

Explanation:

2

6. **(a)** $\tan^{-1}\frac{y}{x} = \log x + C$

Explanation:

We have, $x^2\frac{dy}{dx} = x^2 + xy + y^2$

$\frac{dy}{dx} = 1 + \frac{y}{x} + \frac{y^2}{x^2}$

Let y = vx

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$1 + v + v^2 = v + x\frac{dv}{dx}$

$1 + v^2 = x\frac{dv}{dx}$

$\frac{dx}{x} = \frac{dv}{v^2+1}$

On integrating on both sides, we obtain

$\log x = \tan^{-1}v + C$

$\tan^{-1}\frac{y}{x} = \log x + c$

7.

(b) (0, 8)

Explanation:

Corner points	**Corresponding value of Z = 3x - 4y**
(0, 0)	0
(5, 0)	15
(6, 5)	-2
(6, 8)	-14
(4, 10)	-28
(0, 8)	-32 ← Minimum

Hence, the minimum of Z occurs at (0, 8) and its minimum value is (-32).

8.

(c) $\hat{k}$

Explanation:

$\hat{k}$

9.

(c) $e^x \log x + c$

Explanation:

$\int e^x \left(\frac{x\log x+1}{x}\right) dx$

$\int e^x \left(\log x + \frac{1}{x}\right) dx$

$\because \int e^x \left[f(x) + f'(x)\right] dx = e^x f(x)$

$= e^x \log x + c$

10.

(d) -1

Explanation:

$$\begin{bmatrix} x+y+z \\ x+z \\ y+2z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$$

$\Rightarrow x+y+z=3$ ...(i)

$x+z=2$ ...(ii)

$y+2z=-1$ ...(iii)

from (i) and (ii)

$x + y + z = 3$

$-x \pm z = -2$

$\boxed{y = +1}$

Now put the value of y in eq. (iii) we get

$1+2z=-1$

$2z=-2$

$z=-1$

11.

(d) 41

Explanation:

Corner Point	Z = 0.7x + y
(0, 0)	$0.7 \times 0 + 0 = 0$
(40, 0)	$0.7 \times 40 + 0 = 28$
(30, 20)	$0.7 \times 30 + 20 = 41 \longleftarrow$ Maximum
(0, 40)	$0.7 \times 0 + 40 = 40$

12. **(a)** 2

Explanation:

2

13.

(b) [2,4]

Explanation:

$$A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$$

Apply, $R_1 \rightarrow R_1 + R_3$

$$|A| = \begin{vmatrix} 0 & 0 & 2 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$$

now expanding along R_1

$|A| = 2(\sin^2\theta + 1)$

now,
$-1 \leq \sin\theta \leq 1$
$\Rightarrow 0 \leq \sin^2\theta \leq 1$
$\Rightarrow 1 \leq \sin^2\theta + 1 \leq 2$
$\Rightarrow 2 \leq 2(\sin^2\theta + 1) \leq 4$
$\Rightarrow 2 \leq |A| \leq 4$
So, |A| lies on [2,4]

14.

(b) $\frac{1}{3}$
Explanation:
$\frac{1}{3}$

15.

(b) $\cos x - \sin\left(\frac{y}{x}\right)$
Explanation:
(i.) $f(x, y) = \cos x - \sin\left(\frac{y}{x}\right)$
Replace $x \to \lambda x y \to \lambda y$.
$f(\lambda x, \lambda y) = \cos\lambda x - \sin\left(\frac{\lambda y}{\lambda x}\right)$
$= \cos\lambda x - \sin\left(\frac{y}{x}\right)$
$\neq \lambda^n f(\Delta x, y)$
$\therefore$ Not Homogenous.
(ii)
$f(x, y) = \frac{y}{x}$
$f(\lambda x, \lambda y) = \frac{\lambda y}{\lambda x} = \lambda^0 \frac{y}{x} = \lambda^0 f(x, y)$

Hence it is homog.
(iii) $f(x, y) = \frac{x^2 + y^2}{xy}$
$f(\lambda x, \lambda y) = \frac{\lambda^2 x^2 + \lambda^2 y^2}{\lambda x \lambda y}$
$= \lambda^0 \frac{x^2 + y^2}{xy} = \lambda^0 f(x, y)$
Hence homog.
(r) $\cos^2\left(\frac{x}{y}\right)$
$f(\lambda x, \lambda y) = \cos^2\left(\frac{\lambda x}{\lambda y}\right) = \lambda^0 \cos^2\left(\frac{x}{y}\right)$
Hence homogeneous

16.

(c) $\frac{2}{\sqrt{7}}$
Explanation:
Let:
$\vec{a} = 2\hat{i} - 2\hat{j} + 4\hat{k}$
$\vec{b} = 3\hat{i} + \hat{j} + 2\hat{k}$
$|\vec{a}| = \sqrt{2^2 + (-2)^2 + 4^2}$

$= \sqrt{4+4+16}$

$= \sqrt{24}$

$= 2\sqrt{6}$

$|\vec{b}| = \sqrt{3^2+1^2+2^2}$

$= \sqrt{9+1+4}$

$= \sqrt{14}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 4 \\ 3 & 1 & 2 \end{vmatrix}$$

$= -8\hat{i} + 8\hat{j} + 8\hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{64+64+64}$

$= \sqrt{192}$

$= 8\sqrt{3}$

Let θ be the angle between $\vec{a}$ and $\vec{b}$.

$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$

$\Rightarrow 8\sqrt{3} = (2\sqrt{6})(\sqrt{14}) \sin\theta$

$\Rightarrow \sin\theta = \frac{8\sqrt{3}}{4\sqrt{21}}$

$= \frac{2}{\sqrt{7}}$

$\Rightarrow \theta = \sin^{-1}\left(\frac{2}{\sqrt{7}}\right)$

17.

(c) a = b + $\frac{2}{3}$

Explanation:

Here, f(x) = $\begin{cases} ax+1, & \text{if } x \leq 3 \\ bx+3, & \text{if } x > 3 \end{cases}$

LHL = $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (ax+1)$

Putting x = 3 - h as x $\to 3^-$, h $\to 0$

$\therefore \lim_{h \to 0}$[a(3 - h) + 1] = $\lim_{h \to 0}$ (3a - ah + 1) = 3a +1

RHL = $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+}$ (bx + 3)

Putting x = 3 + h as x $\to 3^+$, h $\to 0$

$\therefore \lim_{h \to 0}$[b(3 + h) + 3] = $\lim_{h \to 0}$ (3b + bh + 3) = 3b + 3

Also, f(3) = 3a + 1 [$\because$ f(x) = ax + 1]

Since, f(x) is continuous at x = 3.

$\therefore$ LHL = RHL = f(3)

$\Rightarrow$ 3a + 1 = 3b + 3

$\Rightarrow$ 3a = 3b + 2

$\Rightarrow$ a = b + $\frac{2}{3}$

18.

(d) $\frac{3\sqrt{2}}{2}$

Explanation:

On comparing the given equations with: $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$, and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$, we get:

$\vec{a_1} = \hat{i} + 2\hat{j} + \hat{k}, \vec{b_1} = \hat{i} - \hat{j} + \hat{k}$, and $\vec{a_2} = 2\hat{i} - \hat{j} - \hat{k}, \vec{b_2} = 2\hat{i} + \hat{j} - 2\hat{k}$

$\therefore S.D. = \left|\frac{(\vec{b_1} \times \vec{b_2}).(\vec{a_2} - \vec{a_1})}{\left|\vec{b_1} \times \vec{b_2}\right|}\right| = \left|\frac{(-3\hat{i}+3\hat{k}).(\hat{i}-3\hat{j}-2\hat{k})}{3\sqrt{2}}\right|$

$= \left|\frac{-3+-6}{3\sqrt{2}}\right| = \left|\frac{-9}{3\sqrt{2}}\right| = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

Assertion: We have, $f(x) = 2x^3 - 9x^2 + 12x - 3$

$\Rightarrow f'(x) = 6x^2 - 18x + 12$

For increasing function, $f'(x) \geq 0$

$\therefore 6(x^2 - 3x + 2) \geq 0$

$\Rightarrow 6(x - 2)(x - 1) \geq 0$

$\Rightarrow x \leq 1$ and $x \geq 2$

$\therefore f(x)$ is increasing outside the interval (1, 2), therefore it is a true statement.

Reason: Now, $f'(x) < 0$

$\Rightarrow 6(x - 2)(x - 1) < 0$

$\Rightarrow 1 < x < 2$

$\therefore$ Assertion and Reason are both true but Reason is not the correct explanation of Assertion.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Assertion: We have, B = {1, 2, 3, 4}

and R = {(1, 1), (2, 2), (3, 3), (1, 3), (3, 1)}

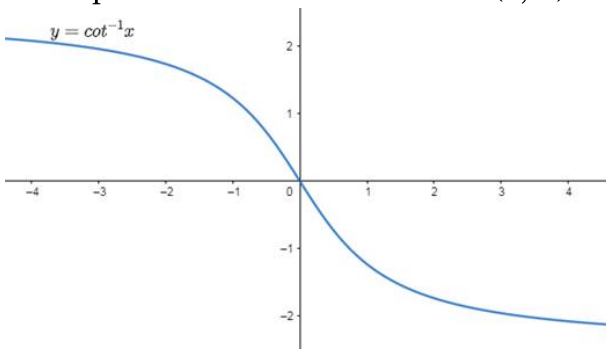
Here, (1, 3), (3, 1) $\in$ R

$\Rightarrow$ (1, 1) $\in$ R

Hence, R is transitive.

Section B

21. Principal value branch of $\cot^{-1} x$ is $(0, \pi)$ and its graph is shown below.



OR

$-1 \leq -x^2 \leq 1 \Rightarrow -1 \leq -x^2 \leq 0$

$\Rightarrow 0 \leq x^2 \leq 1 \Rightarrow -1 \leq x \leq 1$

22. Given: $f(x) = |\sin 4x + 3|$

We know that $-1 \leq \sin 4x \leq 1$

$\Rightarrow 2 \leq \sin 4x + 3 \leq 4$

$\Rightarrow 2 \leq |\sin 4x + 3| \leq 4$

Hence, the maximum and minimum values of **f** are 4 and 2 respectively.

23. Let at any time t, the man be at distances of x and y metres from the foot and top of the tower respectively. Then,

$y^2 = x^2 + (120)^2$ ...(i)

$\Rightarrow \quad 2y\frac{dy}{dt} = 2x\frac{dx}{dt}$

$\Rightarrow \quad \frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt}$

given: $\frac{dx}{dt} = -6 \cdot 5\text{km/hr}$ negative sign due to decreasing,

therefore;

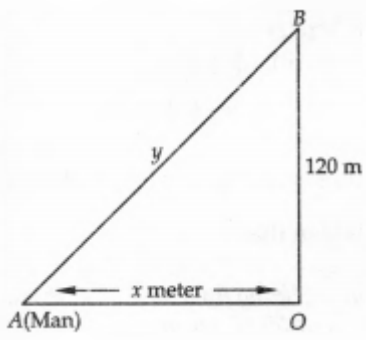
$\frac{dy}{dt} = -\frac{6 \cdot 5x}{y}$ ... (ii)

Putting x = 50 in (i) we get $y = \sqrt{50^2 + 120^2} = 130$

Putting x = 50, y = 130 in (ii), we get

$\frac{dy}{dt} = -\frac{6 \cdot 5 \times 50}{130} = -2 \cdot 5$

Thus, the man is approaching the top of the tower at the rate of 2.5 km/hr.



OR

Given: $f(x) = x^3 - 6x^2 + 9x + 15$

$\Rightarrow f(x) = \frac{d}{dx}\left(x^3 - 6x^2 + 9x + 15\right)$

$\Rightarrow f'(x) = 3x^2 - 12x + 9$

For f(x) lets find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow 3x^2 - 12x + 9 = 0$

$\Rightarrow 3(x^2 - 4x + 3) = 0$

$\Rightarrow 3(x^2 - 3x - x + 3) = 0$

$\Rightarrow x^2 - 3x - x + 3 = 0$

$\Rightarrow (x - 3)(x - 1) = 0$

$\Rightarrow x = 1, 3$

clearly, $f'(x) > 0$ if $x < 1$ and $x > 3$ and $f'(x) < 0$ if $1 < x < 3$

Thus, f(x) increases on the interval $(-\infty, 1) \cup (3, \infty)$

and f(x) is decreasing on interval $x \in (1, 3)$

24. Let $I = \int \frac{(2x-1)}{\sqrt{x^2-x-1}}dx$

Since $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int \frac{\sin x}{(1+\cos x)^2}dx$..... (i)

Let $x^2 - x - 1 = t$

$\Rightarrow \frac{d(x^2-x-1)}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{d(x^2)}{dx} - \frac{d(x)}{dx} - \frac{d(1)}{dx} = \frac{dt}{dx}$

$\Rightarrow (2x-1) = \frac{dt}{dx}$

$\Rightarrow$ (2X-1) dx = dt

Putting this value in equation (i), we get

$I = \int \frac{dt}{t^{\frac{1}{2}}} \left[x^2 - x - 1 = t\right]$

$\Rightarrow \mathrm{I} = \frac{\mathrm{t}^{\frac{1}{2}}}{\frac{1}{2}} + \mathrm{c}$

$\Rightarrow \mathrm{I} = \frac{2\sqrt{\mathrm{t}}}{1} + \mathrm{c}$

$I = \frac{2\sqrt{x^2-x-1}}{1} + c$

25. Given, $\frac{dV}{dt} = 6\ cm^3/sec$. Since, $V = x^3$

$\frac{dV}{dt} = 3x^2\frac{dx}{dt} \Rightarrow 6 = 3x^2\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{2}{x^2}\ cm/sec$

Now, Surface Area $= S = 6x^2 \Rightarrow \frac{dS}{dt} = 12x\frac{dx}{dt} = 3\ cm^2/sec$

Section C

26. Let $I = \int_0^\pi x\sin^3 x dx$

$= \int_0^\pi (\pi - x)\sin^3(\pi - x)dx$

By using property of definite integrals $\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$

$= \int_0^\pi \pi\sin^3 x dx - \int_0^\pi x\sin^3 x dx$

$\therefore I = \int_0^x \pi\sin^3 x dx - l$

$\Rightarrow 2I = \pi\int_0^\pi \sin^3 x dx$

$\Rightarrow 2I = \pi\int_0^x \frac{3\sin x - \sin 3x}{4}dx$

$= \frac{\pi}{4}\int_0^\pi (3\sin x - \sin 3x)dx$

$= \frac{\pi}{4}\left[-3\cos x + \frac{\cos 3x}{3}\right]_0^x$

$= \frac{\pi}{4}\left[\left(-3\cos\pi + \frac{\cos 3\pi}{3}\right) - \left(-3\cos 0 + \frac{\cos 0}{3}\right)\right]$

$= \frac{\pi}{4}\left[\left(3 - \frac{1}{3}\right) - \left(-3 + \frac{1}{3}\right)\right]$

$= \frac{\pi}{4}\left[3 - \frac{1}{3} + 3 - \frac{1}{3}\right]$

$= \frac{\pi}{4}\left[6 - \frac{2}{3}\right]$

$= \frac{\pi}{4} \times \frac{16}{3} = \frac{4\pi}{3}$

$\therefore I = \frac{2\pi}{3}$

OR

Let the given integral be,

$I = \int_0^{\pi/2} \frac{\sin^2 x}{1+\sin x\cos x}dx$ ...(i)

$I = \int_0^{\pi/2} \frac{\sin^2\left(\frac{\pi}{2} - x\right)}{1+\sin\left(\frac{\pi}{2} - x\right)\cos\left(\frac{\pi}{2} - x\right)}dx \left[\text{ Using } : \int_0^a f(x)dx = \int_0^a f(a-x)\right]$

$\Rightarrow \quad I = \int_0^{\pi/2} \frac{\cos^2 x}{1+\cos x\sin x}dx$ ...(ii)

Adding (i) and (ii), we get

$2I = \int_0^{\pi/2} \frac{\sin^2 x + \cos^2 x}{1+\sin x\cos x}dx = \int_0^{\pi/2} \frac{1}{1+\sin x\cos x}dx$

$\Rightarrow \quad 2I = \int_0^{\pi/2} \frac{\sec^2 x}{\sec^2 x + \tan x}dx$ [Dividing numerator and denominator by $\cos^2$ x]

Let $\tan x = t$. Then, $d(\tan x) = dt \Rightarrow \sec^2 x dx = dt$

Also, $x = 0 \Rightarrow t = \tan 0 = 0$ and $x = \frac{\pi}{2} \Rightarrow t = \tan\frac{\pi}{2} = \infty$

$\therefore \quad 2I = \int_0^\infty \frac{dt}{1+t^2+t}$

$\Rightarrow \quad 2I = \int_0^\infty \frac{dt}{(t+1/2)^2+(\sqrt{3/2})^2} dt = \frac{1}{\sqrt{3}/2}\left[\tan^{-1}\left(\frac{t+1/2}{\sqrt{3/2}}\right)\right]_0^\infty$

$\Rightarrow \quad 2I = \frac{2}{\sqrt{3}}\left[\tan^{-1}\left(\frac{2t+1}{\sqrt{3}}\right)\right]_0^\infty = \frac{2}{\sqrt{3}}\left[\tan^{-1}\infty - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right] = \frac{2}{\sqrt{3}}\left[\frac{\pi}{2} - \frac{\pi}{6}\right] = \frac{2\pi}{3\sqrt{3}}$

$\Rightarrow \quad I = \frac{\pi}{3\sqrt{3}}$

27. We have to find the value of $\int \frac{\sin^6 x}{\cos^8 x} dx$.

Let $I = \int \frac{\sin^6 x}{\cos^8 x} dx = \int \tan^6 x \sec^2 x dx$

Put $\tan x = t \Rightarrow \sec^2 x dx = dt$

$\therefore I = \int t^6 dt = \frac{t^7}{7} + C = \frac{\tan^7 x}{7} + C[\text{ put } t = \tan x]$

28. xdy - (y - x^3)dx = 0

This can be written as

xdy = (y - x^3)dx

Divide throughout by x,

$\frac{dy}{dx}$ = $\frac{y}{x}$ – x^2

$\frac{dy}{dx}$ - $\frac{y}{x}$ = -x^2

This is a linear differential equation of the form,

$\frac{dy}{dx}$ + Py = Q

The integrating factor I.F is

$e^{\int Pdx}$ = $e^{\int \frac{-1}{x} dx}$ dx = $e^{-\log x}$ = $e^{\log(\frac{1}{x})}$ = $\frac{1}{x}$

The required solution is

$ye^{\int Pdx} = \int Qe^{\int Pdx}.dx + c$

$y \cdot \left(\frac{1}{x}\right) = -\int x^2 \times \frac{1}{x} dx + c$

$\frac{y}{x}$ = -$\int x dx + c$

$\frac{y}{x} = \frac{-x^2}{2}$ + c

$\frac{y}{x} + \frac{x^2}{2}$ = c

2y + x^3 = 2cx

$\Rightarrow$ x^3 – 2cx + 2y = 0 is the required solution.

OR

Given differential equation is a linear order differential equation with:

$P = -2x, Q = 3x^2 e^{x^2}$

$\text{Integrating Factor } = e^{\int -2x dx} = e^{-x^2}$

The general solution is: $y \cdot e^{-x^2} = \int e^{-x^2} \cdot 3x^2 e^{x^2} dx + C \Rightarrow y \cdot e^{-x^2} = x^3 + C$

Putting $x = 0, y = 5$, we get, $C = 5$

$\therefore$ The Particular solution is: $y \cdot e^{-x^2} = x^3 + 5$ or $y = \left(x^3 + 5\right) e^{x^2}$

29. We have linear constraints as

$x + 2y \geq 100$

$2x - y \leq 0$

$2x + y \leq 200$

$x \geq 0, y \geq 0$

and objective function is min or max $(Z) = x + 2y$

Now, reducing the above inequations into equations and finding their point of intersections i.e.,

$x + 2y = 100$ ...(i)

$2x - y = 0$ ...(ii)

$2x + y = 200$ ...(iii)

$x = 0, y = 0$...(iv)

Equations	Point of Intersection
(i) and (ii)	$x = 20$ and $y = 40$
	$\Rightarrow$ point is$(20, 40)$
(i) and (iii)	$x = 100$ and $y = 0$
	$\Rightarrow$ point is $(100, 0)$
(i) and (iv)	when$x = 0 \Rightarrow y = 50$
	$\Rightarrow$ point is $(0, 50)$
	when $y = 0 \Rightarrow x = 100$
	$\Rightarrow$ point $(100, 0)$
(i) and (iii)	$x = 50, y = 100$
	$\Rightarrow$ point is $(50, 100)$
(i) and (iv)	$x = 0, y = 0$
	$\Rightarrow$ point is $(0, 0)$
(i) and (iv)	when x = 0 $\Rightarrow$ y = 200
	$\Rightarrow$ point is (0, 200)
	when $y = 0, x = 100$
	$\Rightarrow$ point is $(100, 0)$

Now for feasible region, using origin testing method for each constraint

For $x + 2y \geq 100$, let $x = 0, y = 0$

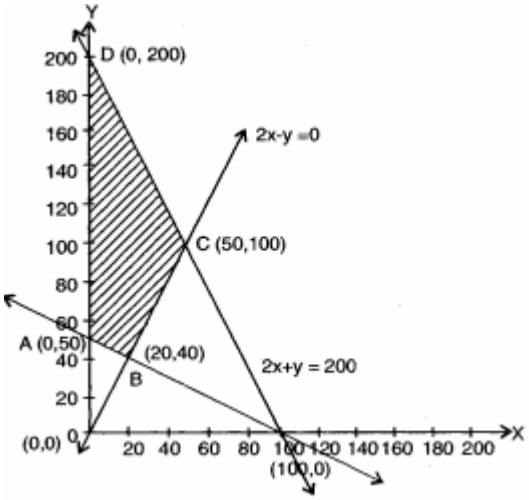
$\Rightarrow 0 \geq 100$ i.e., true $\Rightarrow$ the shaded region will be away from the origin

For $2x + y \leq 200$, let $x = 0, y = 0$

$\Rightarrow 0 \leq 200$ i.e., true $\Rightarrow$ The shaded region will be toward the origin.

Also, non-negative restrictions $x \geq 0, y \geq 0$ indicates that the feasible region will be exist in first quadrant.

Now, corner points are $A(0, 50), B(20, 40), C(50, 100)\ and\ D(0, 200)$



For optimal solution substituting the value of all comer points in $Z = x + 2y$

Corner points	

	$Z = x + 2y$	
$A(0, 50)$	$Z = 0 + 2 \times 50 = 100$	Minimum
$B(20, 40)$	$Z = 20 + 2 \times 40 = 100$	
$C(50, 100)$	$Z = 50 + 2 \times 100 = 250$	
$D(0, 200)$	$Z = 0 + 2 \times 200 = 400$	$\rightarrow$ Maximum

Hence, minimum (Z) = 100 at all points on the line segment joining the points A(0, 50) and B(20, 40) : Maximum (Z) = 400 at (0, 200)

OR

First, we will convert the given inequations into equations, we obtain the following equations and solving then we get the

x + y = 1,10x + y = 5, x + 10y = 1, x = 0 and y = 0

Region represented by x + y $\geq$ 1: The line x + y = 1 meets the coordinate axes at A(1,0) and B(0,1) respectively. By joining these points we obtain the line x + y = 1 Clearly (0,0) does not satisfies the inequation x + y $\geq$ 1 . So, the region in x y plane which does not contain the origin represents the solution set of the inequation x + y $\geq$ 1 Region represented by 10x + y $\geq$ 5 :

The line 10x + y = 5 meets the coordinate axes at $C\left(\frac{1}{2}, 0\right)$ and D(0,5) respectively. By joining these points we obtain the

line

10x + y = 5 . Clearly (0,0) does not satisfies the inequation 10x + y $\geq$ 5 . So, the region which does not contains the origin represents the solution set of the inequation 10x + y $\geq$ 5

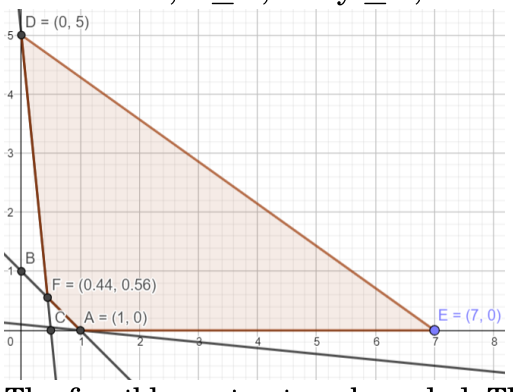
Region represented by x + 10 y $\geq$ 1:

The line x + 10 y = 1 meets the coordinate axes at A(1,0) and $F\left(0, \frac{1}{10}\right)$ respectively. By joining these points we obtain

the line

x + 10 y = 1 . Clearly (0,0) does not satisfies the inequation x + 10 y $\geq$ 1 . So, the region which does not contains the origin represents the solution set of the inequation x + 10 y $\geq$ 1 Region represented by x $\geq$ 0 and y $\geq$ 0 :

since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations x $\geq$ 0, and y $\geq$ 0 The feasible region determined by subject to the constraints are x + y $\geq$ 1,10 x + y $\geq$ 5, x + 10 y $\geq$ 1,and the non-negative restrictions, x $\geq$ 0, and y $\geq$ 0, are as follows.



The feasible region is unbounded. Therefore, the maximum value of objective function is infinity i.e. the solution is unbounded.

30. Let E_1, E_2 and A be the events defined as follows:

E_1 = Selecting a bag from the first group, E_2 = Selecting a bag from the second group and, A = Ball drawn is white

Since there are 5 bags out of which 3 bags belong to first group and 2 bags to second group. Therefore,we have,

$P(E_1) = \frac{3}{5}$, $P(E_2) = \frac{2}{5}$

If E_1 has already occurred, then a bag from the first group is chosen. The bag chosen contains 5 white balls and 3 black balls. Therefore, the probability of drawing a white ball from it is $\frac{5}{8}$.

$\therefore P(\frac{A}{E_1}) = \frac{5}{8}$

Similarly, $P(\frac{A}{E_2}) = \frac{2}{6} = \frac{1}{3}$.

We have to find $P(\frac{E_1}{M})$, i.e. given that the ball drawn is white, what is the probability that it is drawn from a bag of the first group.

By Baye's theorem, required probability is given by,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)} = \frac{\frac{3}{5}\times\frac{5}{8}}{\frac{3}{5}\times\frac{5}{8}+\frac{2}{5}\times\frac{1}{3}} = \frac{45}{61}$$

31. Given function is

$$f(x) = \begin{cases} |x|+3, & x \le -3 \\ -2x, & -3 < x < 3 \\ 6x+2, & x \ge 3 \end{cases} = \begin{cases} -x+3, x \le -3 \\ -2x, -3 < x < 3 \\ 6x+2, x \ge 3 \end{cases}$$

First, we verify continuity at x = - 3 and then at x = 3

Continuity at x = -3

LHL = $\lim\limits_{x\to(-3)^-} f(x) = \lim\limits_{x\to(-3)^-} (-x+3)$

$\Rightarrow \quad \text{LHL} = \lim\limits_{h\to 0}[-(-3-h)+3]$

$= \lim\limits_{h\to 0}(3+h+3)$

= 3 + 3 = 6

and RHL = $\lim\limits_{x\to(-3)^+} f(x) = \lim\limits_{x\to(-3)^+} (-2x)$

$\Rightarrow \quad RHL = \lim\limits_{h\to 0}[-2(-3+h)]$

$= \lim\limits_{h\to 0}(6-2h)$

$\Rightarrow$ RHL = 6

Also, f(-3) = value of f(x) at x = - 3

=-(-3) + 3

= 3 + 3 = 6

$\because$ LHL = RHL = f(-3)

$\therefore$f(x) is continuous at x = - 3 So, x = - 3 is the point of continuity.

Continuity at x = 3

LHL = $\lim\limits_{x\to 3^-} f(x) = \lim\limits_{x\to 3^-} [-(2x)]$

$\Rightarrow \quad LHL = \lim\limits_{h\to 0}[-2(3-h)]$

$= \lim\limits_{h\to 0}(-6+2h)$

$\Rightarrow$ LHL = -6

and RHL = $\lim\limits_{x\to 3^+} f(x) = \lim\limits_{x\to 3^+} (6x+2)$

$\Rightarrow \quad \text{RHL} = \lim\limits_{h\to 0}[6(3+h)+2]$

$\Rightarrow$ RHL = 20

$\because \quad \text{LHL} \neq \text{RHL}$

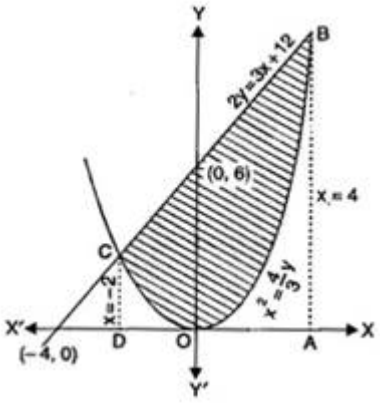
$\therefore$ f is discontinuous at x = 3 Now, as f (x) is a polynomial function for x < -3 , -3< x < 3 and x > 3 so it is continuous in these intervals.

Hence, only x = 3 is the point of discontinuity of f(x).

Section D

32. Equation of the parabola is

$4y = 3x^2$ ...(i)



$\Rightarrow x^2 = \frac{4}{3}y$

Equation of the line is 2y = 3x + 12 ...(ii)

$\Rightarrow y = \frac{3x+12}{2} = \frac{3x}{2} + 6$

In the graph, points of intersection are B (4, 12) and C (-2, 3).

Now, Area ABCD $= \left| \int_{-2}^{4} \left(\frac{3}{2}x + 6\right) dx \right|$

$= \left[\frac{3}{4}x^2 + 6x\right]_{-2}^{4}$

= (12 + 24) - (3 - 12)

= 45 sq units

Again, Area CDO + Area OAB $= \left| \int_{-2}^{4} \left(\frac{3}{4}x^2\right) dx \right|$

$= \left[\frac{3}{4} \cdot \frac{x^3}{3}\right]_{-2}^{4}$

$= \frac{1}{4}[64 - (-8)] = 18$ sq. units

$\therefore$ Required area = Area ABCD - (Area CDO + Area OAB)

= 45 - 18 = 27 sq. units

33. A = R - {3}, B = R - {1}

f : A $\rightarrow$ B is defined as $f(x) = \left(\frac{x-2}{x-3}\right)$.

Let x, y $\in$ A such that f(x) = f(y).

$\Rightarrow \frac{x-2}{x-3} = \frac{y-2}{y-3}$

$\Rightarrow$ (x - 2) (y - 3) = (y - 2) (x - 3)

$\Rightarrow$ xy - 3x - 2y + 6 = xy - 3y - 2x + 6

$\Rightarrow$ -3x - 2y = -3y - 2x

$\Rightarrow$ 3x - 2x = 3y - 2y

$\Rightarrow$ x = y

Therefore, f is one-one.

Let y $\in$ B = R - {1}

Then, y $\neq$ 1.

The function f is onto if there exists x $\in$ A such that f(x) = y.

Now, f (x) = y

$\Rightarrow \frac{x-2}{x-3} = y$

$\Rightarrow$ x - 2 = xy - 3y

$\Rightarrow$ x(1 - y) = -3y + 2

$\Rightarrow x = \frac{2-3y}{1-y} \in \mathbf{A} \qquad [y \neq 1]$

Thus, for any y $\in$ B, there exists $\frac{2-3y}{1-y} \in$ A such that

$f\left(\frac{2-3y}{1-y}\right) = \frac{\left(\frac{2-3y}{1-y}\right)-2}{\left(\frac{2-3y}{1-y}\right)-3} = \frac{2-3y-2+2y}{2-3y-3+3y} = \frac{-y}{-1} = y$

$\therefore$ f is onto.

Hence, function f is one-one and onto.

OR

A = R – {3} and B = R – {1} and $f(x) = \left(\frac{x-2}{x-3}\right)$

Let $x_1, x_2 \in$ A, then $f(x_1) = \frac{x_1-2}{x_1-3}$ and $f(x_2) = \frac{x_2-2}{x_2-3}$

Now, for $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-3}{x_2-3}$

$\Rightarrow (x_1-2)(x_2-3) = (x_2-2)(x_1-3)$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow -3x_1 - 2x_2 = -2x_1 - 3x_2$

$= x_1 = x_2$

$\therefore f$ is one-one function.

Now $y = \frac{x-2}{x-3}$

$\Rightarrow y(x-3) = x-2$

$\Rightarrow xy - 3y = x - 2$

$\Rightarrow x(y-1) = 3y - 2$

$\Rightarrow x = \frac{3y-2}{y-1}$

$\therefore f\left(\frac{3y-2}{y-1}\right) = \frac{\frac{3y-2}{y-1}-2}{\frac{3y-2}{y-1}-3} = \frac{3y-2-2y+2}{2y-2-3y+3} = y$

$\Rightarrow f(x) = y$

Therefore, f is an onto function.

34. We have equation of line is $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$

$\Rightarrow \frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3} = \lambda$

$\Rightarrow x = -2\lambda + 4, y = 6\lambda$ and $z = -3\lambda + 1$

Let the coordinates of L be $(4-2\lambda, 6\lambda, 1-3\lambda)$, then, direction ratios of PL are proportional to $(4-2\lambda-2, 6\lambda-3, 1-3\lambda+8)$ i.e., $(2-2\lambda, 6\lambda-3, 9-3\lambda)$.

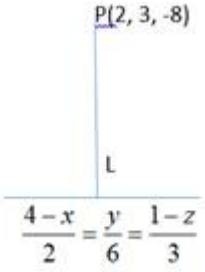
Also, direction ratios are proportional to -2, 6, -3. Since, PL is perpendicular to give line.

$\therefore -2(2-2\lambda) + 6(6\lambda-3) - 3(9-3\lambda) = 0$

$\Rightarrow -4 + 4\lambda + 36\lambda - 18 - 27 + 9\lambda = 0$

$\Rightarrow 49\lambda = 49 \Rightarrow \lambda = 1$

So, the coordinates of L are $(4-2\lambda, 6\lambda, 1-3\lambda)$ i.e., (2, 6, -2).



Also, length of PL $= \sqrt{(2-2)^2+(6-3)^2+(-2+8)^2}$

$= \sqrt{0+9+36} = 3\sqrt{5} units$

OR

$\vec{a}_1 = -\hat{i}-\hat{j}-\hat{k}$

$\vec{a}_2 = 3\hat{i}+5\hat{j}+7\hat{k}$

$\vec{b}_1 = 7\hat{i}-6\hat{j}+1\hat{k}$

$\vec{b}_2 = \hat{i}-2\hat{j}+\hat{k}$

$\vec{a}_2-\vec{a}_1 = 4\hat{i}+6\hat{j}+8\hat{k}$

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$

$= -4\hat{i}-6\hat{j}-8\hat{k}$

$(\vec{a}_2-\vec{a}_1).(\vec{b}_1 \times \vec{b}_2) = (4\hat{i}+6\hat{j}+8\hat{k})(-4\hat{i}-6\hat{j}-8\hat{k}) = -16-36-64 = -116$

$\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{(-4)^2+(-6)^2+(-8)^2}$

$= \sqrt{116}$

$= 2\sqrt{29}$

$d = \left|\frac{(\vec{a}_2-\vec{a}_1).(b_1 \times b_2)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right|$

$= \left|\frac{-116}{2\sqrt{29}}\right|$

$= 2\sqrt{29}$

35. Here, we have:

$A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$

$A^3 = A^2.A$

$A^2 = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+0-6 & 0+0-8 & -2+0-2 \\ -2+2+6 & 0+1+8 & 4-2+2 \\ 3-8+3 & 0-4+4 & -6+8+1 \end{bmatrix} = \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix}$

$A^2.A = \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$

$= \begin{bmatrix} -5^2+16-12 & 0-8+16 & 10-16-4 \\ 6-18+12 & 0-9+16 & -12+18+4 \\ -2-0+9 & 0-0-12 & 4+0+3 \end{bmatrix}$

$= \begin{bmatrix} -1 & -8 & -10 \\ 0 & 7 & 10 \\ 7 & 12 & 7 \end{bmatrix}$

Now, $A^3 - A^2 - 3A - I$

$$= \begin{bmatrix} -1 & -8 & -10 \\ 0 & 7 & 10 \\ 7 & 12 & 7 \end{bmatrix} - \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1+5 & -8+8 & -10+4 \\ 0-6 & 7-9 & 10-4 \\ 7+2 & 12-0 & 7-3 \end{bmatrix} + \begin{bmatrix} -3-1 & -0-0 & 6-0 \\ 6-0 & +3-1 & -6-0 \\ -9-0 & -12+0 & -3-1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & -6 \\ -6 & -2 & 6 \\ 9 & 12 & 4 \end{bmatrix} + \begin{bmatrix} -4 & 0 & 6 \\ 6 & 2 & -6 \\ -9 & -12 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus, $A^3 - A^2 - 3A - I = 0$

Multiply both sides by A^{-1}, we get

$A^{-1}A^3 - A^{-1}A^2 - 3A^{-1}A - IA^{-1} = 0$

$A^2 - A - 3I = A^{-1}$ ...(since $A^{-1}A = I$)

$\Rightarrow A^{-1} = (A^2 - A - 3I)$

$$= \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

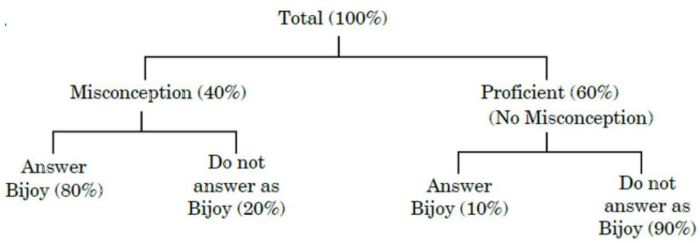
$$= \begin{bmatrix} -5-1-3 & -8-0-0 & -4+2-0 \\ 6+2-0 & 7+1-3 & 4-2-0 \\ -2-3-0 & 0-4-0 & 3-1-3 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

Hence, $A^{-1} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$

Section E

36.



Let E_1 : Student has a misconception

E_2 : Student does not have misconception

A: Student answered Bijoy as correct

$\therefore P(E_1) = \frac{40}{100}, P(E_2) = \frac{60}{100}$

$P(A \mid E_1) = \frac{80}{100}, P(A \mid E_2) = \frac{10}{100}$

$P(\bar{A} \mid E_1) = \frac{20}{100}, P(\bar{A} \mid E_2) = \frac{90}{100}$

i. $P(A \mid E_2) = \frac{10}{100}$ or $\frac{1}{10}$

ii. $P(A) = P(E_1)\,P(A \mid E_1) + P(E_2)\,P(A \mid E_2)$

$= \frac{40}{100} \times \frac{80}{100} + \frac{60}{100} \times \frac{10}{100}$

$= \frac{38}{100}$ or $\frac{19}{50}$

iii. a. $P(E_1 \mid A) = \frac{P(E_1)P(A|E_1)}{P(A)}$

$= \frac{\frac{40}{100} \times \frac{80}{100}}{\frac{38}{100}} = \frac{16}{19}$

OR

b. $P(E_2 \mid A) = \frac{P(E_2)P(A|E_2)}{P(A)}$

$= \frac{\frac{60}{100} \times \frac{10}{100}}{\frac{38}{100}} = \frac{3}{19}$

37. i. Here,

Position vector of A is $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$

Position vector of B is $\vec{b} = 3\hat{i} - 3\hat{j} - 2\hat{k}$

Position vector of C is $\vec{c} = -2\hat{i} + 2\hat{j} + 6\hat{k}$

$\therefore \vec{a} + \vec{b} + \vec{c}$ = (1 + 3 - 2)$\hat{i}$ + (4 - 3 + 2)$\hat{j}$ + (2 - 2 + 6)$\hat{k}$

= $2\hat{i}$ + $3\hat{j}$ + $6\hat{k}$

Thus, $|\vec{a} + \vec{b} + \vec{c}|$ = $\left|\sqrt{(2)^2 + (3)^2 + (6)^2}\,\right|$

= $|\sqrt{4 + 9 + 16}\,|$

= $\sqrt{29}$

ii. Given, $\vec{a} = 4\hat{i} + 6\hat{j} + 12\hat{k}$,

$|\vec{a}| = \sqrt{4^2 + 6^2 + 12^2}$ = 14

Therefore, the unit vector in direction of $\vec{a}$ is given by

$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{4\hat{i} + 6\hat{j} + 12\hat{k}}{14}$

= $\frac{4}{14}\hat{i} + \frac{6}{14}\hat{j} + \frac{12}{14}\hat{k}$

= $\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$

iii. We have, A(1, 4, 2), B(3, -3, -2) and C(-2, 2, 6)

Now, $\overrightarrow{AB} = \vec{b} - \vec{a} = 2\hat{i} - 7\hat{j} - 4\hat{k}$

and $\overrightarrow{AC} = \vec{c} - \vec{a} = -3\hat{i} - 2\hat{j} + 4\hat{k}$

$\therefore \overrightarrow{AB} \times \overrightarrow{AC}$ = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & -4 \\ -3 & -2 & 4 \end{vmatrix}$

= $\hat{i}(-28 - 8) - \hat{j}(8 - 12) + \hat{k}(-4 - 21)$

= - $36\hat{i} + 4\hat{j} - 25\hat{k}$

Now, $|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(-36)^2 + 4^2 + (-25)^2}$

= $|\sqrt{1296 + 16 + 625}\,| = \sqrt{1937}$

$\therefore$ Area of $\triangle$ABC = $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

= $\frac{1}{2}\sqrt{1937}$ sq. units

OR

Triangle law of addition for $\triangle$ABC is given by

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$

If the given points lie on the straight line, then the points will be collinear and so area of $\triangle$ABC = 0

Then, $|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}| = 0$.

Also, if a, b, c are the position vector of the three vertices A, B and C of $\triangle$ABC, then area of triangle is $\frac{1}{2}|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$.

38. i. Capacity = area $\times$ depth = $x^2h = 250 \Rightarrow x^2 = \frac{250}{h}$

$C(\text{cost}) = 500x^2 + 4000h^2$

$\Rightarrow C = 500\left(\frac{250}{h}\right) + 4000h^2 = \frac{125000}{h} + 4000h^2$

ii. $\frac{dC}{dh} = -\frac{125000}{h^2} + 8000h$

$\frac{dC}{dh} = 0 \Rightarrow h = \frac{5}{2}$ m or 2.5 m

iii. $\frac{d^2C}{dh^2} = -125000\left(\frac{-2}{h^3}\right) + 8000 = \frac{250000}{h^3} + 8000$

$\left.\frac{d^2C}{dh^2}\right|_{h=2.5m} > 0 \Rightarrow$ Cost is minimum when h = 2.5 m

Minimum cost = $C = \frac{125000}{\left(\frac{5}{2}\right)} + 4000\left(\frac{5}{2}\right)^2 =$ ₹ 75,000

OR

iii. we already have found above that $h = \frac{5}{2}$ m when $\frac{dC}{dh} = 0$

for the values of h less than $\frac{5}{2}$ and close to $\frac{5}{2}$, $\frac{dC}{dh} < 0$

and, for the values of h more than $\frac{5}{2}$ and close to $\frac{5}{2}$, $\frac{dC}{dh} > 0$

By first derivative test, there is a minimum at $h = \frac{5}{2}$

Now, $x^2 = \frac{250}{h} \Rightarrow x^2 = \frac{250}{\left(\frac{5}{2}\right)} = 100 \Rightarrow x = 10$ m

also, x = 4h