Class XII Session 2026-27
Subject - Mathematics
Sample Question Paper - 2

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculators is not allowed.

Section A

1. If A = {7, 8, 9}, then the relation R = {(8, 9)} in A is **[1]**

   a) Non-symmetric
   b) Symmetric only
   c) Equivalence
   d) Reflexive only

2. The value of $\cos^{-1}\left(\cos\frac{3\pi}{2}\right)$ is equal to **[1]**

   a) $\frac{\pi}{2}$
   b) $\frac{7\pi}{2}$
   c) $\frac{5\pi}{2}$
   d) $\frac{3\pi}{2}$

3. If A is square matrix of order 3×3 such that $|A| = 2$, then write the value of $|\text{adj (adj A)}|$. **[1]**

   a) 16
   b) 2
   c) 0
   d) -16

4. If $y = \frac{\log x}{x}$, then $\frac{d^2y}{dx^2} =$ **[1]**

a) $\frac{2\log x-3}{x^4}$ b) $\frac{2\log x+3}{x^3}$

c) $\frac{3-2\log x}{x^3}$ d) $\frac{2\log x-3}{x^3}$

5. A line is perpendicular to two lines having direction ratios 1, -2, -2 and 0, 2, 1. The direction cosines of the line are **[1]**

a) $\frac{-2}{3}, \frac{1}{3}, \frac{2}{3}$ b) $\frac{2}{3}, \frac{1}{3}, \frac{-1}{3}$

c) $\frac{1}{3}, \frac{-1}{3}, \frac{2}{3}$ d) $\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}$

6. Solution of $x\ dy - ydx = \sqrt{x^2+y^2}\ dx$ is **[1]**

a) $y+\sqrt{x^2+y^3} = Cx^2$ b) $y+\sqrt{x^2+y^2} = Cx^2$

c) $y+\sqrt{x^3+y^2} = Cx^2$ d) $y+\sqrt{3+y^2} = Cx^4$

7. Maximize Z = 5x+3y , subject to constraints x + y ≤ 300 , 2x + y ≤ 360, x ≥ 0, y ≥ 0. **[1]**

a) 1040 b) 1020

c) 1050 d) 1030

8. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined at an angle θ such that $|\vec{a}+\vec{b}| < 1$, then **[1]**

a) $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$ b) $\theta > \frac{2\pi}{3}$

c) $\theta = \frac{2\pi}{3}$ d) $\theta < \frac{\pi}{3}$

9. $\int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1}x}{(1-x^2)^{\frac{3}{2}}}dx = ?$ **[1]**

a) $\frac{1}{2}(\pi - \log 2)$ b) $\left(\frac{\pi}{4} - \frac{1}{2}\log 2\right)$

c) $\left(\frac{\pi}{2} - \frac{1}{3}\log 2\right)$ d) $\left(\frac{\pi}{2} - 2\log 2\right)$

10. If A is any square matrix then which of the following is not symmetric? **[1]**

a) A^tA b) $A - A^t$

c) AA^t d) $A+A^t$

11. The point at which the maximum value of x + y, subject to the constraints x + 2y ≤ 70, 2x + y ≤ 95, x, y ≥ 0 is obtained, is **[1]**

a) (30, 25) b) (20, 35)

c) (35, 20) d) (40,15)

12. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\left(2\vec{a}+\vec{b}\right)$ and $\left(\vec{a}-3\vec{b}\right)$ externally in the ratio 1 : 2. Also, show that P is the mid point of the line segment RQ. **[1]**

a) $5\vec{a}+5\vec{b}$ b) $3\vec{a}+3\vec{b}$

c) $3\vec{a}+5\vec{b}$ d) $5\vec{a}+3\vec{b}$

13. For an invertible square matrix of order 3 with real entries $A^{-1} = A^2$, then det. A = **[1]**

a) $\frac{1}{3}$ b) 1

c) 4 d) 3

14. If A and B are events such that P(A | B) = P(B | A), then **[1]**

a) $A \cap B = \emptyset$ b) A = B

c) P(A) = P(B) d) $A \subset B$ but $A \neq B$

15. Solution of the differential equation $\frac{dx}{x} + \frac{dy}{y} = 0$ is **[1]**

a) xy = c b) log x . log y = c

c) x + y = c d) $\frac{1}{x} + \frac{1}{y} = c$

16. A relation R defined on set A = $\{x : x \in Z \text{ and } 0 \leq x \leq 10\}$ as $R = \{(x, y) : x = y\}$ is given to be an equivalence relation. The number of equivalence classes is: **[1]**

a) 11 b) 10

c) 2 d) 1

17. If $y = x^{x^{x+\infty}}$ then $\frac{dy}{dx} = ?$ **[1]**

a) yx^{y+1} b) $\frac{y}{x(1+y\log x)}$

c) yx^{y-1} d) $\frac{y^2}{x(1-y\log x)}$

18. If the direction cosines of a line are $\sqrt{3}k$, $\sqrt{3}k$, $\sqrt{3}k$, then the value of k is: **[1]**

a) ± 3 b) ± 1

c) $\pm\frac{1}{3}$ d) $\pm\sqrt{3}$

19. **Assertion (A):** The average rate of change of the function y = 15 - x^2 between x = 2 and x = 3 is -5. **[1]**

Reason (R): Average rate of change $\delta_y = y_{\text{at x = 3}} - y_{\text{at x = 2}}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. If $\vec{a} + \vec{b} = \hat{i}$ and $\vec{a} = 2\hat{i} - 2\hat{j} + 2\hat{k}$, then $|\vec{b}|$ equals: **[1]**

a) 3 b) $\sqrt{12}$

c) $\sqrt{17}$ d) $\sqrt{14}$

Section B

21. Evaluate: $\sec^2\left(\tan^{-1}\frac{1}{2}\right) + \text{cosec}^2\left(\cot^{-1}\frac{1}{3}\right)$ **[2]**

OR

Find the value of $\sin\left[2\cot^{-1}\left(\frac{-5}{12}\right)\right]$

22. Check the differentiability of $f(x) = \begin{cases} x^2 + 1, & 0 \leq x < 1 \\ 3 - x, & 1 \leq x \leq 2 \end{cases}$ at $x = 1$. **[2]**

23. The length x of a rectangle is decreasing at the rate of 2 cm/sec and the width y is increasing at the rate of 2 cm/sec. When x = 12 cm and y = 5 cm, find the rate of change of **[2]**

i. the perimeter and

ii. the area of the rectangle.

OR

An edge of a variable cube is increasing at the rate of 3 cm per second. How fast is the volume of the cube increasing when the edge is 10 cm long?

24. Integrate the functions $(4x+2)\sqrt{x^2+x+1}$ **[2]**

25. If A = $\begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$, find A^{16}. **[2]**

Section C

26. Evaluate the area bounded by the ellipse $\frac{x^2}{4}+\frac{y^2}{9}$ = 1 above the x-axis. **[3]**

27. If $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ represent two adjacent sides of a parallelogram, find unit vectors parallel to the diagonals of the parallelogram. **[3]**

28. Evaluate: $\int \frac{x^2+1}{(x^2+4)(x^2+25)}dx$ **[3]**

OR

Evaluate: $\int \frac{(2x+1)}{(x+2)(x-3)}dx$.

29. Solve the differential equation: y dx + $\left\{x\log\left(\frac{y}{x}\right)\right\}$ dy - 2x dy = 0 **[3]**

OR

Find the particular solution of the differential equation $(\tan^{-1} y - x)dy = (1 + y^2)dx$, given that x = 1 when y = 0.

30. Maximize Z = x + 2y subject to the constraints **[3]**

$x-y\geq 0, 2y\leq x+2, x, y\geq 0$

OR

Solve the following LPP graphically:

Maximize Z = 5x + 3y

Subject to

3x + 5y $\leq$ 15

5x + 2y $\leq$ 10

and, x, y $\geq$ 0

31. If x = $a\left(\cos t+\log\tan\frac{t}{2}\right)$, y = a sin t, then evaluate $\frac{d^2y}{dx^2}$ at $t=\frac{\pi}{3}$. **[3]**

Section D

32. Integrate the function $\frac{x+2}{\sqrt{x^2+2x+3}}$ **[5]**

33. Let R be a relation on $N\times N$, defined by (a, b) R (c, d) $\Leftrightarrow$ a + d = b + c for all (a, b), (c, d) $\in N\times N$. Show that R is an equivalence relation. **[5]**

OR

Show that the function f: $R_0 \rightarrow R_0$, defined as f (x) = $\frac{1}{x}$, is one-one onto, where R_0 is the set non-zero real numbers. Is the result true, if the domain R_0 is replaced by N with co-domain being same as R_0?

34. Solve the system of the following equations: (Using matrices): **[5]**

$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4; \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1; \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2;$

35. Show that the straight lines whose direction cosines are given by 2l + 2m - n = 0 and **[5]**

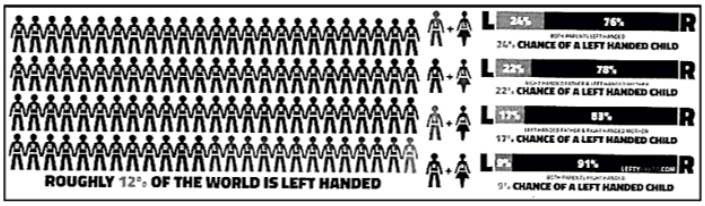
mn + nl + lm = 0 are at right angles.

OR

Find the shortest distance between the given lines. $\vec{r} = (6\hat{i} + 3\hat{k}) + \lambda(2\hat{i} - \hat{j} + 4\hat{k})$, $\vec{r} = (-9\hat{i} + \hat{j} - 10\hat{k}) + \mu(4\hat{i} + \hat{j} + 6\hat{k})$

Section E

36. Recent studies suggest that roughly 12% of the world population is left handed. **[4]**



Depending upon the parents, the chances of having a left handed child are as follows:

A. When both father and mother are left handed:
Chances of left handed child is 24%.

B. When father is right handed and mother is left handed:
Chances of left handed child is 22%.

C. When father is left handed and mother is right handed:
Chances of left handed child is 17%.

D. When both father and mother are right handed:
Chances of left handed child is 9%.

Assuming that P(A) = P(B) = P(C) = P(D) = $\frac{1}{4}$ and L denotes the event that child is left handed.

i. Find $P\left(\frac{L}{C}\right)$. (1)

ii. Find $P\left(\frac{L}{\bar{A}}\right)$. (1)

iii. Find $P\left(\frac{A}{L}\right)$. (2)

OR

Find the probability that a randomly selected child is left handed given that exactly one of the parents is left handed. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Three schools A, B and C organized a mela for collecting funds for helping the rehabilitation of flood victims. They sold handmade fans, mats, and plates from recycled material at a cost of ₹ 25, ₹ 100 and ₹ 50 each. The number of articles sold by school A, B, C are given below.



Article	School	A	B	C
Fans		40	25	35

Mats	50	40	50
Plates	20	30	40

i. Represent the sale of handmade fans, mats and plates by three schools A, B and C and the sale prices (in ₹) of given products per unit, in matrix form. (1)

ii. Find the funds collected by school A, B and C by selling the given articles. (1)

iii. If they increase the cost price of each unit by 20%, then write the matrix representing new price. (2)

OR

Find the total funds collected for the required purpose after 20% hike in price. (2)

38. A carpenter needs to make a wooden cuboidal box, closed from all sides, which has a square base and fixed volume. Since he is short of the paint required to paint the box on completion, he wants the surface area to be minimum. **[4]**

On the basis of the above information, answer the following questions :

i. Taking length = breadth = x m and height = y m, express the surface area (S) of the box in terms of x and its volume (V), which is constant. **(1)**

ii. Find $\frac{dS}{dx}$. **(1)**

iii. a. Find a relation between x and y such that the surface area (S) is minimum. **(2)**

OR

b. If surface area (S) is constant, the volume (V) $= \frac{1}{4}\left(Sx - 2x^3\right)$, x being the edge of base. Show that volume (V) is maximum for $x = \sqrt{\frac{S}{6}}$. **(2)**

Solution

Section A

1. **(a)** Non-symmetric
   Explanation:
   Non-symmetric

2. **(a)** $\frac{\pi}{2}$
   Explanation:
   We have, $\cos^{-1} \cos \frac{3\pi}{2}$
   We know that,
   $\cos \frac{3\pi}{2} = 0$
   So, $\cos^{-1} \cos \frac{3\pi}{2} = \cos^{-1} 0$
   Let, $\cos^{-1} 0 = \theta$
   $\Rightarrow \cos \theta = 0$
   Principal value of $\cos^{-1} x$ is $[0, \pi]$
   For, $\cos \theta = 0$
   So, $\theta = \frac{\pi}{2}$

3. **(a)** 16
   Explanation:
   Since, A is a square matrix of order 3 i.e. n = 3
   we know that
   $|\text{adj A}| = (\text{A})^{\text{n-1}}$
   $|\text{adj (adj A)}| = |A|^{(n-1)^2}$
   $= 2^{(3-1)^2}$ ...($\because |\text{A}| = 2$)
   $= 2^4$
   $= 16$

4. **(d)** $\frac{2 \log x - 3}{x^3}$
   Explanation:
   $\frac{d}{dx}\left(\frac{\log x}{x}\right) = \frac{x.\frac{1}{x} - \log x.1}{x^2} = \frac{1 - \log x}{x^2}$
   $\Rightarrow \frac{d}{dx}\left(\frac{1 - \log x}{x^2}\right)$
   $= \frac{x^2.(-\frac{1}{x}) - (1 - \log x).2x}{x^4} = \frac{-x - 2x + 2x \log x}{x^4} = \frac{x(2 \log x - 3)}{x^4} = \frac{2 \log x - 3}{x^3}$

5. **(d)** $\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}$
   Explanation:
   If a line is perpendicular to two given lines we can find out the parallel vector by cross product of the given two vectors.
   $\vec{a} = \hat{i} - 2\hat{j} - 2\hat{k}$
   $\overrightarrow{\text{b}} = 2\hat{\text{j}} + \hat{\text{k}}$
   $\vec{a} \times \vec{b} = (\hat{i} - 2\hat{j} - 2\hat{k}) \times (2\hat{j} + \hat{k})$
   $= 2\,\hat{i} - \hat{j} + 2\hat{k}$
   So,

$|\vec{a} \times \vec{b}| = \sqrt{2^2 + 1 + 2^2}$ =3

So, Director cosine
$\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$

6.

(b) $y + \sqrt{x^2 + y^2} = Cx^2$

Explanation:

$\frac{dy}{dx} = \frac{\sqrt{x^2+y^2}+y}{x}$

Put y=vx, we have ; $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = \sqrt{1+v^2} + v \Rightarrow \int \frac{1}{\sqrt{1+v^2}} dv = \int \frac{1}{x} dx$

$\Rightarrow \log|\sqrt{1+v^2} + v| = \log|x| + \log C \Rightarrow |\sqrt{1+v^2} + v| = Cx$

$\Rightarrow (y + \sqrt{x^2+y^2})^2 = C^2x^4$

$\Rightarrow (y + \sqrt{x^2+y^2}) = Cx^2$

7.

(b) 1020

Explanation:

Here , Maximize Z = 5x+3y , subject to constraints x + y ≤ 300 , 2x + y ≤ 360, x ≥ 0, y ≥ 0.

Corner points	Z = 5x +3 y
P(0, 300)	900
Q(180, 0)	900
R(60, 240)	1020……..(Max.)
S(0, 0)	0

Hence, the maximum value is 1020

8. **(a)** $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$

Explanation:

We know that,

If $\vec{a}$and$\vec{b}$ are two-unit vectors inclined at an angle θ

$|\vec{a} + \vec{b}| = \left|2\cos\frac{\theta}{2}\right|$

According to question,

$|\vec{a} + \vec{b}| < 1$ or -1< $\vec{a} + \vec{b}$<1

$\Rightarrow \left|2\cos\frac{\theta}{2}\right| < 1$

$\implies -1 < 2cos\frac{\theta}{2} < 1$

$\Rightarrow \frac{-1}{2} < \cos\frac{\theta}{2} < \frac{1}{2}$

$\Rightarrow \frac{2\pi}{3} > \frac{\theta}{2} > \frac{\pi}{3}$

$\Rightarrow \frac{2\pi}{3} < \theta < \frac{4\pi}{3}$

9.

(b) $\left(\frac{\pi}{4} - \frac{1}{2}\log 2\right)$

Explanation:

The given integral is $\int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1} x}{(1-x^2)^{\frac{3}{2}}} dx$

Put $\sin^{-1}$ x = t

$dt = \frac{dx}{\sqrt{1-x^2}}$

x = sin t

$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

= t;

and $\sin^{-1} 0 = 0$

= t

Limit changes to,

$\int_0^{\frac{\pi}{4}} \frac{tdt}{1-\sin^2 t} = \int_0^{\frac{\pi}{4}} t \sec^2 t dt$

$= t \tan t - \int_0^{\frac{\pi}{4}} \tan t dt$

= [t tan t + log cos t] (0 to $\frac{\pi}{4}$)

$= \frac{\pi}{4} - \frac{1}{2}\log 2$

10.

(b) A - A^t

Explanation:

For every square matrix (A – A') is always skew – symmetric.

11.

(d) (40,15)

Explanation:

We need to maximize the function z = x + y Converting the given inequations into equations, we obtain

x + 2y = 70, 2x + y = 95, x = 0 and y = 0

Region represented by $x + 2y \leq 70$:

The line x + 2y = 70 meets the coordinate axes at A(70, 0) and B(0, 35) respectively. By joining these points we obtain the line x + 2y = 70. Clearly (0, 0) satisfies the inequation $x + 2y \leq 70$. So, the region containing the origin represents the solution set of the inequation $x + 2y \leq 70$.

Region represented by $2x + y \leq 95$:

The line 2x + y = 95 meets the coordinate axes at $C\left(\frac{95}{2}, 0\right)$ respectively. By joining these points we

obtain the line 2x + y = 95

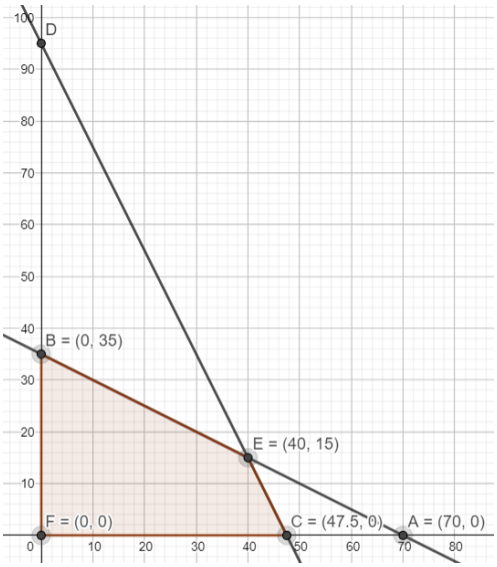
Clearly (0, 0) satisfies the inequation $2x + y \leq 95$. So, the region containing the origin represents the solution set of the inequation $2x + y \leq 95$

Region represented by $x \geq 0$ and $y \geq 0$:

since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations $x \geq 0$, and $y \geq 0$

The feasible region determined by the system of constraints $x + 2y \leq 70$, $2x + y \leq 95$, $x \geq 0$, and $y \geq 0$

are as follows.



The corner points of the feasible region are O(0, 0), C($\frac{95}{2}$, 0) E(40, 15) and B(0, 35).

The value fo Z at these corner points are as follows.

Corner point : z = x + y

O(0, 0) : 0 + 0 = 0

$C\left(\frac{95}{2}, 0\right)$: $\frac{95}{2} + 0 = \frac{95}{2}$

E(40, 15) : 40 + 15 = 55

B(0, 35) : 0 + 35 = 35

We see that maximum value of the objective function Z is 55 which is at (40, 15).

12.

(c) $3\vec{a} + 5\vec{b}$

Explanation:

Let position vector of point R be $\overrightarrow{r}$. As point R divides externally the line segment PQ in the ratio 1:2 .therefore ,

$\overrightarrow{r} = \frac{1\left(\overrightarrow{a} - 3\overrightarrow{b}\right) - 2\left(2\overrightarrow{a} + \overrightarrow{b}\right)}{1-2} = \frac{\left(-3\overrightarrow{a} - 5\overrightarrow{b}\right)}{-1}$

$\overrightarrow{r} = 3\overrightarrow{a} + 5\overrightarrow{b}$

Also , mid point of the line segment RQ is : $= \frac{3\overrightarrow{a} + 5\overrightarrow{b} + \overrightarrow{a} - 3\overrightarrow{b}}{2} = \frac{4\overrightarrow{a} + 2\overrightarrow{b}}{2} = 2\overrightarrow{a} + \overrightarrow{b}$,which is the position vector of point P.Therefore , P is the mid point of line segment RQ.

13.

(b) 1

Explanation:

$A^2 = I \Rightarrow A^2A^{-1} = IA^{-1} \Rightarrow A = A^{-1}$ and it is possible only if A is an identity matrix and determinant of the identity matrix is equal to 1

14.

(c) P(A) = P(B)

Explanation:

It is given that : P(A | B) = P(B | A)

$\Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{P(B \cap A)}{P(A)}$

$\Rightarrow \frac{1}{P(B)} = \frac{1}{P(A)} \Rightarrow P(A) = P(B)$

15. **(a)** xy = c

Explanation:

From the given equation, we get log x + log y = log c giving xy = c.

16. **(a)** 11

Explanation:

The number of equivalence classes for the relation R defined on the set A={x:x∈Z and 0≤x≤10} with the property that x=y is the same as the number of distinct elements in the set A. Since R is an equivalence relation, it is reflexive, symmetric, and transitive. Thus, each element forms an equivalence class with itself.

Given that A contains all integers from 0 to 10, there are 11 distinct elements, and hence, there will be 11 equivalence classes. The equivalence classes would be: {0}, {1}, {2}, ... , {10}.

The correct answer to the question is 11.

17. **(d)** $\frac{y^2}{x(1-y\log x)}$

Explanation:

Given:

$y = x^{x^{x+\infty}}$

We can write it as

$\Rightarrow y = x^y$

Taking log of both sides we obtain

$\log y = y \log x$

Differentiating with respect to x,we get

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}\log x + y \cdot \frac{1}{x}$

$\Rightarrow \left(\frac{1}{y} - \log x\right)\frac{dy}{dx} = \frac{y}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x}\left(\frac{y}{1-y\log x}\right)$

$\Rightarrow \frac{dy}{dx} = \frac{y^2}{x(1-y\log x)}.$

Which is the required solution.

18. **(c)** $\pm\frac{1}{3}$

Explanation:

$\pm\frac{1}{3}$

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Let y = f(x) = 15 - x^2

If x changes from 2 to 3 then δ_x = 3 - 2 = 1

Again f(3) = 15 - 9 = 16 and f(2) = 15 - 4 = 11

Therefore δ_y = f(3) - f(2) = 6 - 11 = -5

20. **(a)** 3

Explanation:

3

Section B

21. $\sec^2\left(\tan^{-1}\frac{1}{2}\right) + \text{cosec}\left(\cot^{-1}\frac{1}{3}\right)$

$= \left[1 + \tan^2\left(\tan^{-1}\frac{1}{2}\right)\right] + \left[1 + \cot^2\left(\cot^{-1}\frac{1}{3}\right)\right]$

$= \left[1+\left(\frac{1}{2}\right)^2\right] + \left[1+\left(\frac{1}{3}\right)^2\right]$

$= \frac{85}{36}$

OR

Let $\cot^{-1}\left(\frac{-5}{12}\right) = y$

Then $\cot y = \frac{-5}{12}$

Now,

$\sin\left[2\cot^{-1}\left(\frac{-5}{12}\right)\right] = \sin 2y$

$= 2\sin y \cos y = 2\left(\frac{12}{13}\right)\left(\frac{-5}{13}\right)\left[\text{since } \cot y < 0, \text{ so } y \in \left(\frac{\pi}{2}, \pi\right)\right]$

$= \frac{-120}{169}$

22. LHD at $x = 1$

$= \lim_{h\to 0} \frac{f(1-h)-f(1)}{-h} = \lim_{h\to 0} \frac{\left[(1-h)^2+1\right]-2}{-h} = 2$

RHD at $x = 1$

$= \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{[3-(1+h)]-2}{h} = -1$

as $LHD \neq RHD$, so $f(x)$ is not differentiable at $x = 1$

23. Let P be the perimeter and A be the area of the rectangle at any time t. Then,

It is given that x = 12 cm , y = 5 cm, $\frac{dx}{dt} = -2\text{cm/s}$ and $\frac{dy}{dt} = 2\text{cm/s}$

i. We have,

$P = 2(x+y)$

$\Rightarrow \quad \frac{dP}{dt} = 2\left(\frac{dx}{dt} + \frac{dy}{dt}\right) = 2(-2+2) = 0\text{cm/sec}$ i.e. the perimeter remains constant.

ii. We have,

$A = xy$

$\Rightarrow \quad \frac{dA}{dt} = x\left(\frac{dy}{dt}\right) + y\left(\frac{dx}{dt}\right)$

$\Rightarrow \quad \frac{dA}{dt} = 12 \times 2 - 2 \times 5$

$\Rightarrow \quad \frac{dA}{dt} = 14\text{cm}^2/\text{sec}$

OR

Let x cm be the edge and y be the volume of the variable cube at any time t.

Rate of increase of edge = 3 cm/sec

$\Rightarrow \frac{dx}{dt}$ is positive and = 3 cm/sec

$\Rightarrow \frac{dx}{dt} = 3cm/\text{ sec}$...(i)

$\Rightarrow y = x^3$

$\therefore$ Rate of change of volume of cube $= \frac{dy}{dt} = \frac{d}{dt}x^3$

$= 3x^2\frac{dx}{dt} = 3x^2\ (3)$ [from (1)]

$= 9x^2 cm^3/\text{ sec}$

Putting x = 10cm (given),

$\frac{dy}{dt} = 9(10)^2 = 900cm^3/\text{ sec}$

Since $\frac{dy}{dt}$ is positive, therefore volume of cube is increasing at the rate of 900 cm^3/sec.

24. Let $I = \int (4x+2)\sqrt{x^2+x+1}\,dx$

$= \int 2\,(2x+1)\sqrt{x^2+x+1}\,dx$

$= \int 2\sqrt{x^2+x+1}\ (2x+1)dx$ ...(i)

Putting $x^2 + x + 1 = t$

$\Rightarrow (2x+1) = \frac{dt}{dx}$

$\Rightarrow (2x+1)\,dx = dt$

$\therefore$ From eq. (i), $I = \int 2\sqrt{t}dt$

$= 2\int t^{\frac{1}{2}}dt$

$= 2\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$

$= \frac{4}{3}t^{\frac{3}{2}} + c$

$= \frac{4}{3}\left(x^2+x+1\right)^{\frac{3}{2}} + c$

25. Given: A = $\begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$

We will find A^2,

$A^2 = A \times A = \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$

$\Rightarrow A^2 = 0$

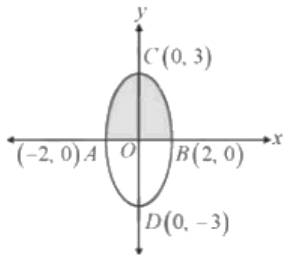
Hence, $A^{16} = (A^2)^8 = (0)^8 = 0$

Hence A^{16} is a null matrix.

Section C

26. The given equation, $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is a right handed ellipse with its vertex at the origin and the point of intersection $(\pm 2, 0)$ and $(0, \pm 3)$. Also, $\frac{x^2}{4} + \frac{y^2}{9} = 1$ contains only even power of x and y. so, it is symmetrical about the x-axis, y-axis respectively.

Therefore Required area of shaded region= 2 area of OBC = $2\int_0^2 ydx = 2\int_0^2 \frac{3}{2}\sqrt{4-x^2}$

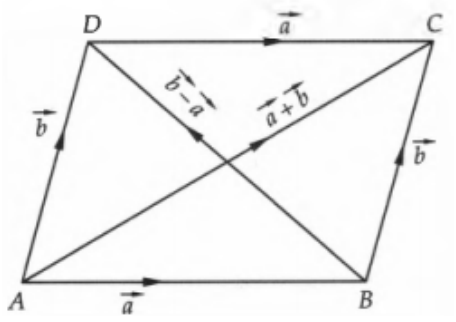


$= 3\int_0^2 \sqrt{(2)^2-(x)^2}\,dx = 3\left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$

$= 3\left[\left\{\frac{2}{2}\sqrt{4-4} + \frac{4}{2}\sin^{-1}\left(\frac{2}{2}\right)\right\} - \left\{\frac{0}{2}\sqrt{4-0} + 2\sin^{-1}(0)\right\}\right]$

= 3 [2 $\sin^{-1}$ (1)] = $3.2 \cdot \frac{\pi}{2}$ sq. units = 3πsq. units

27. Let ABCD be a parallelogram such that $\overrightarrow{AB} = \vec{a}$ and $\overrightarrow{BC} = \vec{b}$. Then,



In ΔABC,

$\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$

$\Rightarrow \quad \overrightarrow{AC}=\vec{a}+\vec{b}=3\hat{i}+6\hat{j}-2\hat{k}$

Again, in ΔABD,

$\overrightarrow{AB}+\overrightarrow{BD}=\overrightarrow{AD}$

$\Rightarrow \quad \overrightarrow{BD}=\overrightarrow{AD}-\overrightarrow{AB}$

$\Rightarrow \quad \overrightarrow{BD}=\vec{b}-\vec{a}=\hat{i}+2\hat{j}-8\hat{k}$

Now, $\overrightarrow{AC}=3\hat{i}+6\hat{j}-2\hat{k}$

$\Rightarrow \quad |\overrightarrow{AC}|=\sqrt{9+36+4}=7$

and $\overrightarrow{BD}=\hat{i}+2\hat{j}-8\hat{k}$

$\Rightarrow \quad |\overrightarrow{BD}|=\sqrt{1+4+64}=\sqrt{69}$

$\therefore$ Unit vector along $\overrightarrow{AC}=\frac{\overrightarrow{AC}}{|\overrightarrow{AC}|}=\frac{1}{7}(3\hat{i}+6\hat{j}-2\hat{k})$

and Unit vector along $\overrightarrow{BD}=\frac{\overrightarrow{BD}}{|\overrightarrow{BD}|}=\frac{1}{\sqrt{69}}(\hat{i}+2\hat{j}-8\hat{k})$

28. Consider the integral

I = $\int \frac{x^2+1}{(x^2+4)(x^2+25)}dx$

Let y = x^2

Thus,

$\frac{x^2+1}{(x^2+4)(x^2+25)}$ = $\frac{y+1}{(y+4)(y+25)}$

$\Rightarrow \frac{y+1}{(y+4)(y+25)}$ = $\frac{A}{y+4}+\frac{B}{y+25}$

by using partial fraction

$\Rightarrow \frac{y+1}{(y+4)(y+25)}$ = $\frac{A(y+25)+B(y+4)}{(y+4)(y+25)}$

$\Rightarrow$ y + 1 = Ay + 25A + By + 4B

Comparing the coefficients,we have,

A + B = 1 and 25A + 4B = 1

Solving the above equation, We have,

A = $\frac{-1}{7}$ and B = $\frac{8}{7}$

Thus using values of A, and B, we get, I = $\int \frac{x^2+1}{(x^2+4)(x^2+25)}dx$

= $\int \frac{\frac{-1}{7}}{x^2+4}dx$ + $\int \frac{\frac{8}{7}}{x^2+25}dx$

= $\frac{-1}{7}\int \frac{1}{x^2+4}dx+\frac{8}{7}\int \frac{1}{x^2+25}dx$

= $\frac{-1}{7}\times\frac{1}{2}\tan^{-1}\frac{x}{2}+\frac{8}{7}\times\frac{1}{5}\tan^{-1}\frac{x}{5}+C$

= $\frac{-1}{14}\tan^{-1}\frac{x}{2}+\frac{8}{35}\tan^{-1}\frac{x}{5}+c$

OR

Let, I = $\int \frac{(2x+1)}{(x+2)(x+3)}dx$

Using partial fractions we have,

$\frac{2x+1}{(x+2)(x-3)}=\frac{A}{x+2}+\frac{B}{x-3}$ .... (i)

Which implies 2x + 1 = A(x - 3) + B(x + 2)

Now put x - 3 = 0, x = 3

$2\times 3+1$ = A(0) + B (3 + 2)

So $B = \frac{1}{5}$

Now put x + 2 = 0, x = -2

-4 + 1 = A(-2 - 3) + B(0)

So $A = \frac{3}{5}$

From equation (1), we get,

$\frac{2x+1}{(x+2)(x-3)} = \frac{3}{5} \times \frac{1}{x+2} + \frac{7}{5} \times \frac{1}{x-3}$

$\int \frac{2x+1}{(x+2)(x-3)} dx = \frac{3}{5} \int \frac{1}{x+2} dx + \frac{7}{5} \int \frac{1}{x-3} dx$

$= \frac{3}{5} \log |x+2| + \frac{7}{5} \log |x-3| + c$

29. The given differential equation is,

ydx + $\left\{x \log\left(\frac{y}{x}\right)\right\}$ dy - 2x dy = 0

y + x log $\left(\frac{y}{x}\right) \frac{dy}{dx} - 2x \frac{dy}{dx}$ = 0

$\frac{dy}{dx} = \frac{y}{2x - x \log\left(\frac{y}{x}\right)}$

It is a homogeneous equation

Put, y = vx and $\frac{dy}{dx} = v + x \frac{dv}{dx}$

So, $v + x \frac{dv}{dx} = \frac{vx}{2x - x \log\left(\frac{vx}{x}\right)}$

$x \frac{dv}{dx} = \frac{v}{2 - \log v} - v$

$x \frac{dv}{dx} = \frac{v - 2v + v \log v}{2 - \log v}$

$\int \frac{\log v - 2}{v(\log v - 1)} dv = -\int \frac{dx}{x}$

Let log v - 1 = t

$\frac{1}{v}$ dv = dt

$\int \left(\frac{t-1}{t}\right) dt = -\int \frac{dx}{x}$

t - log |t| = log $\left|\frac{c}{x}\right|$

log v - 1 log (log v - 1) = log $\left|\frac{c}{x}\right|$

log $e^{\log v - 1}$ - log |log v - 1| = log $\left|\frac{c}{x}\right|$

$e^{\log\left(\frac{v}{e}\right)} = \frac{c}{x}$ |log v - 1|

$\frac{v}{e} = \frac{c}{x}$ |log v - 1|

$y = c_1 \left\{\log\left|\frac{y}{x}\right| - 1\right\}$

OR

Given $(\tan^{-1}y - x)dy = (1 + y^2)dx$

or $\frac{dx}{dy} = \frac{\tan^{-1} y - x}{1+y^2}$

or $\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1} y}{1+y^2}$ ..(i)

This is a linear differential equation with :

P = $\frac{1}{1+y^2}$ and Q = $\frac{\tan^{-1} y}{1+y^2}$

So I.F. = $e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1} y}$

Multiplying both sides of eqn. (i) by I.F. = $e^{\tan^{-1} y}$, we get

x . IF = $\int Q.IF dy$

x$e^{\tan^{-1} y} = \int \frac{\tan^{-1} y}{1+y^2} e^{\tan^{-1} y} dy$ + C

Let t = $\tan^{-1}$y, dt = $\frac{1}{1+y^2}$ dy

Thus,

$xe^t = \int te^t dt + C$

$xe^t = e^t(t - 1) + C$

$xe^{\tan^{-1} y} = e^{\tan^{-1} y}(\tan^{-1}y - 1) + C$ .....(ii)

It is given that y(0) = 0 i.e., y = 0 when x = 0

Putting x = 0, y = 0 in eqn. (ii), we get

$0 = e^0 (0 - 1) + C$ or C = 1

Putting C = 1 in eq (ii), we get

$xe^{\tan^{-1} y} = e^{\tan^{-1} y} (\tan^{-1}y - 1) + 1$

or $(x - \tan^{-1}y + 1)e^{\tan^{-1} y} = 1$

30. According to the question,

Maximize $Z = x + 2y$

Subject to the constraints

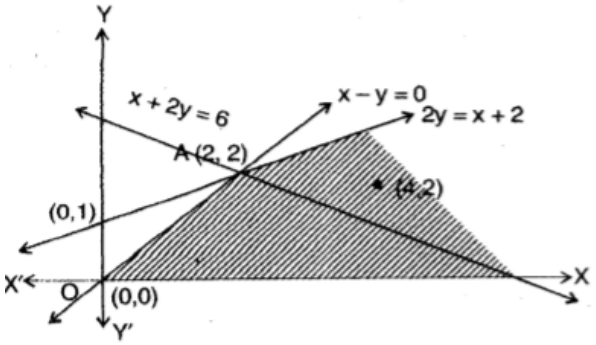
$x - y \geq 0$

$2y \leq x + 2$

such that $x \geq 0, y \geq 0$

We draw line $x - y = 0$ and $2y = x + 2$ and shaded the feasible region with respect to constraints sign.

We observe that the feasible region is unbounded and corner points are $O(0, 0), A(2, 2)$ only. Evaluating Z at all corner points,



Corner Points	**Z = x + 2y**
$(0, 0)$	$Z = 0\ (Smallest)$
$(2, 2)$	$Z = 6\ (largest)$

We are to determine the maximum value. As the feasible region is unbounded we can not say whether the largest value Z = 6 is maximum or not.

We draw the half plane $x + 2y > 6$ and notice that it has common points with feasible region. There does not exist any maximum value. For testing we let $x = 4, y = 2$, this point lie in the feasible region and at $(4, 2)$ the value of $Z = 8$ i.e., greater than '6'.

OR

Converting the given in-equations into equations,

We obtain the following equations:

3x + 5y = I5, 5x + 2y =10, x = 0 and y = 0

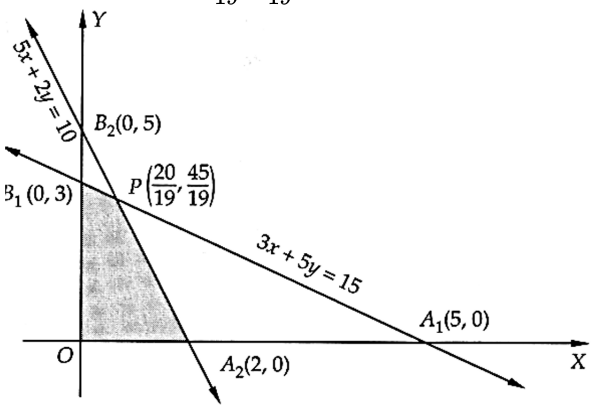
Region represented by $3x + 5y \leq 15$: The line 3x + 5y = 15 meets the coordinate axes at A_1 (5, 0) and B_1(0, 3) respectively. Join these points to obtain the line 3x + 5y = 15. Clearly, (0,0) satisfies the inequation $3x + 5y \leq 15$. So, the region containing the origin represents the solution set of the inequation 3x + 5y < 15.

Region Represented by $5x + 2y \leq 10$: The line 5x + 2y = 10 meets the coordinate axes at A_2 (2, 0) and B_2 (0 ,5) respectively. Join these points to obtain the graph of the line 5x + 2y = 10. Clearly, (0, 0) satisfies the inequation 5x + 2y < 10. So, the region containing the origin represents the solution

set of this inequation.

The region represented by $x \geq 0$ and $y \geq 0$:

So graph will make in first quadrant, graph every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations $x \geq 0$ and $y \geq 0$. The shaded region $OA_2\ PB_1$ in Figure given below represents the common region of the above inequations. This region is the feasible region of the given LPP and the LPP has optimal solysolution The coordinates of the vertices (comer-points) of the shaded feasible region are O (0, 0), A_2 (2, 0), P ($\frac{20}{19}$, $\frac{45}{19}$) and B_1 (0, 3)



These points have been obtained by solving the equations of the corresponding intersecting lines, simultaneously.

The values of the objective function at these points are given in the following table:

Point (x, y)	Value of the objective function Z = 5x + 3 y
O (0, 0)	$Z = 5 \times 0 + 3 \times 0 = 0$
A_2 (2, 0)	$Z = 5 \times 2 + 3 \times 0 = 10$
$P\left(\frac{20}{19}, \frac{45}{19}\right)$	$Z = 5 \times \frac{20}{19} + 3 \times \frac{45}{19} = \frac{235}{19}$
B_1 (0, 3)	$Z = 5 \times 0 + 3 \times 3 = 9$

Clearly, the objective function Z has maximum at P ($\frac{20}{19}$, $\frac{45}{19}$). Hence, $x = \frac{20}{19}$, $y = \frac{45}{19}$ is the optimal solution of the given LPP and the optimal value of Z is $\frac{235}{19}$.

31. Given, $x = a\left(\cos t + \log \tan \frac{t}{2}\right)$ .....(i)

and $y = a \sin t$.....(ii)

Therefore, on differentiating both sides w.r.t t, we get,

$$\frac{dx}{dt} = a\left[\frac{d}{dt}(\cos t) + \frac{d}{dt}\log \tan \frac{t}{2}\right]$$

$$= a\left[-\sin t + \frac{1}{\tan \frac{t}{2}} \frac{d}{dt}\left(\tan \frac{t}{2}\right)\right] \text{ [by using chain rule of derivative]}$$

$$= a\left[-\sin t + \frac{1}{\tan \frac{t}{2}} \cdot \sec^2 \frac{t}{2} \frac{d}{dt}\left(\frac{t}{2}\right)\right]$$

$$= a\left[-\sin t + \frac{1}{\tan \frac{t}{2}} \times \sec^2 \frac{t}{2} \times \frac{1}{2}\right]$$

$$= a\left[-\sin t + \frac{1}{\frac{\sin t/2}{\cos t/2}} \times \frac{1}{\cos^2 \frac{t}{2}} \times \frac{1}{2}\right]$$

$$= a\left[-\sin t + \frac{1}{2 \sin \frac{t}{2} \cdot \cos \frac{t}{2}}\right]$$

$$= a\left[-\sin t + \frac{1}{\sin t}\right] \quad [\because \sin 2\theta = 2 \sin \theta \cos \theta]$$

$$= a\left(\frac{1 - \sin^2 t}{\sin t}\right)$$

$\Rightarrow \quad \frac{dx}{dt} = a\left(\frac{\cos^2 t}{\sin t}\right)\left[\because 1-\sin^2\theta = \cos^2\theta\right]$ .....(iii)

Again,on differentiating both sides of (ii) w.r.t t, we get,

$\frac{dy}{dt}$ = a cos t.....(iv)

Therefore, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a\cos t}{a\left(\frac{\cos^2 t}{\sin t}\right)}$ [from Eqs(iii) and (iv)]

$= \frac{a\cos t}{a\cos^2 t} \times \sin t$ = tan t

Therefore,on differentiating both sides of above equation w.r.t x, we get,

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(\tan t)$

$= \frac{d}{dt}(\tan t)\frac{dt}{dx}\left[\because \frac{d}{dx}f(t) = \frac{d}{dt}f(t)\cdot\frac{dt}{dx}\right]$

$\Rightarrow \quad \frac{d^2y}{dx^2} = \sec^2 t \times \frac{\sin t}{a\cos^2 t}$ [From Eq.(iii)]

$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{\sin t \sec^4 t}{a}$

Therefore, on putting t = $\frac{\pi}{3}$,we get,

$\left[\frac{d^2y}{dx^2}\right]_{t=\frac{\pi}{3}}$ = $\frac{\sin\frac{\pi}{3}\times\sec^4\frac{\pi}{3}}{a}$ = $\frac{\frac{\sqrt{3}}{2}\times(2)^4}{a}$

= $\frac{8\sqrt{3}}{a}$

Section D

32. Let $I = \int \frac{x+2}{\sqrt{x^2+2x+3}}dx$

Let Linear = A $\frac{d}{dx}$ (Quadratic) + B

$\Rightarrow x+2 = A\frac{d}{dx}\left(x^2+2x+3\right)+B$

$\Rightarrow$ x + 2 = A(2x + 2) + B ...(ii)

$\Rightarrow$ x + 2 = 2Ax + 2A + B

Comparing coefficients of x, 2A = 1 $\Rightarrow A = \frac{1}{2}$

Comparing constants, 2A + B = 2

On solving, we get $A = \frac{1}{2}$, B = 1

Putting the values of A and B in eq. (ii),

$x+2 = \frac{1}{2}(2x+2)+1$

Putting this value of x+2 in eq. (i),

$I = \int \frac{\frac{1}{2}(2x+2)+1}{\sqrt{x^2+2x+3}}dx$

$I = \frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+3}}dx + \int \frac{1}{\sqrt{x^2+2x+3}}dx$

$\Rightarrow I = \frac{1}{2}I_1 + I_2$ ...(iii)

Now $I_1 = \int \frac{2x+2}{\sqrt{x^2+2x+3}}dx$

Putting x^2 + 2x + 3 = t

$\Rightarrow 2x+2 = \frac{dt}{dx}$

$\Rightarrow$ (2x + 2)dx = dt

$\therefore I_1 = \int \frac{dt}{\sqrt{t}} = \int t^{\frac{-1}{2}}dt$

$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} = 2\sqrt{t}$

$= 2\sqrt{x^2+2x+3}$ ...(iv)

Again $I_2 = \int \frac{1}{\sqrt{x^2+2x+3}}dx$

$= \int \frac{1}{\sqrt{x^2+2x+1+2}}$

$= \int \frac{1}{\sqrt{(x+1)^2+(\sqrt{2})^2}} dx$

$= \log\left|x+1+\sqrt{(x+1)^2+\left(\sqrt{2}\right)^2}\right|$

$= \log\left|x+1+\sqrt{x^2+2x+3}\right|$ ...(v)

Putting values of I_1 and I_2 in eq. (iii),

$I = \sqrt{x^2+2x+3} + \log\left|x+1+\sqrt{x^2+2x+3}\right| + c$

33. Here R is a relation on $N \times N$, defined by (a, b) R (c, d) $\Leftrightarrow$ a + d = b + c for all (a, b), (c, d) $\in N \times N$

We shall show that R satisfies the following properties

i. Reflexivity:

We know that a + b = b + a for all a, b $\in$ N.

$\therefore$ (a, b) R (a, b) for all (a, b) $\in (N \times N)$

So, R is reflexive.

ii. Symmetry:

Let (a, b) R (c, d). Then,

(a, b) R (c, d) $\Rightarrow$ a + d = b + c

$\Rightarrow$ c + b = d + a

$\Rightarrow$ (c, d) R (a, b).

$\therefore$ (a, b) R (c, d) $\Rightarrow$ (c, d) R (a, b) for all (a, b), (c, d) $\in N \times N$

This shows that R is symmetric.

iii. Transitivity:

Let (a, b) R (c, d) and (c, d) R (e, f). Then,

(a, b) R (c, d) and (c, d) R (e, f)

$\Rightarrow$ a + d = b + c and c + f = d + e

$\Rightarrow$ a + d + c + f = b + c + d + e

$\Rightarrow$ a + f = b + e

$\Rightarrow$ (a, b) R (e, f).

Thus, (a, b) R (c, d) and (c, d) R (e, f) $\Rightarrow$ (a, b) R (e, f)

This shows that R is transitive.

$\therefore$ R is reflexive, symmetric and transitive

Hence, R is an equivalence relation on $N \times N$

OR

We observe the following properties of f.

Injectivity: Let x, y $\in R_0$ such that f(x) = f(y). Then,

$f(x) = f(y) \Rightarrow \frac{1}{x} = \frac{1}{y} \Rightarrow x = y$

So, f : $R_0 \to R_0$ is one-one.

Surjectivity: Let y be an arbitrary element of R_0 (co-domain) such that f(x) = y. Then,

$f(x) = y \Rightarrow \frac{1}{x} = y \Rightarrow x = \frac{1}{y}$

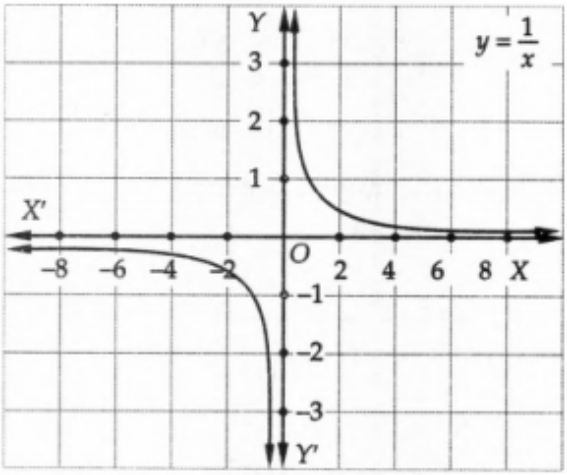
Clearly, $x = \frac{1}{y} \in R_0$ (domain) for all y $\in R_0$ (co-domain).

Thus, for each y $\in R_0$ (co-domain) there exits $x = \frac{1}{y} \in R_0$(domain) such that $f(x) = \frac{1}{x} = y$

So, f : $R_0 \to R_0$ is onto.

Hence, f: $R_0 \to R_0$ is one-one onto.

This is also evident from the graph of f(x) as shown in fig.



Let us now consider f : N $\rightarrow$ R_0 given by $f(x) = \frac{1}{x}$

For any x, y $\in$ N, we find that

f(x) = f(y) $\Rightarrow \frac{1}{x} = \frac{1}{y} \Rightarrow x = y$

So, f: N $\rightarrow$ R_0 is one-one.

We find that $\frac{2}{3}, \frac{3}{5}$ etc. in co-domain R_0 do not have their pre-image in domain N. So, f: N $\rightarrow R_0$ is not onto.

Thus, f: N $\rightarrow R_0$ is one-one but not onto.

34. Put $\frac{1}{x} = u$, $\frac{1}{y} = v$ and $\frac{1}{z} = w$ in the given equations,

$2u + 3v + 10w = 4;\ 4u - 6v + 5w = 1;\ 6u + 9v - 20w = 2$

$\therefore$ The matrix form of given equations is $\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix} \begin{bmatrix} x \\ v \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$ [AX= B]

Here, $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X \begin{bmatrix} x \\ v \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix}$

$= 2(120 - 45) - 3(-80 - 30) + 10(36 + 36)$

$= 150 + 330 + 750 = 1200 \neq 0$

$\therefore A^{-1}$exists and unique solution is $X = A^{-1}B$....(i)

Now $A_{11} = 75, A_{12} = 110, A_{13} = 72$ and $A_{21} = 150, A_{22} = -100, A_{23} = 0$ and $A_{31} = 75, A_{32} = 30, A_{33} = -24$

$\therefore adj.\ A = \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}' = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$

And $A^{-1} = \frac{adj.A}{|A|} = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$

$\therefore$ From eq. (i),

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{1200} \begin{bmatrix} 300+150+150 \\ 440-100+60 \\ 288+0-48 \end{bmatrix}$

$= \frac{1}{1200} \begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$

$\therefore u = \frac{1}{2}, v = \frac{1}{3}, w = \frac{1}{5}$

$\Rightarrow x = \frac{1}{u} = 2, y = \frac{1}{v} = 3, z = \frac{1}{w} = 5$

35. We have, 2l + 2m - n = 0 .... (i)

And mn + nl + lm = 0 .... (ii)

Eliminating m from the both equations, we get

$m = \frac{n-2l}{2}$ [from Eq. (i)]

$\Rightarrow \left(\frac{n-2l}{2}\right) n + nl + l\left(\frac{n-2l}{2}\right) = 0$

$\Rightarrow \frac{n^2-2nl+2nl+nl-2l^2}{2} = 0$

$\Rightarrow$ n^2 + nl - $2l^2$ = 0

$\Rightarrow$ n^2 + 2nl - nl - $2l^2$ = 0

$\Rightarrow$ (n + 2l)(n - l) = 0

$\Rightarrow$ n = -2l and n = l

$\therefore m = \frac{-2l-2l}{2}, m = \frac{l-2l}{2}$

$\Rightarrow m = -2l, m = \frac{-l}{2}$

Thus, the direction ratios of two lines are proportional to l, -2l, -2 and $l, \frac{-l}{2}, l$.

$\Rightarrow$ 1, -2, -2 and $1, \frac{-1}{2}, 1$

$\Rightarrow$ 1, -2, -2 and 2, -1, 2

Also, the vectors parallel to these lines are $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$ respectively.

$\therefore \cos\theta = \frac{\vec{a} \cdot \vec{b}}{\left|\vec{a}\right|\left|\vec{b}\right|} = \frac{(i-2\hat{j}-2\hat{k})\cdot(2\hat{i}-\hat{j}+2\hat{k})}{3\cdot 3}$

$= \frac{2+2-4}{9} = 0$

$\therefore \theta = \frac{\pi}{2}\left[\because \cos\frac{\pi}{2} = 0\right]$

OR

Here,it is given that

$\vec{r} = (6\hat{i} + 3\hat{k}) + \lambda(2\hat{i} - \hat{j} + 4\hat{k})$

$\vec{r} = (-9\hat{i} + \hat{j} - 10\hat{k}) + \mu(4\hat{i} + \hat{j} + 6\hat{k})$

Here,we have

$\vec{a_1} = 6\hat{i} + 3\hat{k}$

$\vec{\mathrm{b}_1} = 2\hat{\mathrm{i}} - \hat{\mathrm{j}} + 4\hat{\mathrm{k}}$

$\vec{a_2} = -9i + j - 10\hat{k}$

$\vec{\mathrm{b}_2} = 4\hat{i} + \hat{j} + 6\hat{\mathrm{k}}$

Thus,

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 4 \\ 4 & 1 & 6 \end{vmatrix}$$

$= i(-6-4) - j(12-16) + \hat{k}(2+4)$

$\therefore \vec{b_1} \times \vec{b_2} = -10\hat{i} + 4\hat{j} + 6\hat{k}$

$\therefore |\vec{b_1} \times \vec{b_2}| = \sqrt{(-10)^2 + 4^2 + 6^2}$

$= \sqrt{100 + 16 + 36}$

$= \sqrt{152}$

$\vec{a_2} - \vec{a_1} = (-9-6)\hat{1} + (1-0)\hat{j} + (6-3)\hat{k}$

$\therefore \vec{a_2} - \vec{a_1} = -15\hat{i} + \hat{j} + 3\hat{k}$

Now,

$(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1}) = (-10\hat{i} + 4\hat{j} + 6\hat{k}) \cdot (-15\hat{i} + \hat{j} + 3\hat{k})$

$= ((-10) \times (-15)) + (4 \times 1) + (6 \times 3)$

= 150 + 4 + 18

= 172

Thus, the shortest distance the given lines is

$d = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$

$\therefore d = \left| \frac{172}{\sqrt{152}} \right|$

$\therefore d = \frac{172}{2\sqrt{38}}$

$\therefore d = \frac{86}{\sqrt{38}}$

$d = \frac{86}{\sqrt{38}}$ units

Section E

36. i. $P\left(\frac{L}{\bar{C}}\right)$ = $\frac{17}{100}$

ii. $P\left(\frac{L}{A}\right)$ = 1 - $P\left(\frac{L}{A}\right)$ = 1 - $\frac{24}{100}$ = $\frac{76}{100}$ or $\frac{19}{25}$

iii. $P\left(\frac{A}{L}\right)$ = $\frac{\frac{1}{4} \times \frac{24}{100}}{\frac{1}{4} \times \frac{24}{100} + \frac{1}{4} \times \frac{22}{100} + \frac{1}{4} \times \frac{17}{100} + \frac{1}{4} \times \frac{9}{100}}$ = $\frac{24}{72} = \frac{1}{3}$

Probability that a randomly selected child is left-handed given that exactly one of the parents is left-handed.

= $P\left(\frac{L}{B \cup C}\right)$ = $\frac{22}{100} + \frac{17}{100}$ = $\frac{39}{100}$

37. i. P = $\begin{matrix} \\ A \\ B \\ C \end{matrix} \begin{matrix} \begin{matrix} Fans & Mats & Plates \end{matrix} \\ \begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix} \end{matrix}$

Q = $\begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} \begin{matrix} Fans \\ Mats \\ Plates \end{matrix}$

ii. Clearly, total funds collected by each school is given by the matrix

$$PQ = \begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix} \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix}$$

$$= \begin{bmatrix} 1000+5000+1000 \\ 625+4000+1500 \\ 875+5000+2000 \end{bmatrix} = \begin{bmatrix} 7000 \\ 6125 \\ 7875 \end{bmatrix}$$

$\therefore$ Funds collected by school A is ₹7000.

Funds collected by school B is ₹6125.

Funds collected by school C is ₹7875.

$$New\ price\ matrix\ Q = 20\% \times \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} \begin{matrix} Fans \\ Mats \\ Plates \end{matrix}$$

iii. $$\Rightarrow Q = \begin{bmatrix} 25+25\times 0.20 \\ 100+100\times 0.20 \\ 50+50\times 0.20 \end{bmatrix} \begin{matrix} Fans \\ Mats \\ Plates \end{matrix}$$

$$Q = \begin{bmatrix} 30 \\ 120 \\ 60 \end{bmatrix} \begin{matrix} Fans \\ Mats \\ Plates \end{matrix}$$

$$\text{New price matrix Q} = 20\% \times \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} \begin{matrix} \text{Fans} \\ \text{Mats} \\ \text{Plates} \end{matrix}$$

$$\Rightarrow \text{Q} = \begin{bmatrix} 25+25\times 0.20 \\ 100+100\times 0.20 \\ 50+50\times 0.20 \end{bmatrix} \begin{matrix} \text{Fans} \\ \text{Mats} \\ \text{Plates} \end{matrix}$$

$$\text{Q} = \begin{bmatrix} 30 \\ 120 \\ 60 \end{bmatrix} \begin{matrix} \text{Fans} \\ \text{Mats} \\ \text{Plates} \end{matrix}$$

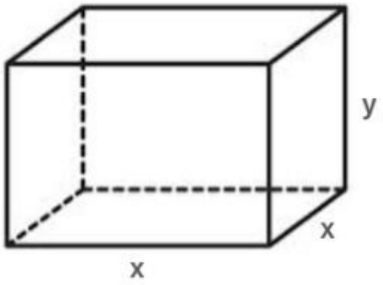
OR

$$PQ = \begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix} \begin{bmatrix} 30 \\ 120 \\ 60 \end{bmatrix}$$

$$PQ = \begin{bmatrix} 1200+6000+1200 \\ 750+4800+1800 \\ 1050+6000+2400 \end{bmatrix} = \begin{bmatrix} 8400 \\ 7350 \\ 9450 \end{bmatrix}$$

Total fund collected = 8400 + 7350 + 9450 = ₹25,200

38.



i. $V = x^2y \Rightarrow y = \frac{V}{x^2}$ ...(I)

Hence, $S = 2x^2 + 4xy = 2x^2 + \frac{4V}{x}$

ii. $\frac{dS}{dx} = 4\left(x - \frac{V}{x^2}\right)$

iii. a. $\frac{dS}{dx} = 0 \Rightarrow V = x^3 \Rightarrow x^2y = x^3 \Rightarrow y = x \frac{d^2\ s}{dx^2} = 4\left(1 + \frac{2\ V}{x^3}\right) > 0 \Rightarrow S$ is minimum if $y = x$.

OR

b. $V = \frac{1}{4}\left(Sx - 2x^3\right) \Rightarrow \frac{dV}{dx} = \frac{1}{4}\left(\ S - 6x^2\right)$

Put $\frac{dV}{dx} = 0 \Rightarrow x = \sqrt{\frac{s}{6}}$

$\left(\frac{d^2V}{dx^2}\right)_{x=\sqrt{\frac{s}{6}}} = -3\sqrt{\frac{s}{6}} < 0 \Rightarrow$ Volume is maximum for $x = \sqrt{\frac{s}{6}}$