Class XII Session 2026-27
Subject - Mathematics
Sample Question Paper - 1

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If A and B are two non-zero square matrices of same order such that $(A + B)^2 = A^2 + B^2$, then: **[1]**

   a) AB = O
   b) AB = BA
   c) AB = - BA
   d) BA = O

2. $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 15^\circ & \cos 15^\circ \end{vmatrix}$ = ? **[1]**

   a) $\frac{\sqrt{3}}{4}$
   b) $\frac{1}{2}$
   c) $\frac{\sqrt{3}}{2}$
   d) 1

3. If A is a non singular matrix and A' denotes the transpose of A, then **[1]**

   a) $|A| - |A'| \neq 0$
   b) $|AA'| \neq |A^2|$
   c) $|A| + |A'| \neq 0$
   d) $|A| \neq |A'|$

4. If $y = \frac{\cos x - \sin x}{\cos x + \sin x}$, then $\frac{dy}{dx}$ is: **[1]**

   a) $-\sec^2\left(\frac{\pi}{4} - x\right)$
   b) $-\log\left|\sec\left(\frac{\pi}{4} - x\right)\right|$
   c) $\log\left|\sec\left(\frac{\pi}{4} - x\right)\right|$
   d) $\sec^2\left(\frac{\pi}{4} - x\right)$

5. The angle between a line with direction ratios 2 : 2 : 1 and a line joining (3, 1, 4) to (7, 2, 12) **[1]**

a) $\tan^{-1}\left(-\frac{2}{3}\right)$ b) $\cos^{-1}\left(\frac{3}{2}\right)$

c) $\tan^{-1}\left(-\frac{3}{2}\right)$ d) $\cos^{-1}\left(\frac{2}{3}\right)$

6. $x \log x \frac{dy}{dx} + y = 2 \log x$ is an example of a: **[1]**

a) homogeneous differential equation. b) first order linear differential equation.

c) differential equation whose degree is not defined. d) variable separable differential equation.

7. Corner points of the feasible region determined by the system of linear constraints are (0, 3), (1, 1) and (3, 0). Let Z = px+qy, where p, q > 0. Condition on p and q so that the minimum of Z occurs at (3, 0) and (1, 1) is **[1]**

a) p = 3q b) p = 2q

c) $p = \frac{q}{2}$ d) p = q

8. If $\cos\left(\sin^{-1}\frac{2}{\sqrt{5}} + \cos^{-1}\mathrm{x}\right) = 0$, then x is equal to **[1]**

a) 1 b) $\frac{1}{\sqrt{5}}$

c) $-\frac{2}{\sqrt{5}}$ d) $\frac{2}{\sqrt{5}}$

9. $\int \frac{1}{x^2} e^{-1/x}\, dx = ?$ **[1]**

a) $e^{-1/x} + C$ b) $\frac{e^{-1/x}}{x} + C$

c) $-\frac{e^{-1/x}}{x} + C$ d) $-e^{-1/x} + C$

10. For the matrix A = $\begin{vmatrix} 3 & 1 \\ 7 & 5 \end{vmatrix}$, find x and y so that $A^2 + xI = yA$ **[1]**

a) (0 , 8) b) (8, 8)

c) (-8, -0) d) (-8, -8)

11. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let F = 4x + 6y be the objective function. Maximum of F – Minimum of F = **[1]**

a) 18 b) 60

c) 42 d) 48

12. What is the value of λ for which **[1]**
$(\lambda\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) \times (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) = (2\hat{\mathbf{i}} - 11\hat{\mathbf{j}} - 7\hat{\mathbf{k}})$

a) 7 b) -2

c) 1 d) 2

13. For any 2 × 2 matrix A if A(adj A) = $\begin{pmatrix} 12 & 0 \\ 0 & 12 \end{pmatrix}$, then $|A|^3$ equals **[1]**

a) 12^3 b) 12^2

c) 12^4 d) 12^5

14. A meeting will be held only if all three members A, B and C are present. The **[1]**

probability that member A does not turn up is 0.10, member B does not turn up is 0.20 and member C does not turn up is 0.05. The probability of the meeting being cancelled is:

a) 0.001 b) 0.316

c) 0.65 d) 0.35

15. The general solution of the differential equation $\frac{dy}{dx}$ + y cot x = cosec x, is **[1]**

a) y sin x = x + C b) y + x (sin x + cos x) = C

c) x + y sin x = C d) x + y cos x = C

16. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors, no two of which are collinear and the vector $\vec{a} + \vec{b}$ is collinear with $\vec{c}$, $\vec{b} + \vec{c}$ is collinear with $\vec{a}$, then $\vec{a} + \vec{b} + \vec{c} =$ **[1]**

a) $\vec{b}$ b) None of these

c) $\vec{a}$ d) $\vec{c}$

17. If f(x) = 2|x| + 3 |sin x| + 6, then the right hand derivative of f(x) at x = 0 is: **[1]**

a) 5 b) 6

c) 3 d) 2

18. The Cartesian equation of a line passing through the point with position vector $\vec{a} = \hat{i} - \hat{j}$ and parallel to the line $\vec{r} = \hat{i} + \hat{k} + \mu(2\hat{i} - \hat{j})$, is **[1]**

a) $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{0}$ b) $\frac{x-1}{2} = \frac{y}{-1} = \frac{z-1}{0}$

c) $\frac{x-2}{1} = \frac{y+1}{0} = \frac{z}{1}$ d) $\frac{x+1}{2} = \frac{y+1}{-1} = \frac{z}{0}$

19. **Assertion (A):** If the circumference of the circle is changing at the rate of 10 cm/s, then the area of the circle changes at the rate 30 cm^2/s, if radius is 3 cm. **[1]**

Reason (R): If A and r are the area and radius of the circle, respectively, then rate of change of area of the circle is given by $\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$.

a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

20. Let R be the relation in the set of integers Z given by R = {(a, a) : 2 divides (a - a)} **[1]**

Assertion (A): R is a reflexive relation.

Reason (R): A relation is said to be reflexive. if x R x, $\forall$ x $\in$ z

a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

Section B

21. Write the interval for the principal value of function and draw its graph: $\tan^{-1}$ x. **[2]**

OR

Evaluate $\cos\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$

22. Find the intervals on which the function f(x) = $2x^3 - 3x^2 - 36x + 7$ is **[2]**

a. strictly increasing

b. strictly decreasing.

23. A man 1.6m tall walks at the rate of 0.3 m/s away from a street light is 4 m above the ground. At what rate is the tip of his shadow moving? At what rate is his shadow lengthening? **[2]**

OR

Write the minimum value of f(x) = x^x.

24. Evaluate: $\int \frac{\sin(x-a)}{\sin x} dx$ **[2]**

25. Using determinants, find the area of the triangle whose vertices are (- 2, 4), (2, - 6) and (5,4). Are the given points collinear? **[2]**

Section C

26. Evaluate: $\int_0^{\pi} \log(1+\cos x)dx$ **[3]**

27. It is known that 20% of the students in a school have above 90% attendance and 80% of the students are irregular. Past year results show that 80% of students who have above 90% attendance and 20% of irregular students get 'A' grade in their annual examination. At the end of a year, a student is chosen at random from the school and is found to have an 'A' grade. What is the probability that the student is irregular? **[3]**

28. Evaluate: $\int \frac{x^2}{(x^2+1)(3x^2+4)} dx$. **[3]**

OR

Integrate the function $\frac{\sqrt{x^2+1}\left[\log\left(x^2+1\right)-2\log x\right]}{x^4}$

29. Find the particular solution of the differential equation $(1 + y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$, given that y = 0 when x = 1. **[3]**

OR

Find the particular solution of the differential equation

$\left(xe^{\frac{y}{x}} + y\right) dx = xdy$, given that $y = 1$ when $x = 1$.

30. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = 3, \left|\vec{b}\right| = 4, |\vec{c}| = 5$and each one of them being $\perp$ to the sum of the other two, find $\left|\vec{a} + \vec{b} + \vec{c}\right|$ **[3]**

OR

Find the distance of the point P(-2, - 4, 7) from the point of intersection Q of the line $\overrightarrow{r} = (3\hat{i} - 2\hat{j} + 6\hat{k}) + \lambda(2\hat{i} - \hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 6$. Also, write the vector equation of the line PQ.

31. Differentiate the function $\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right], 0 < x < \frac{\pi}{2}$ w.r.t. x. **[3]**

Section D

32. Using integration, find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ bounded between the lines $x = -\frac{a}{2}$ to $x = \frac{a}{2}$. **[5]**

33. Let R be a relation on $N \times N$, defined by (a, b) R (c, d) $\Leftrightarrow$ a + d = b + c for all (a, b), (c, d) $\in N \times N$. Show that R is an equivalence relation. **[5]**

OR

Let A = [-1, 1]. Then, discuss whether the following functions defined on A are one-one, onto or bijective:

i. $f(x) = \frac{x}{2}$

ii. g(x) = |x|

iii. h(x) = x|x|

iv. $k(x) = x^2$

34. If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that $I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$. **[5]**

35. Show that semi – vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$ **[5]**

OR

Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.

Section E

36. A shop selling electronic items sells smartphones of only three reputed companies A, B and C because chances of their manufacturing a defective smartphone are only $5\%, 4\%$ and 2% respectively. In his inventory he has 25% smartphones from company A, 35% smartphones from company B and 40% smartphones from company C. **[4]**

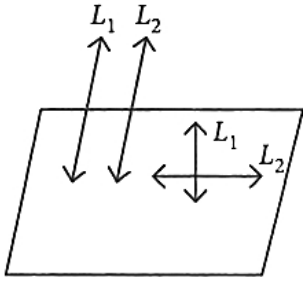
A person buys a smartphone from this shop.

i. Find the probability that it was defective. **(2)**

ii. What is the probability that this defective smartphone was manufactured by company B? **(2)**

37. **Read the following text carefully and answer the questions that follow:** **[4]**

If a_1, b_1, c_1 and a_2, b_2, c_2 are direction ratios of two lines say L_1 and L_2 respectively. Then $L_1 \parallel L_2$ iff $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ and $L_1 \perp L_2$ iff $a_1a_2 + b_1b_2 + c_1c_2 = 0$.



i. Find the coordinates of the foot of the perpendicular drawn from the point A(1, 2, 1) to the line joining B(1, 4, 6) and C(5, 4, 4). (1)

ii. Find the direction ratios of the line which is perpendicular to the lines with direction ratios proportional to (1, -2, -2) and (0, 2, 1). (1)

iii. What is the relation between lines $\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-2}{0}$ and $\frac{x-1}{1} = \frac{y+\frac{3}{2}}{\frac{3}{2}} = \frac{z+5}{2}$. (2)

OR

If l_1, m_1, n_1 and l_2, m_2, n_2 are direction cosines of L_1 and L_2 respectively, then what is the condition for L_1 parallel to L_2. (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Dinesh is having a jewelry shop at Green Park, normally he does not sit on the shop as he remains busy in political meetings. The manager Lisa takes care of jewelry shop where she sells earrings and necklaces. She gains profit of ₹30 on pair of earrings & ₹40 on neckless. It takes 30 minutes to make a pair of earrings and 1 hour to make a necklace, and there are 10 hours a week to make jewelry. In addition, there are only enough materials to make 15 total of jewelry items per week.



i. Formulate the above information mathematically. (1)

ii. Graphically represent the given data. (1)

iii. To obtain maximum profit how many pair of earing and neckleses should be sold? (2)

OR

What would be the profit if 5 pairs of earrings and 5 necklaces are made? (2)

Solution

Section A

1.

(c) AB = - BA

Explanation:

AB = - BA

2.

(c) $\frac{\sqrt{3}}{2}$

Explanation:

$\Delta = \left(\cos^2 15^\circ - \sin^2 15^\circ\right) = \cos(2 \times 15^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$.

3.

(c) |A| + |A'| $\neq 0$

Explanation:

Because, the determinant of a matrix and its transpose are always equal that is |A| = |A'|

4. **(a)** -$\sec^2\left(\frac{\pi}{4} - x\right)$

Explanation:

-$\sec^2\left(\frac{\pi}{4} - x\right)$

y = $\frac{\cos x - \sin x}{\cos x + \sin x}$

y = $\frac{cosx(1-\frac{sinx}{cosx})}{cosx(1+\frac{sinx}{cosx})}$

y = $\frac{1-tanx}{1+tanx}$

y = $\frac{tan\frac{\pi}{4}-tanx}{1+tan\frac{\pi}{4}tanx}$

y = tan ($\frac{\pi}{4}$-x)

$\frac{dy}{dx} = -sec^2(\frac{\pi}{4} - x)$

5.

(d) $\cos^{-1}\left(\frac{2}{3}\right)$

Explanation:

The angle between a line with direction ratios 2 : 2 : 1 and a line joining (3, 1, 4) to (7, 2, 12)

Direction ratios of the line joining the points A(3, 1, 4), B(7, 2, 12) is <x2-x1 , y2-y1, z2-z1> = < 7-3 , 2-1 , 12-4> = <4 ,1 ,8>

Now as the angle between two lines having direction ratios <a1,b1,c1> and <a2,b2,c2> is given by

$\text{Cos}^{-1}\frac{a1a2+b1b2+c1c2}{\sqrt{a1^2+b1^2+c1^2}\sqrt{a2^2+b2^2+c2^2}}$

Using the vuales we have

$\cos^{-1}\frac{2\times4+2\times1+1\times8}{\sqrt{2^2+2^2+1^2}\sqrt{4^2+1^2+8^2}}$

$= \cos^{-1}\frac{18}{27} = \cos^{-1}\frac{2}{3}$

6.

(b) first order linear differential equation.

Explanation:

first order linear differential equation.

7.

(c) $p = \frac{q}{2}$

Explanation:

We have Z = px + qy , At (3, 0) Z = 3p .....(1)

At (1 , 1) Z = p + q .....(2) Therefore , from (1) and (2) : We have : p = q/2 .

8.

(d) $\frac{2}{\sqrt{5}}$

Explanation:

$\frac{2}{\sqrt{5}}$

9. **(a)** $e^{-1/x} + C$

Explanation:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c; \int \sec^2 x dx = \tan x$

Therefore ,

Put $-\frac{1}{x} = t, \frac{1}{x^2} dx = dt$

$= \int e^t dt$

$= e^t + c$

$= e^{-\frac{1}{x}} + c$

10.

(b) (8, 8)

Explanation:

$A = \begin{vmatrix} 3 & 1 \\ 7 & 5 \end{vmatrix}, A^2 = \begin{vmatrix} 3 & 1 \\ 7 & 5 \end{vmatrix} \begin{vmatrix} 3 & 1 \\ 7 & 5 \end{vmatrix} = \begin{vmatrix} 16 & 8 \\ 56 & 32 \end{vmatrix}$

$\because A^2 + xI = yA$

$\Rightarrow \begin{vmatrix} 16 & 8 \\ 56 & 32 \end{vmatrix} + \begin{vmatrix} x & 0 \\ 0 & x \end{vmatrix} = \begin{vmatrix} 3y & y \\ 7y & 5y \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 16+x & 8 \\ 56 & 32+x \end{vmatrix} = \begin{vmatrix} 3y & y \\ 7y & 5y \end{vmatrix}$

$\Rightarrow$ y = 8, 16 + x = 3y = 24 $\Rightarrow$ x = 24 - 16 = 8

$\therefore$ x = 8, y = 8

11.

(b) 60

Explanation:

Here the objective function is given by : F = 4x +6y .

Corner points	Z = 4x +6 y
(0, 2)	12(Min.)
(3,0)	12.(Min.)
(6,0)	24

(6 , 8)	72
(0 , 5)	30

Maximum of F – Minimum of F = 72 – 12 = 30 .

12.

(d) 2

Explanation:

Given,

$(\lambda\hat{i}+\hat{j}-\hat{k})\times(3\hat{i}-2\hat{j}+4\hat{k})$

$=(2\hat{i}-11\hat{j}-7\hat{k})$

$\Rightarrow\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \lambda & 1 & -1\\ 3 & -2 & 4\end{vmatrix}=(2\hat{i}-11\hat{j}-7\hat{k})$

$\Rightarrow 2\hat{i}-(4\lambda+3)\hat{j}+(-2\lambda-3)\hat{k}\ =2\hat{i}-11\hat{j}-7\hat{k}$

On comparing the coefficient of, we get

$(4\lambda+3)$ = 11 $\Rightarrow\lambda$ = 2

13. **(a)** 12^3

Explanation:

A(adj A) = |A|I

$\Rightarrow$ |A| I = A(adj A) = $\begin{vmatrix}12 & 0\\ 0 & 12\end{vmatrix}$ = $12\begin{vmatrix}1 & 0\\ 0 & 1\end{vmatrix}$ = 12I

$\Rightarrow$ |A| = 12

$\therefore$ $|A|^3=(12)^3$

14.

(b) 0.316

Explanation:

0.316

15. **(a)** y sin x = x + C

Explanation:

We have,

$\frac{dy}{dx}$ + y cot x = cosec x

Comparing with $\frac{dy}{dx}$ + Py = Q of the above equation then, we get

$\Rightarrow$ P = cot x, Q = cosec x

I.F. = $e^{\int Pdx}=e^{\int\cot xdx}=e^{\log\sin x}$ = sin x

Multiplying on both sides by sin x

sin x $\frac{dy}{dx}$ + y cos x = 1

$\Rightarrow\frac{d}{dx}(y\sin x)$ = 1

$\Rightarrow$ y sin x = $\int 1dx$

$\Rightarrow$ y sin x = x + C

16.

(b) None of these

Explanation:

Given that $\vec{a}+\vec{b}$ is collinear with $\vec{c}$

$\therefore \vec{a}+\vec{b}=x\vec{c}$ ...(i)

where x is scalar and $x \neq 0$

$\vec{b}+\vec{c}$ is collinear with $\vec{a}$

$\vec{b}+\vec{c}=y\vec{a}$ ...(ii)

y is scalar and $y \neq 0$

Subtracting (ii) from (i) we get

$\vec{a}-\vec{c}=x\vec{c}-y\vec{a}$

$\vec{a}+y\vec{a}=x\vec{c}+\vec{c}$

$\vec{a}(1+y)=(1+x)\vec{c}$

As given

$\vec{a},\vec{c}$ are not collinear. (no two vecotors are collinear)

$\therefore$ 1 + y = 0 and 1 + x = 0

y = -1 and x = -1

Putting value of x in equation (i)

$\vec{a}+\vec{b}=-\vec{c}$

$\vec{a}+\vec{b}+\vec{c}=0$

17. **(a)** 5

    Explanation:

    5

18. **(a)** $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z}{0}$

    Explanation:

    $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z}{0}$

19. **(a)** Both A and R are true and R is the correct explanation of A.

    Explanation:

    Circumference of circle with radius r is given by C = 2πr

    Differentiating w.r.t. 't', we get

    $\frac{dC}{dt}=2\pi\frac{dr}{dt}$

    Given, $\frac{dC}{dt}=10\ \text{cm/s}$

    $\therefore 10=2\pi\frac{dr}{dt}$

    $\Rightarrow \frac{dr}{dt}=\frac{5}{\pi}\text{cm/s}$

    Now, Area of circle, A = πr^2

    $\therefore \frac{dA}{dt}=2\pi r\frac{dr}{dt}$

    Substituting r = 3 cm and $\frac{dr}{dt}=\frac{5}{\pi}\text{cm/s}$, we get

    $\frac{dA}{dt}=2\pi\times 3\times\frac{5}{\pi}$

    $\therefore \frac{dA}{dt}=30\ \text{cm}^2/\text{s}$

20. **(a)** Both A and R are true and R is the correct explanation of A.

    Explanation:

    By definition, a Relation in R us to be refelexive if X R X, $\forall$ x $\in$ Z

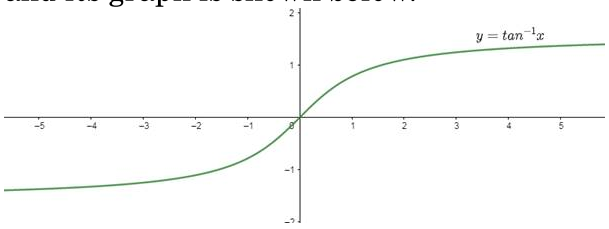
    So R is true.

    a - a = 0 $\Rightarrow$ 2 divides a - a $\Rightarrow$ a R a

    Hence, R is reflexive and A is true.

Section B

21. Principal value branch of $\tan^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
and its graph is shown below.



OR

We have, $\cos\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)+\frac{\pi}{6}\right]$

$\cos\left[\cos^{-1}\left(-cos\frac{\pi}{6}\right)+\frac{\pi}{6}\right]$

$= \cos\left[\cos^{-1}\left(\cos\frac{5\pi}{6}\right)+\frac{\pi}{6}\right]$

$= \cos\left(\frac{5\pi}{6}+\frac{\pi}{6}\right)\{\because \cos^{-1}\cos x = x,\ x \in [0,\pi]\}$

$= \cos\left(\frac{6\pi}{6}\right)$

$= \cos(\pi) = -1$

22. The given function is,

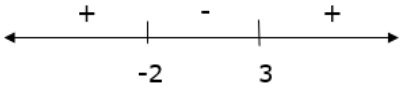
$f(x) = 2x^3 - 3x^2 - 36x + 7$

$f'(x) = 6x^2 - 6x - 36$

$f'(x) = 6(x^2 - x - 6)$

$f'(x) = 6(x - 3)(x + 2)$

f'(x) is 0 at x = 3 and x = -2



F'(x) > 0 for $x \in [-\infty, -2] \cup [3, \infty]$

Hence in this interval function is increasing.

F'(x) < 0 for $x \in$ (-2 ,3)

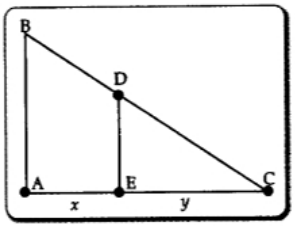
Hence in this interval function is decreasing.

23. Let AB represent the height of the street light from the ground. At any time t seconds, let the man represented as ED of height 1.6 m be at a distance of x m from AB and the length of his shadow EC by y m.

Using similarity of triangles, we have

$\frac{4}{1.6} = \frac{x+y}{y}$

$\Rightarrow$ 3y = 2x



Differentiating both sides w.r.t. t, we get

$3\frac{dy}{dt} = 2\frac{dx}{dt}$

$\frac{dy}{dt} = \frac{2}{3} \times 0.3 \Rightarrow \frac{dy}{dt} = 0.2$

At any time t seconds, the tip of his shadow is at a distance of (x + y) m from AB.

The rate at which the tip of his shadow moving $= \left(\frac{dx}{dt} + \frac{dy}{dt}\right)$ m/s = 0.5 m/s

The rate at which his shadow is lengthening $= \frac{dy}{dt}$ m/s $= 0.2$ m/s

OR

Given:

$f(x) = x^x$

$\because f'(x) = x^x (\log x + 1)$

For maxima and minima,

$f'(x) = 0$

$\Rightarrow x^x (\log x + 1) = 0$

$\Rightarrow x = e^{-1}$

Now,

$f''(x) = x^x (\log x + 1)^2 + \frac{x^x}{x}$

At $x = \frac{1}{e}$

$f''(x) > 0$ as $x^x (\log x + 1)^2 + \frac{x^x}{x} > 0$

$\because x = \frac{1}{e}$ is the point of local minima.

Hence, the minimum value of the function = $f\left(\frac{1}{e}\right) = \left(\frac{1}{e}\right)^{\frac{1}{e}} = e^{\frac{-1}{e}}$

24. Let $I = \int \frac{\sin(x-a)}{\sin x} dx$. Then,

$I = \int \frac{\sin x \cos a - \cos x \sin a}{\sin x} dx$

$\Rightarrow I = \int \cos a \, dx - \int \sin a \cot x \, dx$

$\Rightarrow I = \cos \int 1.dx - \sin a \int \cot x \, dx = x \cos a - \sin a \log | \sin x | + c$

25. Let Δ be the area of the triangle. Then,

$$\Delta = \frac{1}{2}\begin{vmatrix} -2 & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \frac{1}{2}\begin{vmatrix} -2 & 4 & 1 \\ 4 & -10 & 0 \\ 7 & 0 & 0 \end{vmatrix}$$ ...[Appyling $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$]

$$\Rightarrow \Delta = \frac{1}{2}\begin{vmatrix} 4 & -10 \\ 7 & 0 \end{vmatrix}$$ ...[By expanding along C_1]

$\Rightarrow \Delta = \frac{1}{2}(70) = 35$ sq. units

Clearly, $\Delta \neq 0$, therefore given points are not collinear.

Section C

26. Let the given integral be, $I = \int_0^{\pi} \log(1 + \cos x) dx$ ...(i)

Then, $I = \int_0^{\pi} \log(1 + \cos(\pi - x)] dx \ \left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$

$\Rightarrow \quad I = \int_0^{\pi} \log(1 - \cos x) dx$ ...(ii)

Adding (i) and (ii), we get

$2I = \int_0^{\pi} \{\log(1 + \cos x) + \log(1 - \cos x)\} dx$

$\Rightarrow \quad 2I = \int_0^{\pi} \log\left(1 - \cos^2 x\right) dx$

$\Rightarrow \quad 2I = \int_0^{\pi} \log \sin^2 x dx$

$\Rightarrow \quad 2I = 2\int_0^{\pi} \log \sin x dx$

$\Rightarrow \quad I = 2\int_0^{\pi/2} \log \sin x dx$ [Using property X]

$\Rightarrow \quad I = 2x - \frac{\pi}{2}\log_e 2 \left[\because \int_0^{\pi/2} \log \sin x dx = -\frac{\pi}{2}\log_e 2\right]$

$\Rightarrow \quad I = -\pi \log_e 2$

27. Let B : Student getting 'A' grade

E_1 : Student having above 90% attendance

E_2 : Student being irregular

$P(E_1) = \frac{20}{100}; P(E_2) = \frac{80}{100} P(B \mid E_1) = \frac{80}{100}; P(B \mid E_2) = \frac{20}{100}$

$P(E_2 \mid B) = \frac{P(E_2)P(B|E_2)}{P(E_1)P(B|E_1)+P(E_2)P(B|E_2)}$

$= \frac{0.8\times0.2}{0.2\times0.8+0.8\times0.2} = \frac{1}{2}$

28. We will solve this integral by partial fraction

Let $\frac{x^2}{(x^2+1)(3x^2+4)} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(3x^2+4)}$

$\Rightarrow x^2 = (Ax + B)(3x^2 + 4) + (Cx + D)(x^2 + 1)$

$= (3A + C)x^3 + (3B + D)x^2 + (4A + C)x + 4B + D$

Equating similar terms, we get,

$3A + C = 0, 3B + D = 1, 4A + C = 0, 4B + D = 0$

Solving, we get, $A = 0, B = -1, C = 0, D = 4$

Thus,

$I = \int \frac{-dx}{(x^2+1)} + \int \frac{4dx}{(3x^2+4)}$

$= -\tan^{-1}x + \frac{4}{3} \cdot \frac{\sqrt{3}}{2} \tan^{-1}\left(\frac{\sqrt{3}x}{2}\right) + c$

$I = \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{\sqrt{3}x}{2}\right) - \tan^{-1}x + c$

OR

Given integrand is: $\frac{\sqrt{x^2+1}\left[\log\left(x^2+1\right)-2\log x\right]}{x^4}$

Here we can rewrite integrand as; $\frac{\sqrt{x^2+1}\left[\log\left(x^2+1\right)-2\log x\right]}{x^4}$

$= \frac{\sqrt{x^2+1}}{x^4}\left[\log\left(x^2+1\right) - \log x^2\right]$

$= \frac{1}{x^3}\sqrt{\frac{x^2+1}{x^2}}\left[\log\left(\frac{x^2+1}{x^2}\right)\right]$

$= \frac{1}{x^3}\sqrt{1+\frac{1}{x^2}}\left[\log\left(1+\frac{1}{x^2}\right)\right]$

Let, $1+\frac{1}{x^2} = t \Rightarrow -\frac{2}{x^3}dx = dt$

Now, Let $I = \int \frac{\sqrt{x^2+1}\left[\log\left(x^2+1\right)-2\log x\right]}{x^4}dx = \int \frac{1}{x^3}\sqrt{1+\frac{1}{x^2}}\left[\log\left(1+\frac{1}{x^2}\right)\right]dx$

$= \int -\frac{1}{2} \cdot \sqrt{t}[\log(t)]dt$

$= \int -\frac{1}{2} \cdot \sqrt{t}[\log(t)]dt = -\frac{1}{2}\left[\log t. \int \sqrt{t}dt - \int \frac{d}{dt}\log t. \left\{\int \sqrt{t}dt\right\} dt\right]$

$= -\frac{1}{2}\left[\log t \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \int \frac{1}{t} \cdot \left\{\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right\} dt\right]$

$= -\frac{1}{2}\left[\frac{2}{3}t^{\frac{3}{2}} \log t - \int \left\{\frac{t^{\frac{1}{2}-1}}{\frac{3}{2}}\right\} dt\right]$

$= -\frac{1}{2}\left[\frac{2}{3}t^{\frac{3}{2}} \log t - \frac{2}{3}\int t^{\frac{1}{2}} dt\right]$

$= -\frac{1}{2}\left[\frac{2}{3}t^{\frac{3}{2}} \log t - \frac{2}{3} \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right]$

$= \left[-\frac{1}{2} \cdot \frac{2}{3}t^{\frac{3}{2}} \log t + \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot t^{\frac{3}{2}}\right]$

$= -\frac{1}{3}t^{\frac{3}{2}}\left[\log t - \frac{2}{3}\right]$

$\Rightarrow 1 = -\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{\frac{3}{2}}\left[\log\left(1+\frac{1}{x^2}\right) - \frac{2}{3}\right] + C$

29. Given differential equation can be written as

$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{e^{\tan^{-1}y}}{1+y^2}$

L.F. $= e^{\int \frac{dy}{1+y^2}} = e^{\tan^{-1}y}$

Solution is given by

$xe^{\tan^{-1}y} = \int \frac{e^{\tan^{-1}y}}{1+y^2} \times e^{\tan^{-1}y}\, dy = \int \frac{e^{2\tan^{-1}y}}{1+y^2}\, dy$

or $xe^{\tan^{-1}y} = \frac{e^{2\tan^{-1}y}}{2} + c$

when $x = 1, y = 0$ or $c = \frac{1}{2}$

$\therefore$ Solution is given by $xe^{\tan^{-1}y} = \frac{1}{2}e^{2\tan^{-1}y} + \frac{1}{2}$

or $x = \frac{1}{2}\left(e^{\tan^{-1}y} + e^{-\tan^{-1}y}\right)$

OR

$\frac{dy}{dx} = e^{\frac{y}{x}} + \frac{y}{x} = f\left(\frac{y}{x}\right)$ so, its a homogeneous differential equation

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Now, $v + x\frac{dv}{dx} = e^v + v$

$\Rightarrow \int e^{-v}dv = \int \frac{1}{x}dx$

$\Rightarrow -e^{-v} = \log|x| + c \Rightarrow -e^{\frac{-y}{x}} = \log|x| + c \ldots (1)$

Now, $x = 1, y = 1$, gives $c = -e^{-1}$

Thus, $\log|x| + e^{\frac{-y}{x}} = e^{-1}$

30. $\vec{a}.\left(\vec{b}+\vec{c}\right) = 0, \vec{b}.\left(\vec{c}+\vec{a}\right) = 0, \vec{c}.\left(\vec{a}+\vec{b}\right) = 0$ (Given)

$\left|\vec{a}+\vec{b}+\vec{c}\right|^2 = \left(\vec{a}+\vec{b}+\vec{c}\right).\left(\vec{a}+\vec{b}+\vec{c}\right)$

$= \vec{a}.\vec{a} + \vec{a}.\left(\vec{b}+\vec{c}\right) + \vec{b}.\vec{b} + \vec{b}.\left(\vec{a}+\vec{c}\right) + \vec{c}.\vec{c} + \vec{c}.\left(\vec{a}+\vec{b}\right)$

$= |\vec{a}|^2 + \left|\vec{b}\right|^2 + |\vec{c}|^2$

= 9 + 16 + 25

= 50

$\left|\vec{a}+\vec{b}+\vec{c}\right| = \sqrt{50}$

$= 5\sqrt{2}$

OR

Given :

Line: $\vec{r} = (3\hat{i} - 2\hat{j} + 6\hat{k}) + \lambda(2\hat{i} - \hat{j} + 2\hat{k})$

$\vec{r} = (3+2\lambda)\hat{i} + (-2-\lambda)\hat{j} + (6+2\lambda)\hat{k}$

Plane: $\vec{r}\cdot(\hat{i} - \hat{j} + \hat{k}) = 6$

Intersection Point:

$[(3+2\lambda)\hat{i} + (-2-\lambda)\hat{j} + (6+2\lambda)\hat{k}]\cdot(\hat{i} - \hat{j} + \hat{k}) = 6$

$3 + 2\lambda + 2 + \lambda + 6 + 2\lambda = 6$

$5\lambda = -5 \Rightarrow \lambda = -1$

Point Q $(1, -1, 4)$

Point $P(-2, -4, 7)$

$PQ = \sqrt{(1+2)^2 + (-1+4)^2 + (4-7)^2} = \sqrt{9+9+9} = 3\sqrt{3}$

Line PQ : $\overrightarrow{r} = -2\hat{i} - 4\hat{j} + 7\hat{k} + \mu(3\hat{i} + 3\hat{j} - 3\hat{k})$

31. $y = \cot^{-1}\left[\frac{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2} + \sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2} - \sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}\right] \left[\because \sqrt{1 \pm \sin x} = \sqrt{\left(\cos\frac{x}{2} \pm \sin\frac{x}{2}\right)^2}\right]$

$= \cot^{-1}\frac{\cos\frac{x}{2}+\sin\frac{x}{2}+\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}-\cos\frac{x}{2}+\sin\frac{x}{2}}$

$= \cot^{-1}\left(\frac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}\right)$

$= \cot^{-1}\left(\cot\frac{x}{2}\right)$

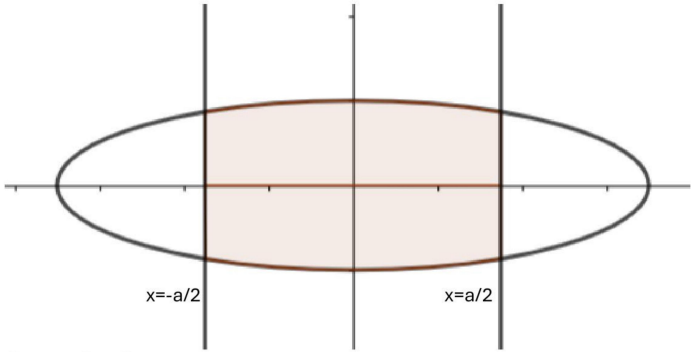
$= \frac{x}{2}$

$y = \frac{x}{2}$

$\frac{dy}{dx} = \frac{1}{2}$

Section D

32.



Required area $= \frac{4b}{a}\int_0^{\frac{a}{2}}\sqrt{a^2-x^2}dx$

$= \frac{4b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^{\frac{a}{2}}$

$= \frac{4b}{a}\left[\frac{a}{4}\cdot\frac{\sqrt{3}a}{2}+\frac{a^2}{2}\cdot\frac{\pi}{6}\right]$

$= ab\left[\frac{\sqrt{3}}{2}+\frac{\pi}{3}\right]$

33. Here R is a relation on $N \times N$, defined by (a, b) R (c, d) $\Leftrightarrow$ a + d = b + c for all (a, b), (c, d) $\in N \times N$

We shall show that R satisfies the following properties

i. Reflexivity:

We know that a + b = b + a for all a, b $\in$ N.

$\therefore$ (a, b) R (a, b) for all (a, b) $\in (N \times N)$

So, R is reflexive.

ii. Symmetry:

Let (a, b) R (c, d). Then,

(a, b) R (c, d) $\Rightarrow$ a + d = b + c

$\Rightarrow$ c + b = d + a

$\Rightarrow$ (c, d) R (a, b).

$\therefore$ (a, b) R (c, d) $\Rightarrow$ (c, d) R (a, b) for all (a, b), (c, d) $\in N \times N$

This shows that R is symmetric.

iii. Transitivity:

Let (a, b) R (c, d) and (c, d) R (e, f). Then,

(a, b) R (c, d) and (c, d) R (e, f)

$\Rightarrow$ a + d = b + c and c + f = d + e

$\Rightarrow$ a + d + c + f = b + c + d + e

$\Rightarrow$ a + f = b + e

$\Rightarrow$ (a, b) R (e, f).

Thus, (a, b) R (c, d) and (c, d) R (e, f) $\Rightarrow$ (a, b) R (e, f)

This shows that R is transitive.

$\therefore$ R is reflexive, symmetric and transitive

Hence, R is an equivalence relation on $N \times N$

OR

Given that A = [-1, 1]

i. $f(x) = \frac{x}{2}$

Let $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1}{2} = \frac{x_2}{2} \Rightarrow x_1 = x_2$

So, f(x) is one-one.

Now, let $y = \frac{x}{2}$

$\Rightarrow x = 2y \notin A,\ \forall y \in A$

As for $y = 1 \in A,\ x = 2 \notin A$

So, f(x) is not onto.

Also, f(x) is not bijective as it is not onto.

ii. g(x) = |x|

Let $g(x_1) = g(x_2)$

$\Rightarrow |x_1| = |x_2| \Rightarrow x_1 = \pm x_2$

So, g(x) is not one-one.

Now, $x = \pm y \notin A$ for all $y \in R$

So, g(x) is not onto, also, g(x) is not bijective.

iii. h(x) = x|x|

$\Rightarrow x_1|x_1| = x_2|x_2| \Rightarrow x_1 = x_2$

So, h(x) is one-one

Now, let y = x|x|

$\Rightarrow y = x^2 \in A, \forall x \in A$

So, h(x) is onto also, h(x) is a bijective.

iv. $k(x) = x^2$

Let $k(x_1) = k(x_2)$

$\Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$

Thus, k(x) is not one-one.

Now, let $y = x^2$

$\Rightarrow x\sqrt{y} \notin A, \forall y \in A\ x = \sqrt{y} \notin A, \forall y \in A$

As for y = -1, $x = \sqrt{-1} \notin A$

Hence, k(x) is neither one-one nor onto.

34. L.H.S. $I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix} = \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$

Now, $I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix} = \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}$

R.H.S. $= (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} \cos\alpha + \sin\alpha\tan\frac{\alpha}{2} & -\sin\alpha + \cos\alpha\tan\frac{\alpha}{2} \\ -\cos\alpha\tan\frac{\alpha}{2} + \sin\alpha & \sin\alpha\tan\frac{\alpha}{2} + \cos\alpha \end{bmatrix}$

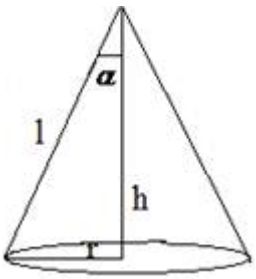
$= \begin{bmatrix} \cos\alpha + \sin\alpha\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & -\sin\alpha + \cos\alpha\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \\ -\cos\alpha\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} + \sin\alpha & \sin\alpha\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} + \cos\alpha \end{bmatrix}$

$$= \begin{bmatrix} \frac{\cos\alpha\cos\frac{\alpha}{2}+\sin\alpha\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \frac{-\sin\alpha\cos\frac{\alpha}{2}+\cos\alpha\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \\ \frac{-\cos\alpha\sin\frac{\alpha}{2}+\sin\alpha\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \frac{\sin\alpha\sin\frac{\alpha}{2}+\cos\alpha\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\cos\left(\alpha-\frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} & \frac{-\sin\left(\alpha-\frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} \\ \frac{\sin\left(\alpha-\frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} & \frac{\cos\left(\alpha-\frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} \end{bmatrix} = \begin{bmatrix} \frac{\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \frac{-\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \\ \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \frac{\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \end{bmatrix} = \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$$

$\therefore$ L.H.S. = R.H.S. Proved.

35.



$s = \pi r^2 + \pi rl$ (given)

$\Rightarrow l = \frac{s-\pi r^2}{\pi r}$

Let v be the volume

$v = \frac{1}{3}\pi r^2 h$

$\Rightarrow v^2 = \frac{1}{9}\pi^2 r^4 h^2 \quad \left[h^2 = l^2 - r^2\right]$

$\Rightarrow v^2 = \frac{1}{9}\pi^2 r^4 \left(l^2 - r^2\right)$

$\Rightarrow v^2 = \frac{1}{9}\pi^2 r^4 \left[\left(\frac{s-\pi r^2}{\pi r}\right)^2 - r^2\right]$

$= \frac{1}{9}\pi^2 r^4 \left[\frac{\left(s-\pi r^2\right)^2}{\pi^2 r^2} - \frac{r^2}{1}\right]$

$= \frac{1}{9} r^2 \left[\left(s-\pi r^2\right)^2 - \pi^2 r^4\right]$

$= \frac{1}{9} r^2 \left[s^2 + \pi^2 r^4 - 2s\pi r^2 - \pi^2 r^4\right]$

$= \frac{1}{9} r^2 \left[s^2 - 2s\pi r^2\right]$

$z = \frac{1}{9}\left[s^2 r^2 - 2s\pi r^4\right]$

$\left[\because v^2 = z\right]$

Now $\frac{dz}{dr} = \frac{1}{9}\left[2rs^2 - 8s\pi r^3\right]$

$0 = \frac{1}{9}\left[2rs^2 - 8s\pi r^3\right]$

$8s\pi r^2 = 2rs^2$

$\implies 4\pi r^2 = s$

Now $\frac{d^2 z}{dx^2} = \frac{1}{9}\left[2s^2 - 24s\pi r^2\right]$

$\left.\frac{d^2 z}{dx^2}\right]_{r^2 = \frac{s}{4\pi}} = \frac{1}{9}\left[25^2 - 24\pi.\frac{5}{4\pi}\right]$

= + ve

Hence minimum

Now $s = 4\pi r^2$

We have $s = \pi rl + \pi r^2$

$4\pi r^2 = \pi rl + \pi r^2$

$\Rightarrow 3\pi r^2 = \pi rl$

$\Rightarrow 3$ r = l

$\Rightarrow \frac{r}{l} = \frac{1}{3}$

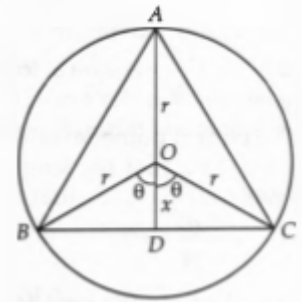
$\Rightarrow \sin\alpha = \frac{1}{3}$

$\therefore \alpha = \sin^{-1}\left(\frac{1}{3}\right)$

OR

Let ABC be a triangle inscribed in a given circle with centre O and radius r.

The area of the triangle will be maximum if its vertex A opposite to the base BC is at a maximum distance from the base BC. This is possible only when A lies on the diameter perpendicular to BC. Thus, AD $\perp$ BC. So, triangle ABC must be an isosceles triangle. Let OD = x. Applying Pythagoras theorem in right triangle ODB, we get



$OB^2 = OD^2 + BD^2$

$\Rightarrow r^2 = x^2 + BD^2$

$\Rightarrow BD = \sqrt{r^2 - x^2}$

$\because BC = 2BD = 2\sqrt{r^2 - x^2}$

Also, AD = AO + OD = r + x.

Let A denote the area of ΔABC, Then,we have,

$A = \frac{1}{2}(BC \times AD)$

$\Rightarrow A = \frac{1}{2} \times 2\sqrt{r^2 - x^2} \times (r + x)$

$\Rightarrow A = (r + x)\sqrt{r^2 - x^2}$

$\Rightarrow \frac{dA}{dx} = \sqrt{r^2 - x^2} - \frac{x(r+x)}{\sqrt{r^2-x^2}}$ [differentiating both sides w.r.t x]

$\Rightarrow \frac{dA}{dx} = \frac{r^2 - rx - 2x^2}{\sqrt{r^2-x^2}}$

The critical points of A are given by $\frac{dA}{dx} = 0$.

$\because \frac{dA}{dx} = 0$

$\Rightarrow \frac{r^2 - rx - 2x^2}{\sqrt{r^2-x^2}} = 0$

$\Rightarrow (r - 2x)(r + x) = 0$

$\Rightarrow r - 2x = 0$

$\Rightarrow x = \frac{r}{2}$

Now, $\frac{dA}{dx} = \frac{r^2 - rx - 2x^2}{\sqrt{r^2-x^2}}$

$\Rightarrow \frac{d^2A}{dx^2} = \frac{(-r-4x)}{\sqrt{r^2-x^2}} + \frac{(r^2 - rx - 2x^2)x}{(r^2-x^2)^{3/2}}$ [Differentiating both sides with respect to x]

$\Rightarrow \left(\frac{d^2A}{dx^2}\right)_{x=r/2} = -2\sqrt{3} < 0$

Therefore, A is maximum when $x = \frac{r}{2}$.

$\because BD = \sqrt{r^2 - x^2} \Rightarrow BD = \frac{\sqrt{3}r}{2}$

In Δ ODB,we have,

$\tan\theta = \frac{BD}{OD} \Rightarrow \tan\theta = \frac{\frac{\sqrt{3}r}{2}}{\frac{r}{2}} = \sqrt{3} \Rightarrow \theta = 60^\circ$

$\because \angle BAC = \theta = 60^\circ$

But, AB = AC Therefore, $\angle B = \angle c = 60^\circ$. Therefore, we obtain $\angle A = \angle B = \angle C = 60^\circ$.

Hence, A is maximum when ΔABC is equilateral.

Section E

36. i. $P(\text{defective smartphone}) = 0.25 \times 0.05 + 0.35 \times 0.04 + 0.40 \times 0.02$

$= 0.0345$

ii. $P(\ B/\ \text{Defective}) = \frac{0.35 \times 0.04}{0.25 \times 0.05 + 0.35 \times 0.04 + 0.40 \times 0.02}$

$= \frac{140}{345}$ or $\frac{28}{69}$

37. i. Equation of line joining B and C is

$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

$\frac{x-1}{4} = \frac{y-4}{0} = \frac{z-6}{-2} = \lambda$

Let coordinates of foot of perpendicular be D$(4\lambda + 1, 4, -2\lambda + 6\ )$ ...(i)

$\therefore$ D.R.'s of AD are $(4\lambda, 2, -2\lambda + 5)$.

Now, $4\,(4\lambda) + 0\,(2) + (-2)\,(-2\lambda + 5) = 0$

$\lambda = \frac{1}{2}$

Putting in eqn (i)

Coordinates of D are (3, 4, 5)

$\therefore$ Required coordinates are (3, 4, 5).

ii. Let a, b, c be the direction ratios of the required line. Since it is perpendicular to the lines whose direction ratios are (1, -2, -2) and (0, 2, 1) respectively.

$\therefore$ a - 2b - 2c = 0 ...(i)

a + 2b + c = 0 ...(ii)

On solving (i) and (ii) by cross-multiplication, we get

$\frac{a}{-2+4} = \frac{b}{0-1} = \frac{c}{2} \Rightarrow \frac{a}{2} = \frac{b}{-1} = \frac{c}{2}$

Thus, the direction ratios of the required line are (2, -1, 2)

iii. Direction ratio of given lines are (3, -2, 0) and $(1, \frac{3}{2}, 2\ )$

Now,

as $3 \cdot 1 + (-2) \cdot \left(\frac{3}{2}\right) + 0 \cdot 2 = 3 - 3 + 0 = 0$

$\therefore$ Given lines are perpendicular to each other.

OR

Since,direction ratio's are proportional to direction cosine's, therefore L_1 will be parallel to L_2, iff

$\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$

38. i. Let number of pairs of earing = x and number of Necklaces = y

As per the given information

$x, y \geq 0$

$0.5x + y \leq 10$

$x + y \leq 15$

Profit function = Z = 30x + 40y

ii. Let number of pairs of earing = x and number of Necklaces = y

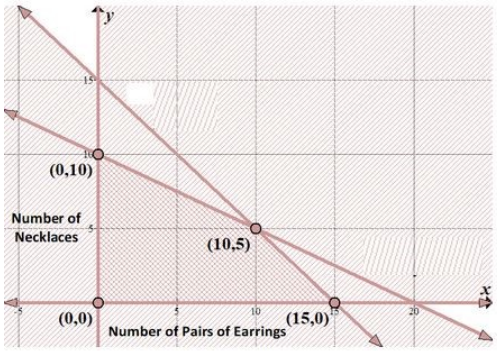
As per the given information

$x, y \geq 0$

$0.5x + y \leq 10$

$x + y \leq 15$

Profit function = Z = 30x + 40y



iii. From graph corner points are (0, 0), (0, 10), (10, 5) and (15, 0).

corner points	**maximum profit = Z = 30x + 40y**
(0, 0)	Z = 0
(0, 10)	Z = ₹400
(10, 5)	**Z = ₹500**
(15, 0)	Z = ₹450

Hence profit is maximum when x = number of pair of Earings = 10 and y = Number of Neckleses

OR

When x = 5 and y = 5

Z = 30x + 40y = 150 + 200 = ₹350