Class X Session 2026-27
Subject - Mathematics (Standard)
Sample Question Paper - 5

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment carrying 04 marks each.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

Section A

1. Lali tosses two different coins simultaneously. The probability that she gets at most one head is: **[1]**

   a) $\frac{1}{2}$ b) $\frac{1}{7}$

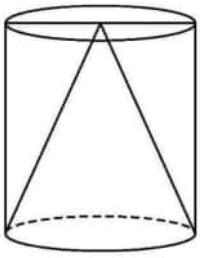
   c) $\frac{3}{4}$ d) 1

2. The roots of the quadratic equation $x^2 - 11x - 10 = 0$ are **[1]**

   a) not real roots b) real and equal

   c) real and distinct d) real and not equal

3. The volume of air in a hollow cylinder is $450cm^3$. A cone of same height and radius as that of cylinder is kept inside it. The volume of empty space in the cylinder is **[1]**



   a) $225cm^3$ b) $300cm^3$

   c) $150cm^3$ d) $250cm^3$

4. The sum of two numbers is 8 and the sum of their reciprocals is $\frac{8}{15}$.Find the numbers. **[1]**

a) 4, 4 b) 7, 1

c) 5, 3 d) 2, 6

5. In an A.P., the third term is 16 and the 7th term exceeds the 5th term by 12, then its first term is **[1]**

a) 3 b) 2

c) 4 d) 1

6. The distance between the points ($\cos\theta$, $\sin\theta$) and ($\sin\theta$, - $\cos\theta$) is **[1]**

a) $\sqrt{2}$ b) 1

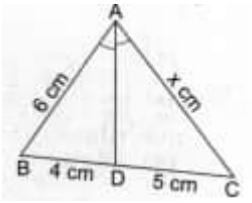
c) 2 d) $\sqrt{3}$

7. If α, β are the zeros of the polynomial f(x) = ax^2 + bx + c, then $\frac{1}{\alpha^2} + \frac{1}{\beta^2} =$ **[1]**

a) $\frac{b^2+2ac}{c^2}$ b) $\frac{b^2-2ac}{a^2}$

c) $\frac{b^2+2ac}{a^2}$ d) $\frac{b^2-2ac}{c^2}$

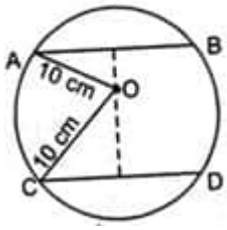
8. In a $\triangle$ ABC it is given that AD is the internal bisector of $\angle$A. If BD = 4 cm, DC = 5 cm and AB = 6 cm, then AC = ? **[1]**



a) 9 cm b) 7.5 cm

c) 8 cm d) 4.5 cm

9. In the given figure, O is the centre of the circle with radius 10 cm. If AB || CD, AB = 16 cm and CD = 12 cm, the distance between the two chords AB and CD is : **[1]**



a) 12 cm b) 14 cm

c) 16 cm d) 20 cm

10. From an external point P, a tangent PA is drawn to a circle. The number of tangents through P parallel to PA is: **[1]**

a) 2 b) 1

c) more than 2 d) 0

11. The value of $\left(\tan^2 A - \frac{1}{\cos^2 A}\right)$ is: **[1]**

a) 1 b) -1

c) more than 1 d) 0

12. A number when divided by 61 gives 27 as quotient and 32 as the remainder, then the number is: **[1]**

a) 1679 b) 1569

c) 1796 d) 1967

13. The angle of depression of a car, standing on the ground, from the top of a 75 m tower, is 30°. The distance of the car from the base of the tower (in metres) is **[1]**

a) $50\sqrt{3}$ b) 150

c) $25\sqrt{3}$ d) $75\sqrt{3}$

14. The area of a sector of a circle of radius 16 cm cut off by an arc, which is 18.5 cm long, is: **[1]**

a) 148 cm^2 b) 154 cm^2

c) 176 cm^2 d) 168 cm^2

15. If the area of a sector of circle of radius 36 cm is $54\pi\ cm^2$, then the length of the corresponding arc of the sector is: **[1]**

a) 5π cm b) 4π cm

c) 3π cm d) 2π cm

16. What is the probability that a leap year has 52 Mondays? **[1]**

a) $\frac{6}{7}$ b) $\frac{4}{7}$

c) $\frac{5}{7}$ d) $\frac{2}{7}$

17. If P (A) denotes the probability of an event A, then **[1]**

a) P(A) < 0 b) $0 \leq$ P(A) ≤ 1

c) -1 $\leq$ P(A) ≤ 1 d) P(A) > 1

18. The class mark of the median class of the following data is: **[1]**

Class Interval	$10-25$	$25-40$	$40-55$	$55-70$	$70-85$	$85-100$
Frequency	2	3	7	6	6	6

a) 47.5 b) 55

c) 62.5 d) 40

19. **Assertion (A):** If we join two hemispheres of same radius along their bases, then we get a sphere. **[1]**

Reason (R): Total Surface Area of a sphere of radius r is $3\pi r^2$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The sum of the series with the nth term. t_n = (9 - 5n) is (465), when no. **[1]**

of terms n = 15.

Reason (R): Given series is in A.P. and sum of n terms of an A.P. is $S_n = \frac{n}{2}[2a + (n-1)d]$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

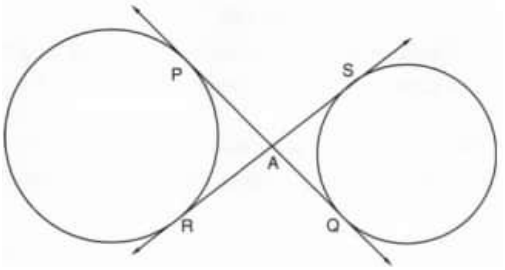
d) A is false but R is true.

Section B

21. In a teachers' workshop, the number of teachers teaching French, Hindi and English are 48, 80 and 144 respectively. Find the minimum number of rooms required if in each room the same number of teachers are seated and all of them are of the same subject. **[2]**

22. In a ΔABC, D and E are points on the sides AB and AC respectively such that $DE \| BC$. If AD = x, DB = x-2, AE = x + 2 and EC = x - 1, find the value of x. **[2]**

23. In fig common tangents PQ and RS to two circles intersect at A. Prove that PQ = RS. **[2]**



24. Find the values of A and $B\,(0 \le A < 90^\circ, 0 \le B < 90^\circ)$, if $\tan(A+B) = 1$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$. **[2]**

OR

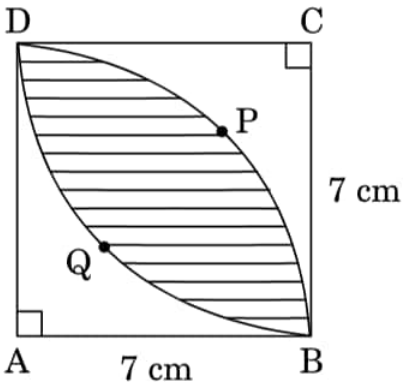
Verify 2 sin 30° cos 30° = sin 60°

25. An umbrella has 8 ribs which are equally spaced (see figure). Assuming umbrella to be a flat circle of radius 45 cm, Find the area between the two consecutive ribs of the umbrella. **[2]**



OR

Calculate the area of the shaded region common between two quadrants of circles of radius 7 cm each (as shown in Figure).



Section C

26. A shopkeeper has 120 litres of petrol, 180 litres of diesel and 240 litres of kerosene. He wants to sell oil by filling the three kinds of oils in tins of equal capacity. What should be the greatest capacity of such a tin? **[3]**

27. A point P divides the line segment joining the points A (3, - 5) and B (- 4, 8) such that $\frac{AP}{PB} = \frac{k}{1}$. If P lies on the line x + y = 0, then find the value of k. **[3]**

28. Solve: $\frac{1}{x+1} + \frac{2}{x+2} = \frac{5}{x+4}, x \neq$ -1, -2, -4 **[3]**

OR

Out of a group of swans, $\frac{7}{2}$ times the square root of the total number of swans are playing on the shore of a tank. Remaining two are playing, with amorous fight, in the water. What is the total number of swans?

29. From an external point P, a tangent PT and a line segment PAB is drawn to a circle with centre O. ON is perpendicular on the chord AB. Prove that. **[3]**

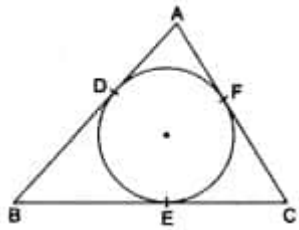
i. $PA.PB = PN^2 - AN^2$

ii. $PN^2 - AN^2 = OP^2 - OT^2$

iii. $PA.PB = PT^2$

OR

In the given figure, a circle inscribed a in a triangle ABC, touches the sides AB, BC and AC at points D, E and F respectively. If AB = 12 cm, BC = 8 cm and AC = 10 cm, find the lengths of AD, BE and CF .



30. Prove that $\frac{\cos A + \sin A - 1}{\cos A - \sin A + 1} = \text{cosec } A - \cot A$ **[3]**

31. A path separates two walls. A ladder leaning against one wall rests at a point on the path. It reaches a height of 90 m on the wall and makes an angle of 60° with the ground. If while resting at the same point on the path, it were made to lean against the other wall, it would have made an angle of 45° with the ground. Find the height it would have reached on the second wall. **[3]**

Section D

32. I am three times as old as my son. Five years later, I shall be two and a half times as **[5]**

old as my son. How old am I and how old is my son?

OR

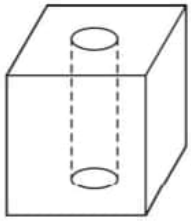
Tara scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each wrong answer, then Tara would have scored 50 marks. Assuming that Tara attempted all questions, find the total number of questions in the test.

33. If the coordinates of the mid-points of the sides of a triangle are (1, 1), (2, -3) and (3,4), find the vertices of the triangle. **[5]**

34. In a cylindrical vessel of radius 10 cm, containing some water, 9000 small spherical balls are dropped which are completely immersed in water which raises the water level. If each spherical ball is of radius 0·5 cm, then find the rise in the level of water in the vessel. **[5]**

OR

In Figure, from a solid cube of side 7 cm, a cylinder of radius 2.1 cm and height 7 cm is scooped out. Find the total surface area of the remaining solid.



35. Along a road lie an odd number of stones placed at intervals of 10 metres. These stones have to be assembled around the middle stone. A person can carry only one stone at a time. A man carried the job with one of the end stones by carrying them in succession. In carrying all the stones he covered a distance of 3 km. Find the number of stones? **[5]**

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

The below picture are few natural examples of parabolic shape which is represented by a quadratic polynomial. A parabolic arch is an arch in the shape of a parabola. In structures, their curve represents an efficient method of load, and so can be found in bridges and in architecture in a variety of forms.






i. In the standard form of quadratic polynomial, $ax^2 + bx + c$, what are a, b and c? (1)

ii. If the roots of the quadratic polynomial are equal, what is the discriminant D? (1)

iii. If α and $\frac{1}{\alpha}$ are the zeroes of the quadratic polynomial are $2x^2 - x + 8k$, then find the value of k? (2)

OR

What is the relation between zeros and coefficient for a quadratic polynomial? (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

BINGO is game of chance. The host has 75 balls numbered 1 through 75. Each player has a BINGO card with some numbers written on it.

The participant cancels the number on the card when called out a number written on the ball selected at random. Whosoever cancels all the numbers on his/her card, says BINGO and wins the game.



The table given below, shows the data of one such game where 48 balls were used before Tara said **BINGO**.

Numbers announced	**Number of times**
0 - 15	8
15 - 30	9
30 - 45	10
45 - 60	12
60 - 75	9

Based on the above information, answer the following:

i. Write the median class. **(1)**

ii. When first ball was picked up, what was the probability of calling out an even number? **(1)**

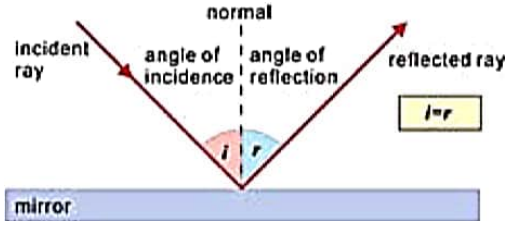
iii. a. Find median of the given data. **(2)**

OR

b. Find mode of the given data. **(2)**

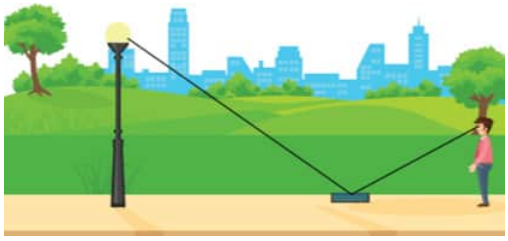
38. **Read the following text carefully and answer the questions that follow:** **[4]**

The law of reflection states that when a ray of light reflects off a surface, the angle of incidence is equal to the angle of reflection.



Suresh places a mirror on level ground to determine the height of a pole (with traffic light fired on it). He stands at a certain distance so that he can see the top of the pole reflected from the mirror. Suresh's eye level is 1.5 m above the ground. The distance

of Suresh and the pole from the mirror are 1.8 m and 6 m respectively.



i. Which criterion of similarity is applicable to similar triangles? (1)

ii. What is the height of the pole? (1)

iii. If angle of incidence is i, find tan i. (2)

OR

Now Suresh move behind such that distance between pole and Suresh is 13 meters. He place mirror between him and pole to see the reflection of light in right position. What is the distance between mirror and Suresh? (2)

Solution

Section A

1.

(c) $\frac{3}{4}$

Explanation:

$\frac{3}{4}$

2.

(c) real and distinct

Explanation:

Here, a = 1, b = -11, c = -10

Then, b^2 - 4ac = $(-11)^2$ - 4 $\times$ 1 $\times$ (-10)

$\Rightarrow$ 121 + 40 = 161

Since, b^2 - 4ac > 0

Therefore, The roots of the quadratic equation x^2 - 11x - 10 = 0 is real and distinct.

3.

(b) $300cm^3$

Explanation:

$300cm^3$

4.

(c) 5, 3

Explanation:

Let the two numbers be x and y.

According to the given conditions,

x + y = 8 ...(i)

and $\frac{1}{x}+\frac{1}{y}=\frac{8}{15}$...(ii)

Putting value of x = 8 - y in (ii), we get

$\frac{1}{8-y}+\frac{1}{y}=\frac{8}{15}\Rightarrow\frac{y+8-y}{y(8-y)}=\frac{8}{15}$

$\Rightarrow y^2$ - 8y + 15 = 0$\Rightarrow y^2$ - 5y - 3y + 15 = 0

$\Rightarrow$(y - 5)(y - 3) = 0$\Rightarrow$y = 5 or y = 3

From (i), x = 3 or x = 5

Thus, the numbers are 5 and 3.

5.

(c) 4

Explanation:

According to question,

a_3 = 16

$\Rightarrow$ a + 2d = 16 ... (i)

And $a_7 = a_5$ + 12

$\Rightarrow$ a + 6d = a + 4d + 12

$\Rightarrow$ a + 6d - a - 4d = 12

$\Rightarrow$ 2d = 12

$\Rightarrow$ d = 6

Putting value of d in eq. (i),

we get a + 2 $\times$ 6 = 16

$\Rightarrow$ a = 4

6. **(a)** $\sqrt{2}$

Explanation:

Distance between $(\cos\theta, \sin\theta)$ and $(\sin\theta, -\cos\theta)$

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-\cos\theta - \sin\theta)^2 + (\sin\theta - \cos\theta)^2}$

$= \sqrt{1+1} = \sqrt{2}$ $\{\because \sin^2\theta + \cos^2\theta = 1\}$

7. **(d)** $\frac{b^2-2ac}{c^2}$

Explanation:

We have to find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

Given α and β are the zeros of the quadratic polynomial f(x) = ax^2 + bx + c

$\alpha + \beta = \frac{-\text{ Coefficient of } x}{\text{Coefficient of } x^3} = \frac{-b}{a}$

$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{c}{a}$

We have,

$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^2 - \frac{2}{\alpha\beta}$

$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \left(\frac{\beta}{\alpha\beta} + \frac{\alpha}{\beta\alpha}\right)^2 - \frac{2}{\alpha\beta}$

$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \left(\frac{\alpha+\beta}{\alpha\beta}\right)^2 - \frac{2}{\alpha\beta}$

$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \left(\frac{\frac{-b}{a}}{\frac{c}{a}}\right)^2 - \frac{2}{\frac{c}{a}}$

$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \left(\frac{-b}{a} \times \frac{a}{c}\right)^2 - \frac{2}{\frac{c}{a}}$

$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \left(\frac{-b}{c}\right)^2 - \frac{2a}{c}$

$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \left(\frac{b^2}{c^2}\right) - \frac{2a\times c}{c\times c}$

$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \left(\frac{b^2}{c^2}\right) - \frac{2ac}{c^2}$

$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \left(\frac{b^2-2ac}{c^2}\right)$

8. **(b)** 7.5 cm

Explanation:

It is given that AD bisects angle A.

Therefore, applying angle bisector theorem, we get:

$\frac{BD}{DC} = \frac{AB}{AC}$

$\Rightarrow \frac{4}{5} = \frac{6}{x}$

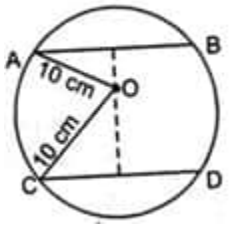
$\Rightarrow x = \frac{5\times6}{4} = 7.5$

Hence, AC = 7.5 cm

9.

(b) 14 cm

Explanation:



Let OP be the perpendicular to chord AB and P bisects the chord AB and OQ be the perpendicular to chord CD and Q bisects the chord CD.

$\therefore$ AP = BP = 8 cm and CQ = DQ = 6 cm

In triangle AOP, $OA^2 = OP^2 + AP^2$

$\Rightarrow (10)^2 = OP^2 + (8)^2$

$\Rightarrow OP^2 = 100 - 64 = 36$

$\Rightarrow$ OP = 6 cm

And in right angled triangle COQ,

$OC^2 = OQ^2 + CQ^2$

$\Rightarrow (10)^2 = OQ^2 + (6)^2$

$\Rightarrow OQ^2 = 100 - 36 = 64$

$\Rightarrow$OQ = 8 cm

Therefore, distance between two chord AB and CD = OP + OQ = 6 + 8 = 14 cm

10.

(d) 0

Explanation:

we cannot draw parallel lines from a unique point, so we cannot draw any parallel tangents to PA through P.

11.

(b) -1

Explanation:

-1

12. **(a)** 1679

Explanation:

The dividend is equal to Divisor $\times$ Quotient + Remainder

Number(dividend) = D $\times$ Q + R

$\therefore$ the number (Dividend) = 61 $\times$ 27 + 32

= 1647 + 32

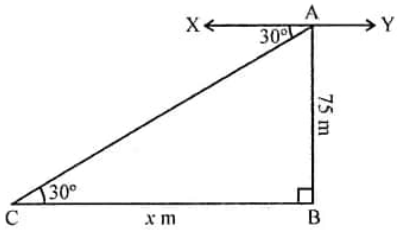
= 1679

13.

(d) $75\sqrt{3}$

Explanation:

AB is as tower and AB = 75 m
From A, the angle of depression of a car C on the ground is 30^o



Let distane BC = x
Now in right $\triangle$ACB,
$\tan\theta = \frac{AB}{BC} \Rightarrow \tan 30^o = \frac{75}{x}$
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{x} \Rightarrow x = 75\sqrt{3}$ m
$\therefore$ BC = $75\sqrt{3}$ m

14. **(a)** 148 cm^2
Explanation:

$$Area\ of\ the\ sector = \frac{Arc\ length}{2\pi r} \times \pi r^2$$

$$= \frac{18.5}{2\pi(16)} \times [\pi 16^2]$$

$$= \mathbf{148\ cm^2}$$

15.
(c) 3π cm
Explanation:
We know that:
Area of a sector $= \frac{\pi r^2\theta}{360^\circ}$
$\frac{\pi\times 36\times 36\times\theta}{360} = 54$
$\Rightarrow \theta = \frac{54\times 360}{36\times 36} = 15^\circ$
$\Rightarrow$ Length of arc $= \frac{\theta}{360^\circ} \times 2\pi r$
$= \frac{15}{360} \times 2\pi \times 36$
= 3π cm

16.
(c) $\frac{5}{7}$
Explanation:
No. of days in a leap year = 366
No. of Mondays = 52
Extra days = 366 - 52 $\times$ 7
= 366 - 364 = 2
$\therefore$ Remaining days in the week = 7 - 2 = 5
$\therefore$ Probability of being 52 Mondays in the leap year = $\frac{5}{7}$

17.

(b) $0 \leq P(A) \leq 1$

Explanation:

The probability of any event is always positive. It could be at the least equal to zero but not less than that. The probability of sure event at the maximum could be equal to 1 so probability lies between 0 and 1 both included.

18.

(c) 62.5

Explanation:

62.5

19.

(c) A is true but R is false.

Explanation:

A is true but R is false.

20.

(d) A is false but R is true.

Explanation:

A is false but R is true.

Section B

21. Minimum number of rooms required means there should be maximum number of teachers in a room. We have to find HCF of 48, 80 and 144.

$48 = 2^4 \times 3$

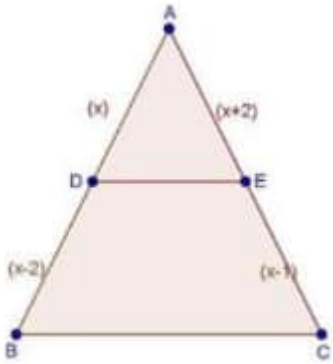
$80 = 2^4 \times 5$

$144 = 2^4 \times 3^2$

HCF (48, 80, 144) = $2^4 = 16$

Therefore, total number of rooms required $= \frac{48}{16} + \frac{80}{16} + \frac{144}{16} = 17$

22. We have,



DE || BC

Therefore, by basic proportionality theorem,

We have,

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$

$\Rightarrow$ x(x - 1) = (x + 2)(x - 2)

$\Rightarrow x^2 - x = x^2 - (2)^2$ [$\because$ (a - b)(a + b) = $a^2 - b^2$]

$\Rightarrow$ -x = -4

$\Rightarrow$ x = 4 cm.

23. We have in given figure common tangents PQ and RS to two circles intersect at A. Since tangents drawn from an external points to a circle are equal.
    $\therefore$ AP = AR
    and AQ = AS
    $\therefore$ AP + AQ = AR + AS [Adding]
    $\Rightarrow$ PQ = RS
    Hence proved.
24. $A + B = 45^\circ$
    $A - B = 30^\circ$
    Solving and getting $A = 37 \cdot 5^\circ$ and $B = 7 \cdot 5^\circ$

    OR

    Consider L.H.S. = 2 sin 30° cos 30°
    = $2 \times (1/2) \times (\sqrt{3}/2)$
    = $\sqrt{3}/2$ = sin 60° = R.H.S.
    $\therefore$ L.H.S. = R.H.S.
    Hence, verified.
25. Here, r = 45 cm and $\theta = \frac{360^\circ}{8} = 45^\circ$
    Area between two consecutive ribs of the umbrella = $\frac{\theta}{360^\circ} \times \pi r^2$
    = $\frac{45^\circ}{360^\circ} \times \frac{22}{7} \times 45 \times 45 \quad = \frac{22275}{28}$ cm^2

    OR

    Area of Shaded Region
    = 2 (Area of one sector ABPD) - Area of square ABCD
    = $2\left(\frac{90^\circ \times \pi \times 7^2}{360^\circ}\right) - 7 \times 7$
    = 28 cm^2

Section C

26. The required greatest capacity is the HCF of 120, 180 and 240.
    $240 = 180 \times 1 + 60$
    $180 = 60 \times 3 + 0$
    HCF is 60.
    Now HCF of 60, 120
    $120 = 60 \times 2 + 0$
    $\therefore$ HCF of 120, 180 and 240 is 60.
    $\therefore$ The required capacity is 60 litres.
27. Given points are A(3, -5) and B(-4, 8).
    P divides AB in the ratio k:1
    Using the section formula, we have:
    Coordinate of point P are $\left\{\left(\frac{-4k+3}{k+1}\right)\left(\frac{8k-5}{k+1}\right)\right\}$
    Now it is given, that P lies on the line x + y =0
    Therefore,
    $\frac{-4k+3}{k+1} + \frac{8k-5}{k+1} = 0$
    $\Rightarrow$ -4k + 3 + 8k - 5 =0
    $\Rightarrow$ -4k + 3 + 8k - 5 = 0
    $\Rightarrow$ 4k - 2 = 0
    $\Rightarrow k = \frac{2}{4}$
    $\Rightarrow k = \frac{1}{2}$
    Thus, the value of k is 1/2.
28. The given equation is:

$\frac{1}{x+1} + \frac{2}{x+2} = \frac{5}{x+4}$

$\Rightarrow \frac{x+2+2x+2}{(x+1)(x+2)} = \frac{5}{x+4}$

$\Rightarrow \frac{3x+4}{x^2+3x+2} = \frac{5}{x+4}$

By cross multiplication method we have,

$(3x + 4)(x + 4) = 5(x^2 + 3x + 2)$

$\Rightarrow 3x^2 + 16x + 16 = 5x^2 + 15x + 10$

$\Rightarrow 2x^2 - x - 6 = 0$

on factorisation the above equation becomes:

$2x^2 - 4x + 3x - 6 = 0$

$\Rightarrow 2x(x - 2) + 3(x - 2) = 0$

$\Rightarrow (x - 2)(2x + 3) = 0$

$\Rightarrow (x - 2)(2x + 3) = 0$

$\Rightarrow x - 2 = 0$ or $2x + 3 = 0$

Therefore, either $x = 2$ or $x = \frac{-3}{2}$.

OR

Let the total number of swans be x.

$\therefore$Number of swans playing on the shore of the tank = $\frac{7}{2}\sqrt{x}$

It is given that there are two remaining swans playing in the water. Hence, total no. of swans = $\frac{7}{2}\sqrt{x}$ +2, which is equal to x.

$Clearly; \quad x = \frac{7}{2}\sqrt{x} + 2$

$\Rightarrow \quad x - \frac{7}{2}\sqrt{x} - 2 = 0$

$\Rightarrow \quad y^2 - \frac{7}{2}y - 2 = 0$, where $y = \sqrt{x} \Rightarrow y^2 = x$

$\Rightarrow 2y^2 - 7y - 4 = 0$

$\Rightarrow 2y^2 - 8y + y - 4 = 0$

$\Rightarrow 2y(y - 4) + (y - 4) = 0$

$\Rightarrow (y - 4)(2y + 1) = 0$

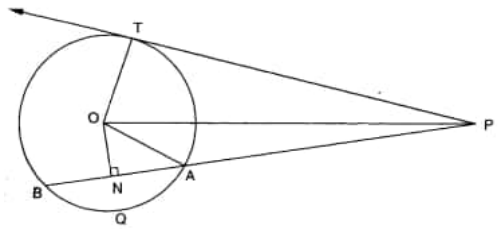
$\Rightarrow y = 4$ or, $y = -\frac{1}{2}$

$\Rightarrow y = 4$ [$\because y = -\frac{1}{2}$ is not possible as y is square root of x]

$\Rightarrow x = y^2 \Rightarrow x = 4^2 = 16$

Hence, the total number of swans = x = 16.

29.



i. PA .PB = (PN - AN)(PN + BN)

= (PN - AN)(PN + AN) $\left[\begin{array}{c} \because ON \perp AB \\ \therefore N \text{ is the mid-point of } AB \\ \Rightarrow AN = BN \end{array}\right]$

= $PN^2 - AN^2$

ii. Applying Pythagoras theorem in right triangle PNO, we obtain

$OP^2 = ON^2 + PN^2$

$\Rightarrow PN^2 = OP^2 - ON^2$

$\therefore PN^2 - AN^2 = (OP^2 - ON^2) - AN^2$

$= OP^2 - (ON^2 + AN^2)$

$= Op^2 - OA^2$ [Using Pythagoras theorem in ΔONA]

$= OP^2 - OT^2$ [$\because OA = OT =$ radius]

iii. From (i) and (ii), we obtain

$PA.PB = PN^2 - AN^2$ and $PN^2 - AN^2 = OP^2 - OT^2$

$\Rightarrow PA.PB = OP^2 - OT^2$

Applying Pythagoras theorem in $\triangle OTP$, we obtain

$OP^2 = OT^2 + PT^2$

$\Rightarrow OP^2 - OT^2 = PT^2$

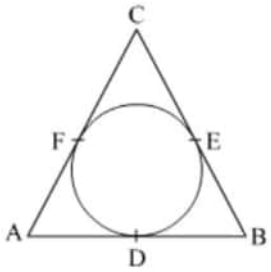
Thus, we obtain

$PA.PB = OP^2 - OT^2$

and $OP^2 - OT^2 = PT^2$

Hence, $PA.PB = PT^2$.

OR



Tangents drawn from an external point to a circle are equal.

$\Rightarrow$ AD = AF, BD = BE, CE = CF.

Let AD = AF = a

BD = BE = b

CE = CF = c

AB = AD + DB = a + b = 8 ..... (1)

BC = BE + EC = b + c = 10 ..... (2)

AC = AF + FC = a + c = 12 ..... (3)

Adding (1), (2) and (3), we get

2 (a + b + c) = 30

$\Rightarrow$ (a + b + c) = 15 ..... (4)

Subtracting (1) from (4), we get c = 7

Subtracting (2) from (4), we get a = 5

Subtracting (3) from (4), we get b = 3

Therefore, AD = a = 5 cm, BE = b = 3 cm, CF = c = 7 cm

30. $LHS = \frac{\cos A + \sin A - 1}{\cos A - \sin A + 1}$

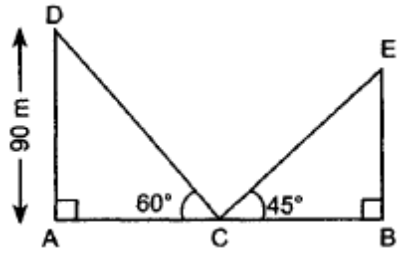
$= \frac{\cot A + 1 - \operatorname{cosec} A}{\cot A - 1 + \operatorname{cosec} A}$

$= \frac{\cot A - \operatorname{cosec} A + \operatorname{cosec}^2 A - \cot^2 A}{\cot A - 1 + \operatorname{cosec} A}$

$= \frac{(\operatorname{cosec} A - \cot A)(-1 + \operatorname{cosec} A + \cot A)}{\cot A - 1 + \operatorname{cosec} A}$

$= \operatorname{cosec} A - \cot A =$ RHS

31. Let AB is path



In rt. $\triangle$DAC, $\frac{DC}{AD} = cosec\ 60^\circ$

$\Rightarrow \quad \frac{\text{DC}}{90} = \frac{2}{\sqrt{3}}$

$\text{DC} = \frac{2}{\sqrt{3}} \times 90\text{m} = \frac{180}{\sqrt{3}}\text{m}$

Now, $DC = CE$

$\therefore \quad \text{CE} = \frac{180}{\sqrt{3}}\text{m}$

In rt. $\triangle$EBC,

$\frac{BE}{CE} = \sin 45^\circ$

$\Rightarrow \quad \text{BE} = \frac{1}{\sqrt{2}} \times \frac{180}{\sqrt{3}}\text{m}$

$\Rightarrow \quad BE = 73.47\text{m}$

Section D

32. Let my age is x years and my son's age is y years.

$\Rightarrow x = 3y$ .....(i)

Five years later, my age will be $(x+5)years$ and my son's age will be $(y+5)years$.

$\therefore x+5 = \frac{5}{2}(y+5)$ [Given]

$\Rightarrow 2x - 5y - 15 = 0$ .....(ii)

Putting x =3y in equation (ii), we get

$6y - 5y - 15 = 0 \Rightarrow y = 15$

Putting $y = 15$ in equation (i), we get

$x = 45$

My present age is 45 years and my son's present age is 15 years.

OR

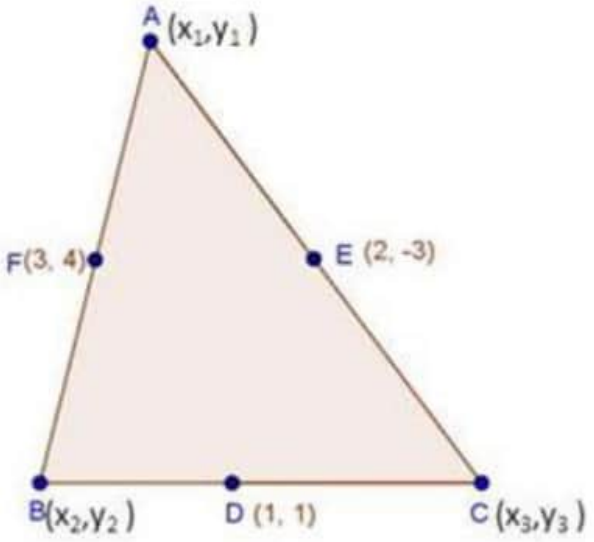
Let number of correct answers be x and
number of incorrect answers be y

3x - y = 40

4x - 2y = 50

Solving, we get x = 15, y = 5

$\therefore$ Total number of questions = 20

33. 

Let A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) be the vertices of ΔABC.

Let D(1, 1), E(2, -3) and F(3, 4) be the mid-points of sides BC, CA and AB respectively.

Since, D is the mid-point of BC

$\therefore \quad \frac{x_2+x_3}{2} = 1$ and $\frac{y_2+y_3}{2} = 1$

$\Rightarrow x_2 + x_3 = 2$ and $y_2 + y_3 = 2$ ...(i)

Similarly E and F are the mid-points of CA and AB respectively.

$\therefore \quad \frac{x_1+x_3}{2} = 2$ and $\frac{y_1+y_3}{2} = -3$

$\Rightarrow x_1 + x_3 = 4$ and $y_1 + y_3 = -6$ ...(ii)

and, $\frac{x_1+x_2}{2} = 3$ and $\frac{y_1+y_2}{2} = 4$

$\Rightarrow x_1 + x_2 = 6$ and $y_1 + y_2 = 8$ ...(iii)

From (i), (ii) and (iii) we get

$x_2 + x_3 + x_1 + x_3 + x_1 + x_2 = 2 + 4 + 6$

and, $y_2 + y_3 + y_1 + y_3 + y_1 + y_2 = 2 + (-6) + 8$

$\Rightarrow 2(x_1 + x_2 + x_3) = 12$ and $2(y_1 + y_2 + y_3) = 4$

$\Rightarrow x_1 + x_2 + x_3 = 6$ and $y_1 + y_2 + y_3 = 2$ ...(iv)

From (i) and (iv) we get

$x_1 + 2 = 6$ and $y_1 + 2 = 2$

$\Rightarrow x_1 = 6 - 2 \Rightarrow y_1 = 2 - 2$

$\Rightarrow x_1 = 4 \Rightarrow y_1 = 0$

So the coordinates of A are (4, 0)

From (ii) and (iv) we get

$x_2 + 4 = 6$ and $y_2 + (-6) = 2$

$\Rightarrow x_2 = 2 \Rightarrow y_2 - 6 = 2$

$\Rightarrow y_2 = 8$

So the coordinates of B are (2, 8)

From (iii) and (iv) we get

$6 + x_3 = 6$ and $8 + y_3 = 2$

$\Rightarrow x_3 = 6 - 6 \Rightarrow y_3 = 2 - 8$

$\Rightarrow x_3 = 0 \Rightarrow y_3 = -6$

So the coordinates of c are (0, -6)

Hence, the vertices of triangle ABC are:

A(4, 0), B(2, 8) and C(0, -6)

34. Volume of raised water in cylinder = Volume of 9000 spherical balls

$\pi(10)^2 H = 9000 \times \frac{4}{3} \times \pi \times (0.5)^3$

$\therefore$ H = 15 cm

OR

We have;

A Cube,

Cube's $\frac{length}{Edge}$, a = 7 cm

A Cylinder:

Cylinder's Radius, r = 2.1 cm or r = $\frac{21}{10}$cm

Cylinder's Height, h = 7 cm

$\because$ A cylinder is scooped out from a cube,

$\therefore$ TSA of the resulting cuboid:

= TSA of whole Cube - 2 $\times$ (Area of upper circle or Area of lower circle) + CSA of the scooped out Cylinder

= $6a^2 + 2\pi rh - 2 \times (\pi r^2)$

= $6 \times (7)^2 + 2 \times (22 \div 7 \times 2.1 \times 7) - 2 \times [22 \div 7 \times (2.1)^2]$

= $6 \times 49 + (44 \div 7 \times 14.7) - (44 \div 7 \times 4.41)$

= 294 + 92.4 - 27.72

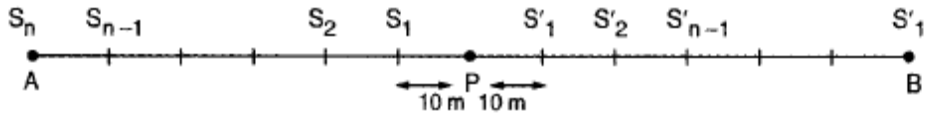
= 294 + 64.68

= 358.68 cm^2

Hence, the total surface area of the remaining solid is 358.68 cm^2

35. Lets suppose there are (2n + 1) stones. Clearly, one stone lies in the middle and n stones on each side of it in a row.

Let P be the middle stone and let A and B be the end stones on the left and right of P respectively.



Clearly, there are n intervals each of length 10 meters on both the sides of P.

Now, suppose the man starts from A. He picks up the end stone on the left of mid-stone and goes to the mid-stone, drops it and goes to (n - 1)th stone on left, picks it up, goes to the mid-stone and drops it. This process is repeated till he collects all stones on the left of the mid-stone at the mid-stone.

So, distance covered in collecting stones on the left of the mid-stones is

$10 \times n + 2 [10 \times (n - 1) + 10 \times (n - 2) + ... + 10 \times 2 + 10 \times 1]$.

After collecting all stones on left of the mid-stone the man goes to the stone B on the right side of the mid-stone, picks it up, goes to the mid-stone and drops it. Then, he goes to $(n - 1)^{th}$ stone on the right and the process is repeated till he collects all stones at the mid-stone.

So that distance covered in collecting the stones on the right side of the mid-stone is equal to

$2 [10 \times n + 10 \times (n - 1) + 10 \times (n - 2) + ... + 10 \times 2 + 10 \times 1]$.

Therefore,total distance covered

= $10 \times n + 2 [10 \times (n - 1) + 10 \times (n - 2) + ... + 10 \times 2 + 10 \times 1]$

+ $2 [10 \times n + 10 \times (n - 1) + ... + 10 \times 2 + 10 \times 1]$

= $4 [10 \times n + 10 \times (n - 1) + ... + 10 \times 2 + 10 \times 1] - 10 \times n$

=

$40\{1 + 2 + 3 + \ldots + n\} - 10n = 40\left\{\frac{n}{2}(1 + n)\right\} - 10n = 20n(n + 1) - 10n = 20n^2 + 10n$

But, the total distance that a man covered in collecting stones is 3 km.i.e;3000m.

Therefore, $20 n^2 + 10n = 3000$

$\Rightarrow 2n^2 + n - 300 = 0. \implies 2n^2 + 25n + 24n - 300 = 0$

$\Rightarrow (n - 12)(2n + 25) = 0$ ...[Therefore, $2n + 25 \neq 0$] $\implies n - 12 = 0$

$\Rightarrow n = 12$.Hence, the number of stones is equal to $2n + 1 = 2(12) + 1 = 25$

So, the total stones carried to the middle is 25.

Section E

36. i. a is a non zero real number and b and c are any real numbers.

ii. D = 0

iii. $2x^2 - x + 8k$

$\alpha \times \frac{1}{\alpha} = \frac{8k}{2}$

$1 = 4k$

$k = \frac{1}{4}$

OR

$\alpha + \beta = \frac{-b}{a}$ i.e., $\left(\frac{-\text{coefficient of x}}{\text{coefficient of } x^2}\right)$

$\alpha\beta = \frac{c}{a}$ i.e., $\left(\frac{\text{constant term}}{\text{coeff of } x^2}\right)$

37.

Number announced	0 - 15	15 - 30	30 - 45	45 - 60	60 - 75
Number of times (f)	8	9	10	12	9
cf	8	17	27	39	48 = N

i. $\frac{N}{2} = 24$

$\therefore$ median class is 30 - 45

ii. P(picking up an even number) = $\frac{37}{75}$

iii. a. Median = $30 + \frac{\left(\frac{48}{2} - 17\right)}{10} \times 15$

= 40.5

OR

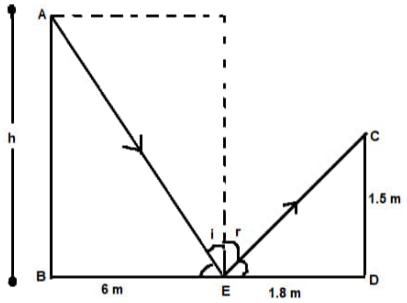
b. Modal class is 45 - 60

Mode = $45 + \frac{12-10}{2\times12-10-9} \times 15$

= 51

38. i. AA criterion

ii.



$\triangle ABE \sim \triangle CDE$ (by AA criteria)

$\frac{AB}{CD} = \frac{BE}{DE}$

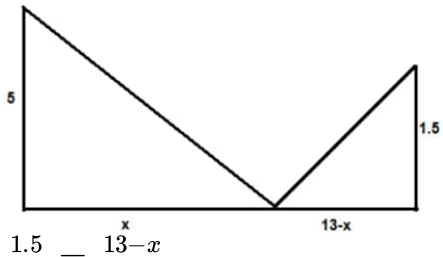
$h = \frac{6\times1.5}{1.8}$

h = 5

i.e., height of pole = 5 m.

iii. $\tan i = \frac{6}{5}$

OR



$\frac{1.5}{5} = \frac{13-x}{x}$

1.5x = 65 - 5x

6.5x = 65

x = $\frac{65}{6.5}$

= 10

∴ distance of Suresh from mirror

= 13 - x

= 13 - 10

= 3 m