

Series : S7PQR



SET~3

प्रश्न-पत्र कोड
Q.P. Code

65/7/3

रोल नं.

Roll No.

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परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।
Candidates must write the Q.P. Code on the title page of the answer-book.

नोट	NOTE
(I) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं।	(I) Please check that this question paper contains 23 printed pages.
(II) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।	(II) Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
(III) कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं।	(III) Please check that this question paper contains 38 questions.
(IV) कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में यथा स्थान पर प्रश्न का क्रमांक अवश्य लिखें। []	(IV) Please write down the Serial Number of the question in the answer-book at the given place before attempting it.
(V) इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।	(V) 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.

गणित

MATHEMATICS

निर्धारित समय : 3 घण्टे

Time allowed : 3 hours

अधिकतम अंक : 80

Maximum Marks : 80

सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख्ती से पालन कीजिए :

- (i) इस प्रश्न-पत्र में 38 प्रश्न हैं। सभी प्रश्न अनिवार्य हैं।
- (ii) यह प्रश्न-पत्र पाँच खण्डों में विभाजित है – क, ख, ग, घ एवं ङ।
- (iii) खण्ड क में प्रश्न संख्या 1 से 18 तक बहुविकल्पीय (MCQ) तथा प्रश्न संख्या 19 एवं 20 अभिकथन एवं तर्क आधारित 1 अंक के प्रश्न हैं।
- (iv) खण्ड ख में प्रश्न संख्या 21 से 25 तक अति लघु-उत्तरीय (VSA) प्रकार के 2 अंकों के प्रश्न हैं।
- (v) खण्ड ग में प्रश्न संख्या 26 से 31 तक लघु-उत्तरीय (SA) प्रकार के 3 अंकों के प्रश्न हैं।
- (vi) खण्ड घ में प्रश्न संख्या 32 से 35 तक दीर्घ-उत्तरीय (LA) प्रकार के 5 अंकों के प्रश्न हैं।
- (vii) खण्ड ङ में प्रश्न संख्या 36 से 38 तक प्रकरण अध्ययन आधारित 4 अंकों के प्रश्न हैं।
- (viii) प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड ख के 2 प्रश्नों में, खण्ड ग के 3 प्रश्नों में, खण्ड घ के 2 प्रश्नों में तथा खण्ड ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।
- (ix) कैल्कुलेटर का उपयोग वर्जित है।

खण्ड क

इस खण्ड में बहुविकल्पीय प्रश्न (MCQ) हैं, जिनमें प्रत्येक प्रश्न 1 अंक का है।

1. यदि $f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ है, तो निम्नलिखित में से कौन-सा कथन सत्य नहीं है ?

- (A) सभी $x < 0$ के लिए f संतत है।
- (B) सभी $x \in \mathbb{R}$ के लिए f संतत है।
- (C) सभी $x > 0$ के लिए f संतत है।
- (D) $x = 0$ को छोड़कर अन्य सभी बिंदुओं पर f संतत है।

2. $\sin^{-1} \left(\sin \frac{3\pi}{5} \right)$ का मान है :

- (A) $\frac{3\pi}{5}$
- (B) $\frac{\pi}{5}$
- (C) $\frac{7\pi}{5}$
- (D) $\frac{2\pi}{5}$

General Instructions :

Read the following instructions very carefully and strictly follow them :

- (i) This question paper contains **38** questions. **All** questions are **compulsory**.
- (ii) This question paper is divided into **five** Sections – **A, B, C, D** and **E**.
- (iii) In **Section A**, Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.
- (iv) In **Section B**, Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.
- (v) In **Section C**, Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.
- (vi) In **Section D**, Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.
- (vii) In **Section E**, Questions no. **36** to **38** are case study based questions carrying **4** marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
- (ix) Use of calculator is **not** allowed.

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then which of the following statements is **not**

correct ?

- (A) f is continuous for every $x < 0$.
- (B) f is continuous for every $x \in \mathbb{R}$.
- (C) f is continuous for every $x > 0$.
- (D) f is continuous for each point except $x = 0$.

2. The value of $\sin^{-1} \left(\sin \frac{3\pi}{5} \right)$ is :

- | | |
|----------------------|----------------------|
| (A) $\frac{3\pi}{5}$ | (B) $\frac{\pi}{5}$ |
| (C) $\frac{7\pi}{5}$ | (D) $\frac{2\pi}{5}$ |

3. $f(x) = -|x + 1| + 5$ के लिए, निम्नलिखित में से कौन-सा कथन सत्य है ?
- (A) $x = 4$ पर उच्चतम $f(x) = 0$ है।
 (B) $x = -1$ पर उच्चतम $f(x) = 5$ है।
 (C) $x = 0$ पर उच्चतम $f(x) = 4$ है।
 (D) $f(x)$ का कोई उच्चतम मान नहीं है।
4. यदि $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$, $A' + A = I$ को संतुष्ट करता है, तो θ का मान है :
- (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$
 (C) $\frac{2\pi}{3}$ (D) $\frac{5\pi}{6}$
5. यदि A एक वर्ग आव्यूह है, तो निम्नलिखित में से कौन-सा सही **नहीं** है ?
- (A) $A + A'$ एक सममित आव्यूह है
 (B) AA' तथा $A'A$ सममित आव्यूह हैं
 (C) $A + A'$ एक विषम सममित आव्यूह है
 (D) $A - A'$ एक विषम सममित आव्यूह है
6. यदि $\begin{vmatrix} 3x & x+2 \\ 3(x-1) & x-2 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$ है, तो x का मान है :
- (A) 2 (B) $\frac{1}{3}$
 (C) 5 (D) 3
7. $\left[\tan^{-1}(-1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right) \right]$ का मान है :
- (A) $\frac{7\pi}{12}$ (B) $\frac{5\pi}{12}$
 (C) $-\frac{5\pi}{12}$ (D) $\frac{\pi}{3}$

3. For $f(x) = -|x + 1| + 5$, which of the following statements is true ?
- (A) $\text{Max } f(x) = 0$ at $x = 4$.
 (B) $\text{Max } f(x) = 5$ at $x = -1$.
 (C) $\text{Max } f(x) = 4$ at $x = 0$.
 (D) $f(x)$ has no maximum value.
4. If $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ satisfies $A' + A = I$, then the value of θ is :
- (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$
 (C) $\frac{2\pi}{3}$ (D) $\frac{5\pi}{6}$
5. If A is a square matrix, then which of the following is **not** true ?
- (A) $A + A'$ is a symmetric matrix
 (B) AA' and $A'A$ are symmetric matrices
 (C) $A + A'$ is a skew symmetric matrix
 (D) $A - A'$ is a skew symmetric matrix
6. If $\begin{vmatrix} 3x & x+2 \\ 3(x-1) & x-2 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$, then the value of x is :
- (A) 2 (B) $\frac{1}{3}$
 (C) 5 (D) 3
7. The value of $\left[\tan^{-1}(-1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right) \right]$ is :
- (A) $\frac{7\pi}{12}$ (B) $\frac{5\pi}{12}$
 (C) $-\frac{5\pi}{12}$ (D) $\frac{\pi}{3}$

8. यदि $A = \begin{bmatrix} 1 & -2 & 3 \\ 3 & 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ -1 & 2 \\ 4 & -5 \end{bmatrix}$ तथा $C = [C_{ij}] = AB$ है, तो C_{12} है :
- (A) 22 (B) -16
(C) 4 (D) 0
9. यदि A तथा B कोटि 3 के वर्ग आव्यूह इस प्रकार हैं कि $|A| = 4$ तथा $|B| = 7$ है, तो $|2AB|$ बराबर है :
- (A) 112 (B) 56
(C) 28 (D) 224
10. यदि $\int_0^1 (9x^2 - 4x - 2k) dx = 0$ है, तो 'k' का मान है :
- (A) 1 (B) 0
(C) $\frac{1}{2}$ (D) $-\frac{1}{2}$
11. एक कण की गति को $S = 2t^3 - 6t^2 + 8$ द्वारा वर्णित किया गया है। त्वरण $\frac{d^2S}{dt^2}$ के शून्य होने पर इसका वेग $\frac{dS}{dt}$ है :
- (A) -6 इकाई/से. (B) 0 इकाई/से.
(C) 18 इकाई/से. (D) 6 इकाई/से.
12. यदि $\int_0^4 \frac{1}{x + \sqrt{x}} dx = k \log 3$ है, तो k का मान है :
- (A) 4 (B) -2
(C) -4 (D) 2

8. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 3 & 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ -1 & 2 \\ 4 & -5 \end{bmatrix}$ and $C = [C_{ij}] = AB$, then C_{12} is :
- (A) 22 (B) -16
(C) 4 (D) 0
9. If A and B are square matrices of order 3 such that $|A| = 4$ and $|B| = 7$, then $|2AB|$ is :
- (A) 112 (B) 56
(C) 28 (D) 224
10. If $\int_0^1 (9x^2 - 4x - 2k) dx = 0$, then the value of 'k' is :
- (A) 1 (B) 0
(C) $\frac{1}{2}$ (D) $-\frac{1}{2}$
11. The motion of a particle is described as $S = 2t^3 - 6t^2 + 8$. Its velocity $\frac{dS}{dt}$, when the acceleration $\frac{d^2S}{dt^2}$ vanishes is :
- (A) -6 unit/sec (B) 0 unit/sec
(C) 18 unit/sec (D) 6 unit/sec
12. If $\int_0^4 \frac{1}{x + \sqrt{x}} dx = k \log 3$, then the value of k is :
- (A) 4 (B) -2
(C) -4 (D) 2

13. $\int e^x \left(\frac{2 + \sin 2x}{1 + \cos 2x} \right) dx$ बराबर है :

(A) $\log \cos 2x + C$

(B) $\sec^2 2x + C$

(C) $e^x \tan x + C$

(D) $e^x \cot x + C$

14. $\sqrt{\cos x}$ के सापेक्ष $\log \sin x$ का अवकलज है :

(A) $-2 \sqrt{\cos x} \cdot \cot x \cdot \operatorname{cosec} x$

(B) $\sqrt{\cos x} \cdot \cot x \cdot \operatorname{cosec} x$

(C) $\frac{2 \sqrt{\cos x}}{\sin x}$

(D) $\frac{1}{\sin x \sqrt{\cos x}}$

15. यदि दो घटनाएँ A तथा B स्वतंत्र हैं, तो निम्नलिखित में से कौन-सा कथन सही नहीं है ?

(A) $\bar{A}$ तथा B स्वतंत्र हैं।

(B) स्वतंत्र होना यह नहीं दर्शाता कि घटनाएँ परस्पर अपवर्जी हैं।

(C) $P(\bar{A} \cap \bar{B}) = [1 - P(\bar{A})] [1 - P(\bar{B})]$

(D) $P(\bar{A} \cap \bar{B}) = [1 - P(A)] [1 - P(B)]$

16. $\cos^{-1}(e^x)$ का x के सापेक्ष अवकलज है :

(A) $\frac{e^x}{\sqrt{1 - e^{2x}}}$

(B) $\frac{-e^x}{\sqrt{1 + e^x}}$

(C) $\frac{-e^x}{\sqrt{1 - e^{2x}}}$

(D) $\frac{e^x}{\sqrt{1 + e^{2x}}}$

17. यदि सदिश $\vec{a} = 3\hat{i} - 2\hat{j} + 4\hat{k}$ तथा $\vec{b} = \lambda\hat{i} + \hat{j}$ परस्पर लंबवत हैं, तो λ का मान है :

(A) 3

(B) -3

(C) $\frac{2}{3}$

(D) $-\frac{2}{3}$

13. $\int e^x \left(\frac{2 + \sin 2x}{1 + \cos 2x} \right) dx$ is equal to :

(A) $\log \cos 2x + C$

(B) $\sec^2 2x + C$

(C) $e^x \tan x + C$

(D) $e^x \cot x + C$

14. Differential of $\log \sin x$ with respect to $\sqrt{\cos x}$ is :

(A) $-2 \sqrt{\cos x} \cdot \cot x \cdot \operatorname{cosec} x$

(B) $\sqrt{\cos x} \cdot \cot x \cdot \operatorname{cosec} x$

(C) $\frac{2 \sqrt{\cos x}}{\sin x}$

(D) $\frac{1}{\sin x \sqrt{\cos x}}$

15. If two events A and B are independent, then which of the following statements is **not** correct ?

(A) $\bar{A}$ and B are independent.

(B) Being independent does not indicate that they are mutually exclusive.

(C) $P(\bar{A} \cap \bar{B}) = [1 - P(\bar{A})] [1 - P(\bar{B})]$.

(D) $P(\bar{A} \cap \bar{B}) = [1 - P(A)] [1 - P(B)]$.

16. Differential of $\cos^{-1}(e^x)$ with respect to x is :

(A) $\frac{e^x}{\sqrt{1 - e^{2x}}}$

(B) $\frac{-e^x}{\sqrt{1 + e^x}}$

(C) $\frac{-e^x}{\sqrt{1 - e^{2x}}}$

(D) $\frac{e^x}{\sqrt{1 + e^{2x}}}$

17. If vectors $\vec{a} = 3\hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{b} = \lambda\hat{i} + \hat{j}$ are perpendicular to each other, then the value of λ is :

(A) 3

(B) -3

(C) $\frac{2}{3}$

(D) $-\frac{2}{3}$

18. यदि $P(A) = 0.3$, $P(B) = 0.7$ तथा $P(B/A) = 0.5$ है, तो $P(A \cup B)$ है :

(A) 0.35

(B) 0.85

(C) 0.5

(D) 0.21

प्रश्न संख्या 19 और 20 अभिकथन एवं तर्क आधारित प्रश्न हैं। दो कथन दिए गए हैं जिनमें एक को अभिकथन (A) तथा दूसरे को तर्क (R) द्वारा अंकित किया गया है। इन प्रश्नों के सही उत्तर नीचे दिए गए कोडों (A), (B), (C) और (D) में से चुनकर दीजिए।

(A) अभिकथन (A) और तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या करता है।

(B) अभिकथन (A) और तर्क (R) दोनों सही हैं, परन्तु तर्क (R), अभिकथन (A) की सही व्याख्या नहीं करता है।

(C) अभिकथन (A) सही है, परन्तु तर्क (R) गलत है।

(D) अभिकथन (A) गलत है, परन्तु तर्क (R) सही है।

19. अभिकथन (A) : माना R एक संबंध है जो समुच्चय A पर है, जहाँ A उस स्कूल की लाइब्रेरी की सभी पुस्तकों का समुच्चय है, तथा संबंध R इस प्रकार परिभाषित है कि

$R = \{(x, y) : x, y \in A, x \text{ तथा } y \text{ के पृष्ठ समान हैं}\}$ तथा समुच्चय A पर एक अन्य संबंध S इस प्रकार है कि $S = \{(p, q) : p \text{ तथा } q \text{ गणित की पुस्तकें हैं}\}$ ।

$R \cap S \neq \phi$ भी प्रदत्त है। तो $R \cap S$ एक तुल्यता संबंध है।

तर्क (R) : किसी समुच्चय पर दो तुल्यता संबंधों का प्रतिच्छेदन उस समुच्चय पर एक तुल्यता संबंध होता है।

20. अभिकथन (A) : बिंदु A(6, -7, 0), B(16, -19, -4) तथा C(1, -1, 2) संरेखी हैं।

तर्क (R) : तीन बिंदु A, B तथा C संरेखी होते हैं, जब बिंदु C बिंदु A तथा B को मिलाने वाली रेखाखण्ड का मध्य-बिंदु हो।

18. If $P(A) = 0.3$, $P(B) = 0.7$ and $P(B/A) = 0.5$, then $P(A \cup B)$ is :

(A) 0.35

(B) 0.85

(C) 0.5

(D) 0.21

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Assertion (A) is false, but Reason (R) is true.

19. Assertion (A) : Let R be a relation on set A of all books in a school library given by $R = \{(x, y) : x, y \in A, x \text{ and } y \text{ have the same number of pages}\}$ and S is a relation on set A from the same library such that $S = \{(p, q) : p \text{ and } q \text{ are Mathematics books}\}$. Also, $R \cap S \neq \phi$. Then, $R \cap S$ is an equivalence relation.

Reason (R) : The intersection of two equivalence relations on a set is an equivalence relation on the set.

20. Assertion (A) : Points A(6, - 7, 0), B(16, -19, - 4) and C(1, - 1, 2) are collinear.

Reason (R) : Three points A, B and C are collinear, when point C is the mid-point of line segment joining points A and B.

खण्ड ख

इस खण्ड में अति लघु-उत्तरीय (VSA) प्रकार के 5 प्रश्न हैं, जिनमें प्रत्येक के 2 अंक हैं।

21. यदि $A = \begin{bmatrix} 6 & 3x \\ 8 & -6 \end{bmatrix}$ तथा $B = \begin{bmatrix} 6 & x^2 + 2 \\ 8 & x^2 - 5x \end{bmatrix}$ तथा $A = B$ है, तो x का मान ज्ञात कीजिए।

22. (क) एक बक्से में 1 से 22 तक की संख्याओं से अंकित कार्ड हैं। बक्से में से बिना प्रतिस्थापना के दो कार्ड एक साथ निकाल कर उन पर लिखी संख्याएँ नोट की गईं। इसकी प्रायिकता क्या है कि दोनों कार्डों की संख्याएँ सम हैं ?

अथवा

(ख) यदि $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{2}$ तथा $P(A \cap B) = \frac{1}{4}$ है, तो $P(\bar{A} / \bar{B})$ ज्ञात कीजिए।

23. (क) यदि $f(x) = \begin{cases} 2, & x \leq 3 \\ ax + b, & 3 < x < 5 \\ 5, & x \geq 5 \end{cases}$ एक संतत फलन है, तो a तथा b के मान ज्ञात कीजिए।

अथवा

(ख) यदि $2y^3 + y = 2x$ है, तो दर्शाइए कि

$$\frac{d^2y}{dx^2} = \frac{-48y}{(6y^2 + 1)^3}.$$

24. नीचे दी गई रेखिक प्रोग्रामन समस्या के लिए, सुसंगत क्षेत्र को ग्राफ पर छायांकित कीजिए तथा इसके कोनीय बिंदुओं के निर्देशांक ज्ञात कीजिए :

व्यवरोधों

$$4x + y \geq 20$$

$$2x + 3y \geq 30$$

$$x, y \geq 0$$

के अंतर्गत $Z = 18x + 10y$ का न्यूनतमीकरण करना।

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. Find the value of x if $A = B$, where

$$A = \begin{bmatrix} 6 & 3x \\ 8 & -6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 6 & x^2 + 2 \\ 8 & x^2 - 5x \end{bmatrix}.$$

22. (a) A box contains cards numbered 1 to 22. Two cards are drawn simultaneously without replacement and their numbers are noted. What is the probability that both numbers are even ?

OR

- (b) If $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{4}$, find $P(\bar{A} / \bar{B})$.

23. (a) Determine a and b if $f(x)$ is a continuous function, where

$$f(x) = \begin{cases} 2, & x \leq 3 \\ ax + b, & 3 < x < 5 \\ 5, & x \geq 5 \end{cases}$$

OR

- (b) If $2y^3 + y = 2x$, then show $\frac{d^2y}{dx^2} = \frac{-48y}{(6y^2 + 1)^3}$.

24. For the following Linear Programming Problem, shade the feasible region on the graph and find the coordinates of its corner points :

Minimize $Z = 18x + 10y$

subject to constraints

$$4x + y \geq 20$$

$$2x + 3y \geq 30$$

$$x, y \geq 0.$$

25. विद्यार्थी कक्षा के नोटिस बोर्ड पर एक ग्राफ शीट पर थंब पिन लगाने का प्रयत्न कर रहे हैं। थंब पिनों के समुच्चय A पर संबंध R इस प्रकार परिभाषित किया गया है :

$$R = \{(P, Q) : P, Q \in A \text{ तथा } P \text{ तथा } Q \text{ के बीच की दूरी } 3 \text{ cm से कम है}\}$$

ज्ञात कीजिए कि क्या R एक तुल्यता संबंध है।

खण्ड ग

इस खण्ड में लघु-उत्तरीय (SA) प्रकार के 6 प्रश्न हैं, जिनमें प्रत्येक के 3 अंक हैं।

26. (क) एक रेखा का समीकरण ज्ञात कीजिए जो बिंदु $(1, -1, 2)$ से होकर जाती है तथा x -अक्ष के समांतर है। इस रेखा की एक अन्य रेखा $\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$ से न्यूनतम दूरी भी ज्ञात कीजिए।

अथवा

- (ख) यदि बिंदुओं A, B, C तथा D के निर्देशांक क्रमशः $(3, -4, 2)$, $(-1, -2, 0)$, $(4, 1, -1)$ तथा $(4, 0, 5)$ हैं, तो रेखाओं AC और BD के समीकरण ज्ञात कीजिए और दर्शाइए कि रेखाएँ AC तथा BD परस्पर लंबवत हैं।

27. यदि $y = \sqrt{\log \left\{ \sin \left(\frac{x^2}{3} + 1 \right) \right\}}$ है, तो दर्शाइए कि $\frac{dy}{dx} = \frac{x \cot \left(\frac{x^2}{3} + 1 \right)}{3y}$.

28. फलन $f(x) = \cos x - \frac{1}{2} \cos 2x$ के, अंतराल $x \in [0, \frac{\pi}{2}]$ में, अधिकतम तथा न्यूनतम मानों का गुणनफल ज्ञात कीजिए।

25. Students are trying to fix thumb pins on a graph sheet on the class notice board. A relation R is defined on the set A of thumb pins as

$$R = \{(P, Q) : P, Q \in A \text{ and distance between P and Q is less than 3 cm}\}$$

Determine if R is an equivalence relation.

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26. (a) Find the equation of a line passing through the point $(1, -1, 2)$ and parallel to x-axis. Also, find the shortest distance between this line and the line $\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$.

OR

- (b) Find the equations of the lines AC and BD, if coordinates of points A, B, C and D respectively, are $(3, -4, 2)$, $(-1, -2, 0)$, $(4, 1, -1)$ and $(4, 0, 5)$. Show that the lines AC and BD are perpendicular to each other.

27. If $y = \sqrt{\log \left\{ \sin \left(\frac{x^2}{3} + 1 \right) \right\}}$, then show that $\frac{dy}{dx} = \frac{x \cot \left(\frac{x^2}{3} + 1 \right)}{3y}$.

28. Find the product of the maximum and minimum values of the function $f(x) = \cos x - \frac{1}{2} \cos 2x$, $x \in [0, \frac{\pi}{2}]$.

29. (क) यदि R , प्राकृत संख्याओं के समुच्चय N पर इस प्रकार परिभाषित संबंध है कि

$$R = \{(x, y) : x, y \in N, x + 2y = 25\},$$

तो संबंध R का प्रांत व परिसर ज्ञात कीजिए। यह भी ज्ञात कीजिए कि क्या यह एक तुल्यता संबंध है।

अथवा

(ख) ज्ञात कीजिए कि क्या फलन $f : \mathbb{R} \rightarrow \mathbb{R}$, जो कि $f(x) = 2x^3 - 7, \forall x \in \mathbb{R}$ (वास्तविक संख्याएँ) द्वारा परिभाषित है, एकैकी आच्छादी फलन है या नहीं।

30. निम्नलिखित रैखिक प्रोग्रामन समस्या को ग्राफ द्वारा हल कीजिए :
व्यवरोधों

$$x + y \geq 3$$

$$x + 2y \geq 4$$

$$x, y \geq 0$$

के अंतर्गत $Z = 4x + y$ का न्यूनतमीकरण करना।

31. (क) आव्यूह $A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$ को एक सममित आव्यूह तथा एक विषम सममित आव्यूह के योगफल के रूप में व्यक्त कीजिए।

अथवा

(ख) xy -तल में दो रेखाएँ, समीकरणों $5x - 7y - 2 = 0$ तथा $7x - 5y - 3 = 0$ द्वारा निरूपित की गई हैं। आव्यूह विधि से इन रेखाओं का प्रतिच्छेदन बिंदु ज्ञात कीजिए।

29. (a) If R is a relation on set of natural numbers N such that

$$R = \{(x, y) : x, y \in N, x + 2y = 25\},$$

then find the Domain and Range of the relation R . Also, determine if it is an equivalence relation.

OR

- (b) Determine whether the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 2x^3 - 7$, $\forall x \in \mathbb{R}$ (Real numbers) is bijective or not.

30. Solve the following Linear Programming Problem graphically :

$$\text{Minimize } Z = 4x + y$$

subject to

$$x + y \geq 3$$

$$x + 2y \geq 4$$

$$x, y \geq 0.$$

31. (a) Express the matrix $A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$ as the sum of a symmetric matrix and a skew symmetric matrix.

OR

- (b) Two lines are represented by equations $5x - 7y - 2 = 0$ and $7x - 5y - 3 = 0$ in the xy -plane. Using matrix method, find their point of intersection.

खण्ड घ

इस खण्ड में 4 दीर्घ-उत्तरीय (LA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 5 अंक हैं।

32. मान ज्ञात कीजिए :

$$I = \int_{-\pi}^{\pi} \frac{2x(1 + \sin x)}{1 + \cos^2 x} dx$$

33. (क) समाकलन विधि के उपयोग से, वक्र $y = x^2$ तथा रेखा $y = 2$ के द्वारा परिबद्ध क्षेत्र का क्षेत्रफल ज्ञात कीजिए।

अथवा

(ख) समाकलन विधि के उपयोग से, वक्र $9x^2 + 16y^2 = 144$ तथा रेखाओं $x = 2$ तथा $x = -2$ के द्वारा परिबद्ध क्षेत्र का क्षेत्रफल ज्ञात कीजिए।

34. अवकल समीकरण $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$ को हल कीजिए।

35. (क) माना $\vec{\alpha}$, $\vec{\beta}$, $\vec{\gamma}$ तीन शून्येतर सदिश इस प्रकार हैं कि $\vec{\alpha} \times \vec{\beta} = \vec{\gamma}$ तथा $\vec{\beta} \times \vec{\gamma} = \vec{\alpha}$ है। दर्शाइए कि α , β , γ परस्पर लंबवत सदिश इस प्रकार हैं कि $|\vec{\beta}| = 1$ तथा $|\vec{\alpha}| = |\vec{\gamma}|$ है।

सदिश $\vec{\beta}$ पर $\vec{\alpha}$ का प्रक्षेपण ज्ञात कीजिए। अपने उत्तर के समर्थन में कारण दीजिए।

अथवा

(ख) यदि बिंदु $A(3, -1, 2)$, $B(6, -3, 6)$, $C(4, -3, 1)$ तथा $D(1, -1, -3)$ एक चतुर्भुज ABCD के शीर्ष हैं, तो इन बिंदुओं A, B, C तथा D के स्थिति सदिश लिखिए तथा दर्शाइए कि ABCD एक समांतर चतुर्भुज है। समांतर चतुर्भुज ABCD का क्षेत्रफल भी ज्ञात कीजिए।

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. Evaluate :

$$I = \int_{-\pi}^{\pi} \frac{2x(1 + \sin x)}{1 + \cos^2 x} dx$$

33. (a) Using integration, find the area bounded by the curve $y = x^2$ and the line $y = 2$.

OR

(b) Using integration, find the area bounded by the curve $9x^2 + 16y^2 = 144$ and the lines $x = 2$, $x = -2$.

34. Solve the differential equation $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$.

35. (a) Let $\vec{\alpha}$, $\vec{\beta}$, $\vec{\gamma}$ be three non-zero vectors, such that $\vec{\alpha} \times \vec{\beta} = \vec{\gamma}$ and $\vec{\beta} \times \vec{\gamma} = \vec{\alpha}$. Show that α , β , γ are mutually perpendicular vectors such that $|\vec{\beta}| = 1$ and $|\vec{\alpha}| = |\vec{\gamma}|$.

Find projection of vector $\vec{\alpha}$ on $\vec{\beta}$. Give reasons to support your answer.

OR

(b) If points A(3, -1, 2), B(6, -3, 6), C(4, -3, 1) and D(1, -1, -3) are vertices of a quadrilateral ABCD, write the position vectors of A, B, C and D and show that ABCD represents a parallelogram. Also, find the area of the parallelogram ABCD.

खण्ड ड

इस खण्ड में 3 प्रकरण-अध्ययन आधारित प्रश्न हैं, जिनमें प्रत्येक के 4 अंक हैं।

प्रकरण अध्ययन – 1

36. एक संस्था को भवन निर्माण कम्पनी का चयन करना है जो विभिन्न शहरों में भवन निर्माण का कार्य कर सके। तीन कम्पनियों X, Y तथा Z को शॉर्टलिस्ट किया गया तथा उनको चुनने की प्रायिकताएँ क्रमशः $\frac{1}{3}$, $\frac{2}{9}$ तथा $\frac{4}{9}$ हैं। यह पाया गया कि इन कम्पनियों द्वारा खराब निर्माण गुणवत्ता देने की प्रायिकताएँ क्रमशः 20%, 40% तथा 30% हैं।

उपर्युक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

- (i) यदि निर्मित भवन मजबूत पाया गया हो, तो क्या प्रायिकता है कि भवन निर्माण अनुबंध का काम कम्पनी X को दिया गया था ? 2
- (ii) यदि नव निर्मित भवन का एक भाग गिर जाता है, तो इसकी क्या प्रायिकता है कि ऐसा कम्पनी Y अथवा Z को चुनने के कारण हुआ हो ? 2

प्रकरण अध्ययन – 2

37. एक स्कूल के घनाभाकार आकार के तरणताल की चौड़ाई रेखा $\frac{-x+2}{-5} = y = z - 3$ द्वारा

निरूपित है। दो विद्यार्थी A तथा B तरणताल में इसकी लंबाई की दिशा में तैर रहे हैं।

उपर्युक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

- (i) यदि विद्यार्थी A का रास्ता रेखा $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-3p}$ द्वारा निरूपित है, तो p का मान ज्ञात कीजिए। 1
- (ii) विद्यार्थी B के तैरने के रास्ते का सदिश समीकरण लिखिए, यदि वह विद्यार्थी A की दिशा के समांतर तैर रहा है और वह किसी समय बिंदु (2, 4, -1) से होकर जाता है। 1
- (iii) (क) (2, 4, -1) से विद्यार्थी A के पथ पर डाले गए लंब का पाद ज्ञात कीजिए। 2

अथवा

- (iii) (ख) यदि एक पक्षी तरणताल के ऊपर से रेखा $\frac{x+4}{2} = \frac{y+3}{8} = \frac{z-5}{-18}$ के रास्ते पर उड़ता है, तो इस रास्ते और विद्यार्थी A के रास्ते के बीच की न्यूनतम दूरी ज्ञात कीजिए। 2

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

- 36.** An organisation needs to select a building construction company to make houses in cities. Three companies X, Y and Z are short-listed with the probability of their selection being $\frac{1}{3}$, $\frac{2}{9}$ and $\frac{4}{9}$ respectively. It was found that there were 20%, 40% and 30% chances respectively, of delivering poor construction quality.

Based on the above information, answer the following questions :

- (i) If the building constructed was found to be strong, then what is the probability that the construction contract was given to the company X ? 2
- (ii) If a part of the newly constructed building falls off, then what is the probability that it was due to the selection of company Y or Z ? 2

Case Study – 2

- 37.** In a school, the swimming pool is shaped as a cuboid, and the breadth of the pool is represented by line $\frac{-x+2}{-5} = y = z - 3$. Two students A and B are swimming along the length of the pool.

Based on the above information, answer the following questions :

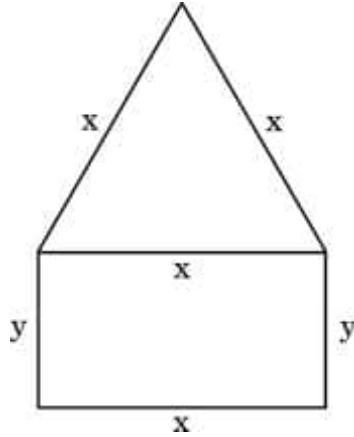
- (i) If the path of student A is represented by line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-3p}$, find the value of p. 1
- (ii) Write the vector equation of the path of student B swimming parallel to the path of student A, if at a certain instant he passes through the point with coordinates (2, 4, -1). 1
- (iii) (a) Find the foot of the perpendicular drawn from (2, 4, -1) to the path represented by student A. 2

OR

- (iii) (b) If a bird flies across the top of the swimming pool with its path represented by $\frac{x+4}{2} = \frac{y+3}{8} = \frac{z-5}{-18}$, then find the shortest distance between its path and the path of student A. 2

प्रकरण अध्ययन – 3

38. एक खिड़की x m लंबाई तथा y m चौड़ाई के आयत के आकार की है जिसकी लंबाई के ऊपर एक समबाहु त्रिभुज है।



उपर्युक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

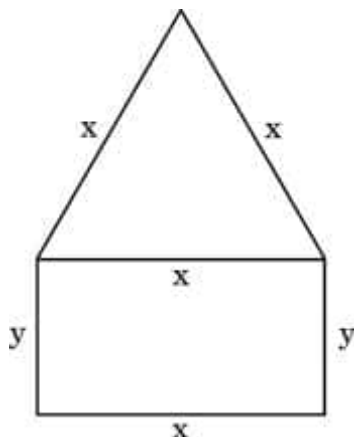
- | | | |
|-------|---|---|
| (i) | यदि खिड़की का परिमाप 18 m है, तो x तथा y के बीच संबंध लिखिए। | 1 |
| (ii) | ऊपर (i) में प्राप्त संबंध से खिड़की के क्षेत्रफल का केवल x के फलन के रूप में एक व्यंजक लिखिए। | 1 |
| (iii) | (क) आयत की वह विमाएँ ज्ञात कीजिए जिससे खिड़की में से अधिकतम रोशनी आ सके। | 2 |

अथवा

- | | | |
|-------|---|---|
| (iii) | (ख) यदि यह दिया गया है कि खिड़की का क्षेत्रफल 100 m^2 है, तो x के पदों में इसके परिमाप का व्यंजक ज्ञात कीजिए। | 2 |
|-------|---|---|

Case Study – 3

38. A window is in the form of a rectangle of length x and breadth y (in metres), surmounted by an equilateral triangle on its length.



Based on the above information, answer the following questions :

- (i) If the perimeter of the window is 18 m, write the relation between x and y . 1
- (ii) From (i), write an expression for the area of the window as a function of x only. 1
- (iii) (a) Find the dimensions of the rectangle that will allow the maximum light through the window. 2

OR

- (iii) (b) If it is given that the area of the window is 100 m^2 , then find the expression for its perimeter in terms of x . 2

Marking Scheme
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Senior Secondary Supplementary School Examination, 2026 (XII)
SUBJECT NAME: MATHEMATICS (041) (Q.P. CODE /Set No: 65/7/3)

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	“Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, evaluation done and several other aspects. Its leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in Newspaper/Website, etc. may invite action under various rules of the Board and IPC.”
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one’s own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In Class-XII, while evaluating two competency-based questions, please try to understand given answer and even if reply is not from marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.
4	The Marking scheme carries only suggested value points for the answers. These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (✓) wherever answer is correct. For wrong answer CROSS ‘X’ be marked. Evaluators will not put right (✓) while evaluating which gives an impression that answer is correct and no marks are awarded. This is most common mistake which evaluators are committing.
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note “Extra Question” .
10	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
11	A full scale of marks 0 to 80 has to be used. Please do not hesitate to award full marks if the answer deserves it.

12	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
13	Ensure that you do not make the following common types of errors committed by the Examiner in the past :- • Leaving answer or part thereof unassessed in an answer book. • Giving more marks for an answer than assigned to it. • Wrong totaling of marks awarded on an answer. • Wrong transfer of marks from the inside pages of the answer book to the title page. • Wrong question wise totaling on the title page. • Wrong totaling of marks of the two columns on the title page. • Wrong grand total. • Marks in words and figures not tallying/not same. • Wrong transfer of marks from the answer book to online award list. • Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) • Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
14	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
15	Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
16	The Examiners should acquaint themselves with the guidelines given in the “ Guidelines for Spot Evaluation ” before starting the actual evaluation.
17	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
18	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME
MATHEMATICS (Subject Code-041)
(PAPER CODE: 65/7/3)

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Steps	Marks
	SECTION A		
	Q. Number 1 to 20 are multiple choice questions of 1 mark each.		
1.	If $f(x) = \begin{cases} \frac{ x }{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then which of the following statements is not correct ? (A) f is continuous for every $x < 0$. (B) f is continuous for every $x \in \mathbb{R}$. (C) f is continuous for every $x > 0$. (D) f is continuous for each point except $x = 0$.		
Sol.	(B) f is continuous for every $x \in \mathbb{R}$.		1
2.	The value of $\sin^{-1}\left(\sin \frac{3\pi}{5}\right)$ is : (A) $\frac{3\pi}{5}$ (B) $\frac{\pi}{5}$ (C) $\frac{7\pi}{5}$ (D) $\frac{2\pi}{5}$		
Sol.	(D) $\frac{2\pi}{5}$		1
3.	For $f(x) = - x + 1 + 5$, which of the following statements is true ? (A) $\text{Max } f(x) = 0$ at $x = 4$. (B) $\text{Max } f(x) = 5$ at $x = -1$. (C) $\text{Max } f(x) = 4$ at $x = 0$. (D) $f(x)$ has no maximum value.		
Sol.	(B) $\text{Max } f(x) = 5$ at $x = -1$.		1
4.	If $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ satisfies $A' + A = I$, then the value of θ is : (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{2\pi}{3}$ (D) $\frac{5\pi}{6}$		
Sol.	(B) $\frac{\pi}{3}$		1

5.	If A is a square matrix, then which of the following is not true ? (A) $A + A'$ is a symmetric matrix (B) AA' and $A'A$ are symmetric matrices (C) $A + A'$ is a skew symmetric matrix (D) $A - A'$ is a skew symmetric matrix		
Sol.	(C) $A + A'$ is a skew symmetric matrix		1
6.	If $\begin{vmatrix} 3x & x+2 \\ 3(x-1) & x-2 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$, then the value of x is : (A) 2 (B) $\frac{1}{3}$ (C) 5 (D) 3		
Sol.	(A) 2		1
7.	The value of $\left[\tan^{-1}(-1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right) \right]$ is : (A) $\frac{7\pi}{12}$ (B) $\frac{5\pi}{12}$ (C) $-\frac{5\pi}{12}$ (D) $\frac{\pi}{3}$		
Sol.	(A) $\frac{7\pi}{12}$		1
8.	If $A = \begin{bmatrix} 1 & -2 & 3 \\ 3 & 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ -1 & 2 \\ 4 & -5 \end{bmatrix}$ and $C = [C_{ij}] = AB$, then C_{12} is : (A) 22 (B) -16 (C) 4 (D) 0		
Sol.	(B) -16		1
9.	If A and B are square matrices of order 3 such that $A = 4$ and $B = 7$, then $2AB$ is : (A) 112 (B) 56 (C) 28 (D) 224		
Sol.	(D) 224		1

10.	If $\int_0^1 (9x^2 - 4x - 2k) dx = 0$, then the value of 'k' is (A) 1 (B) 0 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$		
Sol.	(C) $\frac{1}{2}$		1
11.	The motion of a particle is described as $S = 2t^3 - 6t^2 + 8$. Its velocity $\frac{dS}{dt}$, when the acceleration $\frac{d^2S}{dt^2}$ vanishes is : (A) - 6 unit/sec (B) 0 unit/sec (C) 18 unit/sec (D) 6 unit/sec		
Sol.	(A) - 6 unit/sec		1
12.	If $\int_0^4 \frac{1}{x + \sqrt{x}} dx \equiv k \log 3$, then the value of k is : (A) 4 (B) - 2 (C) - 4 (D) 2		
Sol.	(D) 2		1
13.	$\int e^x \left(\frac{2 + \sin 2x}{1 + \cos 2x} \right) dx$ is equal to : (A) $\log \cos 2x + C$ (B) $\sec^2 2x + C$ (C) $e^x \tan x + C$ (D) $e^x \cot x + C$		
Sol.	(C) $e^x \tan x + C$		1
14.	Differential of $\log \sin x$ with respect to $\sqrt{\cos x}$ is : (A) $-2 \sqrt{\cos x} \cdot \cot x \cdot \operatorname{cosec} x$ (B) $\sqrt{\cos x} \cdot \cot x \cdot \operatorname{cosec} x$ (C) $\frac{2\sqrt{\cos x}}{\sin x}$ (D) $\frac{1}{\sin x \sqrt{\cos x}}$		
Sol.	(A) $-2 \sqrt{\cos x} \cdot \cot x \cdot \operatorname{cosec} x$		1

15.	If two events A and B are independent, then which of the following statements is not correct ? (A) $\bar{A}$ and B are independent. (B) Being independent does not indicate that they are mutually exclusive. (C) $P(\bar{A} \cap \bar{B}) = [1 - P(\bar{A})] [1 - P(\bar{B})]$. (D) $P(\bar{A} \cap \bar{B}) = [1 - P(A)] [1 - P(B)]$.		
Sol.	(C) $P(\bar{A} \cap \bar{B}) = [1 - P(\bar{A})] [1 - P(\bar{B})]$		1
16.	Differential of $\cos^{-1}(e^x)$ with respect to x is : (A) $\frac{e^x}{\sqrt{1 - e^{2x}}}$ (B) $\frac{-e^x}{\sqrt{1 + e^x}}$ (C) $\frac{-e^x}{\sqrt{1 - e^{2x}}}$ (D) $\frac{e^x}{\sqrt{1 + e^{2x}}}$		
Sol.	(C) $\frac{-e^x}{\sqrt{1 - e^{2x}}}$		1
17.	If vectors $\vec{a} = 3\hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{b} = \lambda\hat{i} + \hat{j}$ are perpendicular to each other, then the value of λ is : (A) 3 (B) -3 (C) $\frac{2}{3}$ (D) $-\frac{2}{3}$		
Sol.	(C) $\frac{2}{3}$		1
18.	If $P(A) = 0.3$, $P(B) = 0.7$ and $P(B/A) = 0.5$, then $P(A \cup B)$ is : (A) 0.35 (B) 0.85 (C) 0.5 (D) 0.21		
Sol.	(B) 0.85		1

	Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below. (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.		
19.	Assertion (A) : Let R be a relation on set A of all books in a school library given by $R = \{(x, y) : x, y \in A, x \text{ and } y \text{ have the same number of pages}\}$ and S is a relation on set A from the same library such that $S = \{(p, q) : p \text{ and } q \text{ are Mathematics books}\}$. Also, $R \cap S \neq \phi$. Then, $R \cap S$ is an equivalence relation. Reason (R) : The intersection of two equivalence relations on a set is an equivalence relation on the set.		
Sol.	(D) Assertion (A) is false, but Reason (R) is true.		1
20.	Assertion (A) : Points A(6, - 7, 0), B(16, -19, - 4) and C(1, - 1, 2) are collinear. Reason (R) : Three points A, B and C are collinear, when point C is the mid-point of line segment joining points A and B.		
Sol.	(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).		1
SECTION B			
Q. Numbers 21 to 25 are very short answer questions of 2 marks each.			
21.	Find the value of x if $A = B$, where $A = \begin{bmatrix} 6 & 3x \\ 8 & -6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 6 & x^2+2 \\ 8 & x^2-5x \end{bmatrix}.$		
Sol.	As $A = B$ so $3x = x^2 + 2$ and $-6 = x^2 - 5x$ Thus, $3x - 2 = 5x - 6$ gives $x = 2$	I II III	1 $\frac{1}{2}$ $\frac{1}{2}$

22.	(a) A box contains cards numbered 1 to 22. Two cards are drawn simultaneously without replacement and their numbers are noted. What is the probability that both numbers are even ? OR (b) If $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{4}$, find $P(\bar{A} \bar{B})$.		
Sol.	(a) As, box contains 11 even numbers and 11 odd numbers. $P(\text{Both Even numbers}) = \frac{11}{22} \times \frac{10}{21} \text{ or } \frac{{}^{11}C_2}{{}^{22}C_2}$ $\text{Thus, } P(\text{Both Even numbers}) = \frac{5}{21}$ OR (b) As, $P(A \cup B) = \frac{3+4-2}{8} = \frac{5}{8}$ $\text{So, } P(\bar{A} \bar{B}) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})} = \frac{1-P(A \cup B)}{1-P(B)}$ $\text{Thus, } P(\bar{A} \bar{B}) = \frac{3/8}{1/2} = \frac{3}{4}$	I $\frac{1}{2}$ II 1 III $\frac{1}{2}$ I 1 II $\frac{1}{2}$ III $\frac{1}{2}$	
23.	(a) Determine a and b if f(x) is a continuous function, where $f(x) = \begin{cases} 2, & x \leq 3 \\ ax + b, & 3 < x < 5 \\ 5, & x \geq 5 \end{cases}$ OR (b) If $2y^3 + y = 2x$, then show $\frac{d^2y}{dx^2} = \frac{-48y}{(6y^2 + 1)^3}$.		
Sol.	For getting $3a + b = 2$ For getting $5a + b = 5$ $\text{Thus, } a = \frac{3}{2} \text{ and } b = \frac{-5}{2}$ OR $\text{As, } 2y^3 + y = 2x \Rightarrow (6y^2 + 1) \frac{dy}{dx} = 2$	I $\frac{1}{2}$ II $\frac{1}{2}$ III $\frac{1}{2} + \frac{1}{2}$ I $\frac{1}{2}$	

	Thus, $\frac{dy}{dx} = \frac{2}{6y^2+1}$ On again Differentiating w.r.t. x, we get $\frac{d^2y}{dx^2} = \frac{-24y}{(6y^2+1)^2} \times \frac{dy}{dx}$ $\frac{d^2y}{dx^2} = \frac{-24y}{(6y^2+1)^2} \times \frac{2}{6y^2+1} = \frac{-48y}{(6y^2+1)^3}$	II	$\frac{1}{2}$
		III	$\frac{1}{2}$
		IV	$\frac{1}{2}$
24.	For the following Linear Programming Problem, shade the feasible region on the graph and find the coordinates of its corner points : Minimize $Z = 18x + 10y$ subject to constraints $4x + y \geq 20$ $2x + 3y \geq 30$ $x, y \geq 0.$		

Sol.	Corner Points are (15, 0), (0, 20), (3, 8)	Correct graph	
		I	1
		II	1

25.	Students are trying to fix thumb pins on a graph sheet on the class notice board. A relation R is defined on the set A of thumb pins as $R = \{(P, Q) : P, Q \in A \text{ and distance between P and Q is less than 3 cm}\}$ Determine if R is an equivalence relation.		
Sol.	Let $d(P, Q)$ denotes distance between any point P and Q For any point P on set A, the distance from P to itself is zero. $d(P, P) = 0 < 3$, $(P, P) \in R$. Therefore, R is reflexive. Let $(P, Q) \in R$, then $d(P, Q) < 3$. As, $d(P, Q) = d(Q, P)$. Thus, $d(Q, P) < 3$, which means $(Q, P) \in R$. Therefore, R is symmetric. Let $(P, Q) \in R$, so $d(P, Q) < 3$ and $(Q, S) \in R$ so $d(Q, S) < 3$, then $(P, S) \notin R$ for example Let P, Q and R be any three different points on the same line such that $d(P, Q) = 2$ cm and $d(Q, R) = 2$ cm so $d(P, R) = 4$ cm > 3 Therefore, R is not transitive. Thus, R is not an equivalence relation. **Note: If a student shows only Transitive part correctly (Without mentioning anything about Reflexive and Symmetric part), then also solution deserve full 2 marks. Alternative example are also acceptable.	I	$\frac{1}{2}$
		II	$\frac{1}{2}$
		III	1
SECTION C Q. Numbers 26 to 31 are short answer questions of 3 marks each.			
26.	(a) Find the equation of a line passing through the point $(1, -1, 2)$ and parallel to x-axis. Also, find the shortest distance between this line and the line $\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$. OR (b) Find the equations of the lines AC and BD, if coordinates of points A, B, C and D respectively, are $(3, -4, 2)$, $(-1, -2, 0)$, $(4, 1, -1)$ and $(4, 0, 5)$. Show that the lines AC and BD are perpendicular to each other.		
Sol.	Required eq. of line: $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \lambda(\hat{i})$ Now, $\vec{a}_1 = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b}_1 = \hat{i}$, $\vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{b}_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$	I	1

	So, $\vec{a_2} - \vec{a_1} = \hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{b_1} \times \vec{b_2} = -2\hat{j} - 5\hat{k}$, $\vec{b_1} \times \vec{b_2} = \sqrt{29}$ Thus, Shortest distance = $\left \frac{-4+15}{\sqrt{29}} \right = \frac{11}{\sqrt{29}}$ OR Required eq. of line AC: $\frac{x-3}{1} = \frac{y+4}{5} = \frac{z-2}{-3}$ OR $\frac{x-4}{1} = \frac{y-1}{5} = \frac{z+1}{-3}$ Required eq. of line BD: $\frac{x+1}{5} = \frac{y+2}{2} = \frac{z}{5}$ OR $\frac{x-4}{5} = \frac{y}{2} = \frac{z-5}{5}$ Since, $1 \times 5 + 5 \times 2 + (-3) \times 5 = 5 + 10 - 15 = 0$ Therefore, the lines AC and BD are perpendicular to each other..	II	1½
		III	½
		I	1
		II	1
		III	1
27.	If $y = \sqrt{\log \left\{ \sin \left(\frac{x^2}{3} + 1 \right) \right\}}$, then show that $\frac{dy}{dx} = \frac{x \cot \left(\frac{x^2}{3} + 1 \right)}{3y}$.		
Sol.	(a) $\frac{dy}{dx} = \frac{1}{2\sqrt{\log \left\{ \sin \left(\frac{x^2}{3} + 1 \right) \right\}}} \cdot \frac{d}{dx} \log \left\{ \sin \left(\frac{x^2}{3} + 1 \right) \right\}$ $= \frac{1}{2\sqrt{\log \left\{ \sin \left(\frac{x^2}{3} + 1 \right) \right\}}} \cdot \frac{1}{\sin \left(\frac{x^2}{3} + 1 \right)} \frac{d}{dx} \sin \left(\frac{x^2}{3} + 1 \right)$ $= \frac{1}{2\sqrt{\log \left\{ \sin \left(\frac{x^2}{3} + 1 \right) \right\}}} \cdot \frac{1}{\sin \left(\frac{x^2}{3} + 1 \right)} \cos \left(\frac{x^2}{3} + 1 \right) \cdot \frac{2x}{3}$ $= \frac{x \cot \left(\frac{x^2}{3} + 1 \right)}{3y}$	I	1
		II	1
		III	½
		IV	½
28.	Find the product of the maximum and minimum values of the function $f(x) = \cos x - \frac{1}{2} \cos 2x$, $x \in [0, \frac{\pi}{2}]$.		
Sol.	$f'(x) = -\sin x + \sin 2x$ $f'(x) = 0$ gives $\sin x(2\cos x - 1) = 0$ $\sin x = 0$ gives $x = 0$ $2\cos x - 1 = 0$ gives $x = \frac{\pi}{3}$ $f(0) = \frac{1}{2}$ (minimum) $f(\frac{\pi}{3}) = \frac{3}{4}$ (maximum)	I	1
		II	½
		III	½
		IV	½

	$f\left(\frac{\pi}{2}\right) = \frac{1}{2}$ (minimum) Required product = $\frac{3}{8}$	V	$\frac{1}{2}$
29.	(a) If R is a relation on set of natural numbers N such that $R = \{(x, y) : x, y \in \mathbb{N}, x + 2y = 25\},$ then find the Domain and Range of the relation R. Also, determine if it is an equivalence relation. OR (b) Determine whether the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 2x^3 - 7$, $\forall x \in \mathbb{R}$ (Real numbers) is bijective or not.		
Sol	Relation R $R = \{(23,1), (21,2), (19,3), (17,4), (15,5), (13,6), (11,7), (9,8), (7,9), (5,10), (3,11), (1,12)\}$ Domain of relation R = {23, 21, 19, 17, 15, 13, 11, 9, 7, 5, 3, 1} Range of relation R = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12} Since, $1 \in \mathbb{N}$, but $(1,1) \notin R$, So R is not Reflexive. Therefore, R is not an equivalence relation. OR Let $f(a) = f(b)$, for some $a, b \in \mathbb{R} \Rightarrow 2a^3 - 7 = 2b^3 - 7$ $\Rightarrow a^3 = b^3 \Rightarrow a = b$, thus, f is one-one function. Now, Let $y = f(x) = 2x^3 - 7 \Rightarrow x = \left(\frac{y+7}{2}\right)^{\frac{1}{3}}$ For any real number y, the expression $\left(\frac{y+7}{2}\right)^{\frac{1}{3}}$ is a real number. Therefore, Range = $\mathbb{R}$ = Co-domain, so f is onto function. Since f is both one-one and onto, so it is a bijective function.	I II III IV I II III IV	1 $\frac{1}{2}$ $\frac{1}{2}$ 1 1 $\frac{1}{2}$ 1 $\frac{1}{2}$

30.

Solve the following Linear Programming Problem graphically :

Minimize $Z = 4x + y$

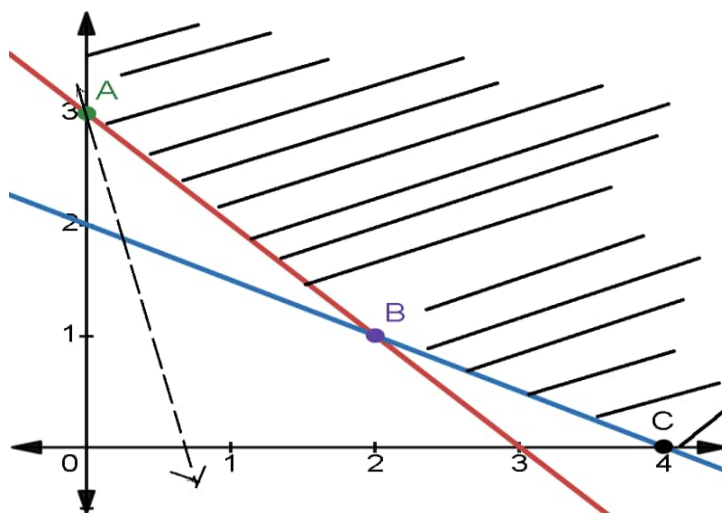
subject to

$$x + y \geq 3$$

$$x + 2y \geq 4$$

$$x, y \geq 0.$$

Sol.



Corner point	Value of $Z = 4x + y$
A (0, 3)	3
B (2, 1)	9
C (4, 0)	16

Because the feasible region is unbounded, $Z = 3$ might not be the absolute minimum.

On plotting, $4x + y < 3$, we observe the region $4x + y < 3$ lies entirely below this dashed line, so there is no overlap.

Since there are no common points, the minimum value is confirmed.

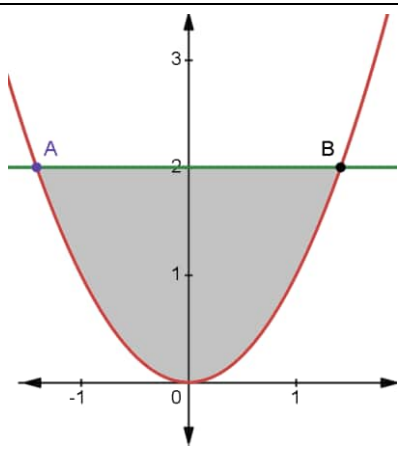
Minimum value of Z is 3 at $x = 0$ & $y = 3$.

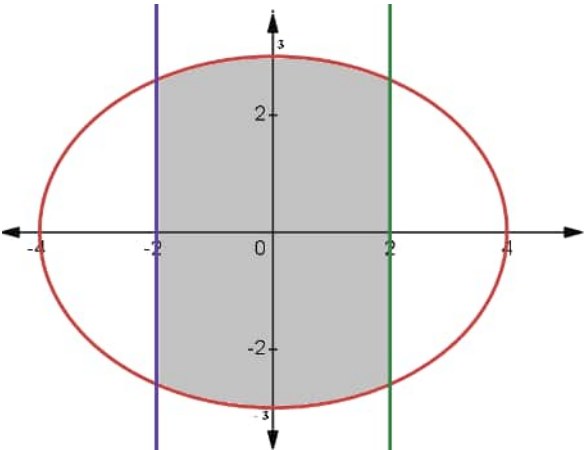
1
For
correct
graph &
shading

1

 $\frac{1}{2}$ $\frac{1}{2}$

31.	(a) Express the matrix $A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$ as the sum of a symmetric matrix and a skew symmetric matrix. OR (b) Two lines are represented by equations $5x - 7y - 2 = 0$ and $7x - 5y - 3 = 0$ in the xy-plane. Using matrix method, find their point of intersection.		
Sol.	(a) As, $A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 2 & 5 \\ 4 & 6 \end{bmatrix}$ So, $A + A' = \begin{bmatrix} 4 & 9 \\ 9 & 12 \end{bmatrix}$ and $A - A' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ Thus, $A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A') = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 6 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$ OR (b) Given equations can be written in matrix form as $\begin{bmatrix} 5 & -7 \\ 7 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ A X B Now, $A = 24 \neq 0, \therefore A^{-1}$ exists $\Rightarrow X = A^{-1}B$ Also, $A^{-1} = \frac{1}{24} \begin{bmatrix} -5 & 7 \\ -7 & 5 \end{bmatrix}$ Thus, $X = \frac{1}{24} \begin{bmatrix} -5 & 7 \\ -7 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} \frac{11}{24} \\ \frac{1}{24} \end{bmatrix} \Rightarrow x = \frac{11}{24}, y = \frac{1}{24}$ $\therefore$ the point of intersection of the two lines is $\left(\frac{11}{24}, \frac{1}{24}\right)$	I II III I II III IV	$\frac{1}{2}$ 1 $1\frac{1}{2}$ $\frac{1}{2}$ 1 1
SECTION D Q. Numbers 32 to 35 are long answer questions of 5 marks each.			

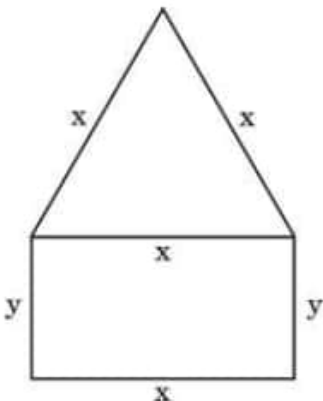
32.	Evaluate : $I = \int_{-\pi}^{\pi} \frac{2x(1 + \sin x)}{1 + \cos^2 x} dx$		
Sol.	$I = \int_{-\pi}^{\pi} \frac{2x}{1 + \cos^2 x} dx + \int_{-\pi}^{\pi} \frac{2x \sin x}{1 + \cos^2 x} dx$ $= 0 + 2 \int_0^{\pi} \frac{2x \sin x}{1 + \cos^2 x} dx$ $= 2 \int_0^{\pi} \frac{2(\pi - x) \sin x}{1 + \cos^2 x} dx$ $I = \int_0^{\pi} \frac{2\pi \sin x}{1 + \cos^2 x} dx$ Put $\cos x = t$ and $\sin x dx = -dt$ $I = 2\pi \int_1^{-1} \frac{-1}{1 + t^2} dt$ $I = 2\pi \int_{-1}^1 \frac{1}{1 + t^2} dt$ $= 2\pi [\tan^{-1} t]_{-1}^1$ $= \pi^2$	I II III IV V VI	1 1 1 $\frac{1}{2}$ 1 $\frac{1}{2}$
33.	(a) Using integration, find the area bounded by the curve $y = x^2$ and the line $y = 2$. OR (b) Using integration, find the area bounded by the curve $9x^2 + 16y^2 = 144$ and the lines $x = 2$, $x = -2$.		
Sol.	 Required Area $= 2 \int_0^2 x dy = 2 \int_0^2 \sqrt{y} dy$	I II	1 (For correct graph) 2

	$= 2 \left(\frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right)_0^2$ $= \frac{4}{3} (2\sqrt{2}) = \frac{8\sqrt{2}}{3}$ OR 	III	1
		IV	1
		I	1 (For correct graph)
		II	2
	Required Area = $4 \int_0^2 y \, dx = 4 \times \frac{3}{4} \int_0^2 \sqrt{4^2 - x^2} \, dx$ $= 3 \times \left(\frac{x\sqrt{4^2 - x^2}}{2} + 8 \sin^{-1} \frac{x}{4} \right)_0^2$ $= 3 \left(2\sqrt{3} + \frac{8 \times \pi}{6} \right) = (6\sqrt{3} + 4\pi)$	III	1
		IV	1
34.	Solve the differential equation $y + \frac{d}{dx} (xy) = x (\sin x + \log x)$.		
Sol.	Given differential equation is $y + x \frac{dy}{dx} + y = x(\sin x + \log x)$ $\frac{dy}{dx} + \frac{2y}{x} = \sin x + \log x$ Integrating Factor = $e^{\int \frac{2}{x} dx} = e^{2 \log x} = x^2$ Solution is $y \cdot x^2 = \int x^2 (\sin x + \log x) dx + C$	I	1
		II	$\frac{1}{2}$
		III	1
		IV	1

	$yx^2 = \int x^2(\sin x)dx + \int x^2(\log x)dx + C$ $= -x^2\cos x + \int 2x(\cos x)dx + \log x \frac{x^3}{3} - \int \frac{1}{x} \cdot \frac{x^3}{3} dx + C$ $= -x^2\cos x + 2x(\sin x) + 2\cos x + \log x \cdot \frac{x^3}{9} + C$	V	1
		VI	$\frac{1}{2}$
35.	(a) Let $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ be three non-zero vectors, such that $\vec{\alpha} \times \vec{\beta} = \vec{\gamma}$ and $\vec{\beta} \times \vec{\gamma} = \vec{\alpha}$. Show that α, β, γ are mutually perpendicular vectors such that $\vec{\beta} = 1$ and $\vec{\alpha} = \vec{\gamma}$. Find projection of vector $\vec{\alpha}$ on $\vec{\beta}$. Give reasons to support your answer. OR (b) If points A(3, -1, 2), B(6, -3, 6), C(4, -3, 1) and D(1, -1, -3) are vertices of a quadrilateral ABCD, write the position vectors of A, B, C and D and show that ABCD represents a parallelogram. Also, find the area of the parallelogram ABCD.		
Sol.	As, $\vec{\alpha} \times \vec{\beta} = \vec{\gamma} \Rightarrow \vec{\gamma} \perp \vec{\alpha}$ and $\vec{\gamma} \perp \vec{\beta}$ Similarly, $\vec{\beta} \times \vec{\gamma} = \vec{\alpha} \Rightarrow \vec{\alpha} \perp \vec{\gamma}$ and $\vec{\alpha} \perp \vec{\beta}$ Thus, $\vec{\alpha}, \vec{\beta}$ and $\vec{\gamma}$ are mutually perpendicular vectors. Now, $\vec{\alpha} \times \vec{\beta} = \vec{\gamma} \Rightarrow \vec{\alpha} \vec{\beta} = \vec{\gamma}$ and $\vec{\beta} \times \vec{\gamma} = \vec{\alpha} \Rightarrow \vec{\beta} \vec{\gamma} = \vec{\alpha}$ Thus, $\vec{\beta} \vec{\alpha} \vec{\beta} = \vec{\alpha} \Rightarrow \vec{\beta} = 1$ (since, $\vec{\alpha} \neq 0$) On putting $\vec{\beta} = 1$ in the $\vec{\beta} \vec{\gamma} = \vec{\alpha}$, we get $\vec{\gamma} = \vec{\alpha}$ Projection of $\vec{\alpha}$ on $\vec{\beta} = 0$, as $\vec{\alpha} \perp \vec{\beta}$. OR Position vector of A = $3\hat{i} - \hat{j} + 2\hat{k}$, Position vector of B = $6\hat{i} - 3\hat{j} + 6\hat{k}$ Position vector of C = $4\hat{i} - 3\hat{j} + \hat{k}$, Position vector of D = $\hat{i} - \hat{j} - 3\hat{k}$ Now, $\vec{AB} = 3\hat{i} - 2\hat{j} + 4\hat{k}$, $\vec{CD} = -3\hat{i} + 2\hat{j} - 4\hat{k} = -\vec{AB}$	I II III IV V VI I II III	1 1 1 1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1

	Also, $\overrightarrow{CD} = \overrightarrow{AB}$ As, AB and CD are parallel and equal so ABCD represents a parallelogram. Since, $\overrightarrow{AB} = 3\hat{i} - 2\hat{j} + 4\hat{k}$, $\overrightarrow{BC} = -2\hat{i} - 5\hat{k}$ $\overrightarrow{AB} \times \overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 4 \\ -2 & 0 & -5 \end{vmatrix} = 10\hat{i} + 7\hat{j} - 4\hat{k}$ Area of Parallelogram = $\sqrt{100 + 49 + 16} = \sqrt{165}$	IV	1
		V	1
		VI	1
	SECTION E This section (Q. 36 to 38) has 3 case study based questions of 4 marks each.		
36.	Case Study - 1 An organisation needs to select a building construction company to make houses in cities. Three companies X, Y and Z are short-listed with the probability of their selection being $\frac{1}{3}$, $\frac{2}{9}$ and $\frac{4}{9}$ respectively. It was found that there were 20%, 40% and 30% chances respectively, of delivering poor construction quality. Based on the above information, answer the following questions : (i) If the building constructed was found to be strong, then what is the probability that the construction contract was given to the company X ? 2 (ii) If a part of the newly constructed building falls off, then what is the probability that it was due to the selection of company Y or Z ? 2		
Sol.	(i) $P(\text{strong construction quality}) = \frac{1}{3} \times \frac{80}{100} + \frac{2}{9} \times \frac{60}{100} + \frac{4}{9} \times \frac{70}{100} = \frac{640}{900}$ So, $P(\frac{\text{Construction Contract to X}}{\text{strong construction quality}}) = \frac{\frac{240}{900}}{\frac{640}{900}} = \frac{3}{8}$ (ii) $P(\text{Defective construction quality}) = \frac{1}{3} \times \frac{20}{100} + \frac{2}{9} \times \frac{40}{100} + \frac{4}{9} \times \frac{30}{100} = \frac{260}{900}$ So, $P(\frac{\text{Construction Contract to Y or Z}}{\text{strong construction quality}}) = 1 - \frac{\frac{260}{900}}{\frac{640}{900}} = \frac{10}{13}$	I II I II	1 1 1 1

37.	Case Study - 2 In a school, the swimming pool is shaped as a cuboid, and the breadth of the pool is represented by line $\frac{-x+2}{-5} = y = z - 3$. Two students A and B are swimming along the length of the pool. Based on the above information, answer the following questions : (i) If the path of student A is represented by line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-3p}$, find the value of p. 1 (ii) Write the vector equation of the path of student B swimming parallel to the path of student A, if at a certain instant he passes through the point with coordinates (2, 4, -1). 1 (iii) (a) Find the foot of the perpendicular drawn from (2, 4, -1) to the path represented by student A. 2 OR (iii) (b) If a bird flies across the top of the swimming pool with its path represented by $\frac{x+4}{2} = \frac{y+3}{8} = \frac{z-5}{-18}$, then find the shortest distance between its path and the path of student A. 2		
Sol.	(i) $5 \times 1 + 1 \times 4 + 1 \times (-3p) = 0 \Rightarrow p = 3$ (ii) Vector equation of student B's path: $\vec{r} = (2\hat{i} + 4\hat{j} - \hat{k}) + \lambda(\hat{i} + 4\hat{j} - 9\hat{k})$ (iii) (a) Let the point be $Q(2, 4, -1)$ Let $P(\lambda - 5, 4\lambda - 3, -9\lambda + 6)$ be the foot of the perpendicular So, drs of PQ = $\langle \lambda - 7, 4\lambda - 7, -9\lambda + 7 \rangle$ Since PQ will be perpendicular to path of student A, so $\lambda - 7 + 16\lambda - 28 + 81\lambda - 63 = 0 \Rightarrow 98\lambda = 98 \Rightarrow \lambda = 1$ Thus, required foot of the perpendicular from Q to path of A is $(-4, 1, -3)$. OR (iii) (b) As, drs of Bird path and drs of student A's path are proportional So, both paths (line) are parallel. Here, $\vec{a}_1 = -5\hat{i} - 3\hat{j} + 6\hat{k}, \vec{a}_2 = -4\hat{i} - 3\hat{j} + 5\hat{k}, \vec{b} = \hat{i} + 4\hat{j} - 9\hat{k}$	I II I II III IV I	1 1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

	Now, $(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ 1 & 4 & -9 \end{vmatrix} = 4\hat{i} + 8\hat{j} + 4\hat{k}$ Thus, required shortest distance = $\frac{\sqrt{96}}{\sqrt{98}}$ or $\frac{4\sqrt{3}}{7}$	II	1
		III	$\frac{1}{2}$
38.	Case Study - 3 A window is in the form of a rectangle of length x and breadth y (in metres), surmounted by an equilateral triangle on its length.  Based on the above information, answer the following questions : (i) If the perimeter of the window is 18 m, write the relation between x and y. 1 (ii) From (i), write an expression for the area of the window as a function of x only. 1 (iii) (a) Find the dimensions of the rectangle that will allow the maximum light through the window. 2 OR (iii) (b) If it is given that the area of the window is 100 m², then find the expression for its perimeter in terms of x. 2		
Sol.	(i) Perimeter = 18 = 3x + 2y (ii) Area, $A = xy + \frac{\sqrt{3}x^2}{4} = \left(9x - \frac{3x^2}{2}\right) + \frac{\sqrt{3}x^2}{4}$ (iii) (a) $\frac{dA}{dx} = 9 - 3x + \frac{\sqrt{3}x}{2} = 0 \Rightarrow 9 = 3x - \frac{\sqrt{3}x}{2}$ So, $x = \frac{18}{6-\sqrt{3}}$ m Now, $\frac{d^2A}{dx^2} = -3 + \frac{\sqrt{3}}{2} < 0$	I	1
		I	1
		I	1
		II	$\frac{1}{2}$

	Thus, area is maximum when $x = \frac{18}{6-\sqrt{3}}$ m, $y = \frac{27-9\sqrt{3}}{6-\sqrt{3}}$ m Thus, Required foot of the perpendicular from Q to path of A is $(-4, 1, -3)$. OR (iii) (b) $xy + \frac{\sqrt{3}x^2}{4} = 100 \Rightarrow y = \frac{100}{x} - \frac{\sqrt{3}x}{4}$ So, Perimeter = $3x + 2y = \left(3x - \frac{\sqrt{3}x}{2} + \frac{200}{x}\right)$ m	III	$\frac{1}{2}$
		I	1
		II	1