



Series : PQSR2

SET-1



प्रश्न-पत्र कोड
Q.P. Code 65/2/1

रोल नं.

Roll No.

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परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।

Candidates must write the Q.P. Code on the title page of the answer-book.



गणित

MATHEMATICS

निर्धारित समय : 3 घण्टे

Time allowed : 3 hours

अधिकतम अंक : 80

Maximum Marks : 80

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं।
- कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में यथा स्थान पर प्रश्न का क्रमांक अवश्य लिखें।
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।
- Please check that this question paper contains 23 printed pages.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 38 questions.
- Please write down the serial number of the question in the answer-book at the given place before attempting it.
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period. {}

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P.T.O.



सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख्ती से पालन कीजिए :

- (i) इस प्रश्न-पत्र में कुल 38 प्रश्न हैं। सभी प्रश्न अनिवार्य हैं।
- (ii) यह प्रश्न-पत्र पाँच खण्डों में विभाजित है – खण्ड-क, ख, ग, घ एवं ङ।
- (iii) खण्ड – क में प्रश्न संख्या 1 से 18 तक बहुविकल्पीय तथा प्रश्न संख्या 19 एवं 20 अभिकथन एवं तर्क आधारित 1 अंक के प्रश्न हैं।
- (iv) खण्ड – ख में प्रश्न संख्या 21 से 25 तक अति लघु-उत्तरीय (VSA) प्रकार के 2 अंकों के प्रश्न हैं।
- (v) खण्ड – ग में प्रश्न संख्या 26 से 31 तक लघु-उत्तरीय (SA) प्रकार के 3 अंकों के प्रश्न हैं।
- (vi) खण्ड – घ में प्रश्न संख्या 32 से 35 तक दीर्घ-उत्तरीय (LA) प्रकार के 5 अंकों के प्रश्न हैं।
- (vii) खण्ड – ङ में प्रश्न संख्या 36 से 38 प्रकरण अध्ययन आधारित 4 अंकों के प्रश्न हैं।
- (viii) प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड – क के अतिरिक्त अन्य खण्डों के कुछ प्रश्नों में आंतरिक विकल्प दिया गया है।
- (ix) कैल्कुलेटर का उपयोग वर्जित है।

खण्ड – क

इस खण्ड में 20 बहुविकल्पीय प्रश्न हैं। प्रत्येक प्रश्न 1 अंक का है।

1. समुच्चय $A = \{1, 2, 3\}$ पर $R = \{(1, 1), (2, 2), (1, 2)\}$ द्वारा प्रदत्त संबंध R 1
 - (A) केवल स्वतुल्य है।
 - (B) स्वतुल्य तथा संक्रामक है।
 - (C) सममित तथा संक्रामक है।
 - (D) केवल संक्रामक है।
2. यदि A तथा B समान कोटि के वर्ग आव्यूह हैं, तो निम्न में से कौन सा/से कथन सदैव सत्य है/हैं? 1
 - (i) $(A + B)(A - B) = A^2 - B^2$
 - (ii) $AB = BA$
 - (iii) $(A + B)^2 = A^2 + AB + BA + B^2$
 - (iv) $AB = 0 \Rightarrow A = 0 \text{ or } B = 0$
 - (A) केवल (i) तथा (iii)
 - (B) केवल (ii) तथा (iii)
 - (C) केवल (iii)
 - (D) केवल (iii) तथा (iv)



General Instructions :

Read the following instructions very carefully and strictly follow them :

- (i) This Question paper contains **38** questions. **All** questions are **compulsory**.
- (ii) Question paper is divided into **FIVE** Sections – Section **A, B, C, D** and **E**.
- (iii) In Section **A** – Question Number **1** to **18** are Multiple Choice Questions (MCQs) and Question Number **19 & 20** are Assertion-Reason based questions of **1** mark each.
- (iv) In Section **B** – Question Number **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.
- (v) In Section **C** – Question Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.
- (vi) In Section **D** – Question Number **32** to **35** are Long Answer (LA) type questions, carrying **5** marks each.
- (vii) In Section **E** – Question Number **36** to **38** are case study based questions, carrying **4** marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in few questions in all the Sections except Section – **A**.
- (ix) Use of calculator is **NOT** allowed.

SECTION – A

This section comprises **20** Multiple Choice Questions (MCQs) of **1** mark each.

1. A relation R on set $A = \{1, 2, 3\}$ defined as $R = \{(1, 1), (2, 2), (1, 2)\}$ is 1
 - (A) Reflexive only
 - (B) Reflexive and Transitive
 - (C) Symmetric and Transitive
 - (D) Transitive only
2. If A and B are square matrices of same order, then which of the following statements is/are always true ? 1
 - (i) $(A + B)(A - B) = A^2 - B^2$
 - (ii) $AB = BA$
 - (iii) $(A + B)^2 = A^2 + AB + BA + B^2$
 - (iv) $AB = 0 \Rightarrow A = 0$ or $B = 0$
 - (A) Only (i) and (iii)
 - (B) Only (ii) and (iii)
 - (C) Only (iii)
 - (D) Only (iii) and (iv)



3. यदि $A = \begin{bmatrix} 1 & a & b \\ -1 & 2 & c \\ 0 & 5 & 3 \end{bmatrix}$ एक सममित आव्यूह है, तो $3a + b + c$ का मान है 1
- (A) 2 (B) 6
(C) 4 (D) 0
4. यदि $A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$ है तथा $A + A' = I$ है, तो $x \in \left[0, \frac{\pi}{2}\right]$ का मान है 1
- (A) 0 (B) $\frac{\pi}{4}$
(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$
5. एक वर्ग आव्यूह A के लिए $(3A)^{-1}$ बराबर है 1
- (A) $3A^{-1}$ (B) $9A^{-1}$
(C) $\frac{1}{3}A^{-1}$ (D) $\frac{1}{9}A^{-1}$
6. यदि $\begin{vmatrix} -1 & -2 & 5 \\ -2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = -86$ है, तो a के सभी मानों का योग है 1
- (A) 4 (B) 5
(C) -4 (D) 9
7. यदि $e^{-x} + e^{-y} = 2$ है, तो $\frac{dy}{dx}$ है 1
- (A) e^{x-y} (B) e^{y-x}
(C) $-e^{x-y}$ (D) $-e^{y-x}$



3. If $A = \begin{bmatrix} 1 & a & b \\ -1 & 2 & c \\ 0 & 5 & 3 \end{bmatrix}$ is a symmetric matrix, then the value of $3a + b + c$ is 1
- (A) 2 (B) 6
(C) 4 (D) 0
4. If $A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$ and $A + A' = I$, then the value of $x \in \left[0, \frac{\pi}{2}\right]$ is 1
- (A) 0 (B) $\frac{\pi}{4}$
(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$
5. For a square matrix A , $(3A)^{-1} =$ 1
- (A) $3A^{-1}$ (B) $9A^{-1}$
(C) $\frac{1}{3}A^{-1}$ (D) $\frac{1}{9}A^{-1}$
6. If $\begin{vmatrix} -1 & -2 & 5 \\ -2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = -86$, then the sum of all possible values of a is 1
- (A) 4 (B) 5
(C) -4 (D) 9
7. If $e^{-x} + e^{-y} = 2$, then $\frac{dy}{dx}$ is 1
- (A) e^{x-y} (B) e^{y-x}
(C) $-e^{x-y}$ (D) $-e^{y-x}$



8. $f(x) = x + \frac{1}{x}$, ($x \neq 0$) के लिए

1

- (A) स्थानीय उच्चतम मान 2 है ।
(B) स्थानीय निम्नतम मान -2 है ।
(C) स्थानीय उच्चतम मान -2 है ।
(D) स्थानीय निम्नतम मान $<$ स्थानीय अधिकतम मान

9. यदि $\int_0^{2a} \frac{1}{1+4x^2} dx = \frac{\pi}{6}$ है, तो a का मान है

1

- (A) $\frac{\sqrt{3}}{4}$ (B) $\frac{\sqrt{3}}{2}$
(C) $\sqrt{3}$ (D) $2\sqrt{3}$

10. निम्नलिखित में से कौन सा व्यंजक वक्र $y = x^2$ तथा रेखा $y = 16$ से घिरे क्षेत्र का क्षेत्रफल देगा ?

1

- (A) $\int_0^4 x^2 dx$ (B) $2 \int_0^4 x^2 dx$
(C) $\int_0^{16} \sqrt{y} dy$ (D) $2 \int_0^{16} \sqrt{y} dy$

11. अवकल समीकरण $\frac{dy}{dx} = \frac{\sqrt{y}}{\sqrt{x}}$ का व्यापक हल है

1

- (A) $\log \sqrt{y} = \log \sqrt{x} + C$ (B) $\sqrt{y} + \sqrt{x} = C$
(C) $\sqrt{y} - \sqrt{x} = C$ (D) $\log \sqrt{y} + \log \sqrt{x} = C$



8. For $f(x) = x + \frac{1}{x}$ ($x \neq 0$) 1

- (A) local maximum value is 2
- (B) local minimum value is -2
- (C) local maximum value is -2
- (D) local minimum value $<$ local maximum value

9. If $\int_0^{2a} \frac{1}{1+4x^2} dx = \frac{\pi}{6}$, then the value of a is 1

- (A) $\frac{\sqrt{3}}{4}$ (B) $\frac{\sqrt{3}}{2}$
- (C) $\sqrt{3}$ (D) $2\sqrt{3}$

10. Which of the following expressions will give the area of region bounded by the curve $y = x^2$ and line $y = 16$? 1

- (A) $\int_0^4 x^2 dx$ (B) $2 \int_0^4 x^2 dx$
- (C) $\int_0^{16} \sqrt{y} dy$ (D) $2 \int_0^{16} \sqrt{y} dy$

11. The general solution of the differential equation $\frac{dy}{dx} = \frac{\sqrt{y}}{\sqrt{x}}$ is 1

- (A) $\log \sqrt{y} = \log \sqrt{x} + C$ (B) $\sqrt{y} + \sqrt{x} = C$
- (C) $\sqrt{y} - \sqrt{x} = C$ (D) $\log \sqrt{y} + \log \sqrt{x} = C$



12. अवकल समीकरण $2x \frac{dy}{dx} - y = 3$ का समाकलन गुणक है 1
- (A) $\sqrt{x}$ (B) $\frac{1}{\sqrt{x}}$
(C) e^x (D) e^{-x}
13. यदि $|\vec{a}| = 5$ तथा $-2 \leq \lambda \leq 1$ है, तो $|\lambda \vec{a}|$ के अधिकतम तथा न्यूनतम मानों का योग है 1
- (A) -5 (B) 5
(C) 10 (D) 15
14. परिमाण 3 वाला एक सदिश जो x तथा y अक्ष से समान कोण बनाता है तथा z -अक्ष के लंबवत है, है 1
- (A) $\hat{i} + 2\sqrt{2} \hat{j}$ (B) $3\hat{k}$
(C) $\frac{3\sqrt{2}}{2} \hat{i} + \frac{3\sqrt{2}}{2} \hat{j}$ (D) $\sqrt{3} \hat{i} + \sqrt{3} \hat{j} + \sqrt{3} \hat{k}$
15. रेखा $\frac{2x-1}{4} = \frac{1-y}{3} = \frac{-z}{6}$ के दिक् कोसाइन हैं 1
- (A) $2, -3, -6$ (B) $\frac{2}{7}, \frac{-3}{7}, \frac{-6}{7}$
(C) $\frac{2}{7}, \frac{-3}{7}, \frac{6}{7}$ (D) $\frac{4}{\sqrt{61}}, \frac{-3}{\sqrt{61}}, \frac{-6}{\sqrt{61}}$
16. एक रैखिक प्रोग्रामन समस्या में रैखिक फलन जिसका अधिकतमीकरण या न्यूनतमीकरण करना होता है, कहलाता है 1
- (A) सुसंगत फलन (B) उद्देश्य फलन
(C) एक इष्टतम फलन (D) एक व्यवरोध

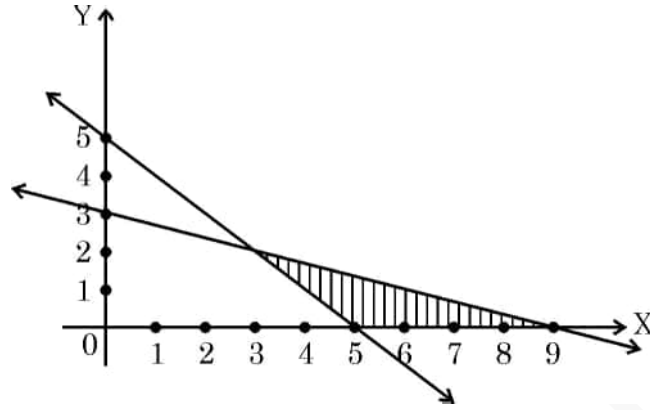


12. The integrating factor of the differential equation $2x \frac{dy}{dx} - y = 3$ is 1
- (A) $\sqrt{x}$ (B) $\frac{1}{\sqrt{x}}$
(C) e^x (D) e^{-x}
13. If $|\vec{a}| = 5$ and $-2 \leq \lambda \leq 1$, then the sum of greatest and the smallest value of $|\lambda \vec{a}|$ is 1
- (A) -5 (B) 5
(C) 10 (D) 15
14. Vector of magnitude 3 making equal angles with x and y axes and perpendicular to z axis is 1
- (A) $\hat{i} + 2\sqrt{2} \hat{j}$ (B) $3\hat{k}$
(C) $\frac{3\sqrt{2}}{2} \hat{i} + \frac{3\sqrt{2}}{2} \hat{j}$ (D) $\sqrt{3} \hat{i} + \sqrt{3} \hat{j} + \sqrt{3} \hat{k}$
15. Direction cosines of the line given by equations : $\frac{2x-1}{4} = \frac{1-y}{3} = \frac{-z}{6}$ are 1
- (A) $2, -3, -6$ (B) $\frac{2}{7}, \frac{-3}{7}, \frac{-6}{7}$
(C) $\frac{2}{7}, \frac{-3}{7}, \frac{6}{7}$ (D) $\frac{4}{\sqrt{61}}, \frac{-3}{\sqrt{61}}, \frac{-6}{\sqrt{61}}$
16. In a linear programming problem, the linear function which has to be maximized or minimized is called 1
- (A) a feasible function (B) an objective function
(C) an optimal function (D) a constraint



17. किसी रैखिक प्रोग्रामन समस्या का सुसंगत क्षेत्र नीचे दर्शाया गया है, इसके अतुच्छ व्यवरोध हैं :

1



- (A) $x + y \leq 5, x + 3y \leq 9$ (B) $x + y \leq 5, x + 3y \geq 9$
 (C) $x + y \geq 5, x + 3y \leq 9$ (D) $x + y \geq 5, 3x + y \leq 9$

18. दो घटनाएँ A तथा B इस प्रकार हैं कि $P(A) \neq 0$ तथा $P(B) \neq 1$ है तो $P(A'/B')$ बराबर है

1

- (A) $1 - P(A/B)$ (B) $1 - P(A'/B)$
 (C) $\frac{1 - P(A \cap B)}{P(B')}$ (D) $\frac{1 - P(A \cup B)}{P(B')}$

अभिकथन – तर्क आधारित प्रश्न

निर्देश : प्रश्न संख्या 19 और 20 अभिकथन (A) एवं तर्क (R) पर आधारित प्रत्येक 1 अंक के प्रश्न हैं। दो कथन दिए गए हैं जिनमें एक को अभिकथन (A) तथा दूसरे को तर्क (R) द्वारा अंकित किया गया है।

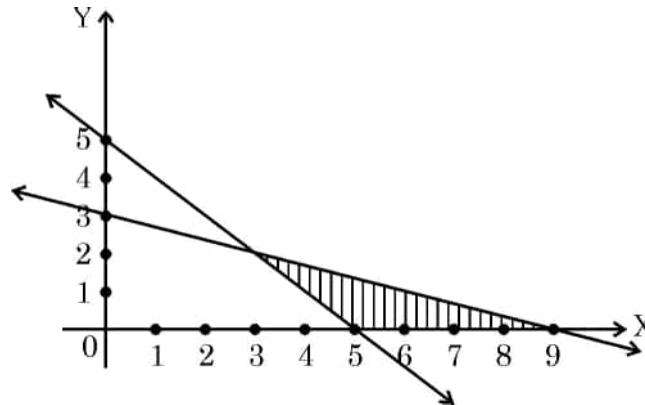
इन प्रश्नों के सही उत्तर नीचे दिए गए कोडों (A), (B), (C) और (D) में से चुनकर दीजिए :

- (A) अभिकथन (A) तथा तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या करता है।
 (B) अभिकथन (A) तथा तर्क (R) दोनों सही हैं, परन्तु तर्क (R), अभिकथन (A) की सही व्याख्या नहीं करता है।
 (C) अभिकथन (A) सही है, परन्तु तर्क (R) गलत है।
 (D) अभिकथन (A) गलत है, परन्तु तर्क (R) सही है।



17. For the feasible region shown below, the non-trivial constraints of the linear programming problem are

1



- (A) $x + y \leq 5, x + 3y \leq 9$ (B) $x + y \leq 5, x + 3y \geq 9$
 (C) $x + y \geq 5, x + 3y \leq 9$ (D) $x + y \geq 5, 3x + y \leq 9$
18. For two events A and B such that $P(A) \neq 0$ and $P(B) \neq 1$, $P(A'/B') =$

1

- (A) $1 - P(A/B)$ (B) $1 - P(A'/B)$
 (C) $\frac{1 - P(A \cap B)}{P(B')}$ (D) $\frac{1 - P(A \cup B)}{P(B')}$

Assertion – Reason Based Questions

Direction : Question numbers 19 and 20 are Assertion (A) and Reason (R) based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R).

Select the correct answer from the codes (A), (B), (C) and (D) as given below.

- (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
 (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
 (C) Assertion (A) is true, but Reason (R) is false.
 (D) Assertion (A) is false, but Reason (R) is true.



19. दो सदिशों $\vec{a}$ तथा $\vec{b}$ के लिए

अभिकथन (A) : $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$ 1

तर्क (R) : $|\vec{a} \times \vec{b}| = (\vec{a} \cdot \vec{b}) \tan \theta, \left(\theta \neq \frac{\pi}{2} \right)$

20. अभिकथन (A) : एक रेखा के दिक्-कोसाइन $\langle 1, 1, 1 \rangle$ हो सकते हैं। 1

तर्क (R) : $\theta = 0$ के लिए $\cos \theta = 1$ संभव है।

खण्ड – ख

इस खण्ड में अति लघु-उत्तरीय प्रकार के प्रश्न हैं। प्रत्येक प्रश्न के 2 अंक हैं।

21. (a) जाँच कीजिए कि फलन $f : \mathbb{R} - \{3\} \rightarrow \mathbb{R}$, जो कि $f(x) = \frac{x-2}{x-3}$ द्वारा परिभाषित है, आच्छादक है या नहीं। 2

अथवा

(b) जाँच कीजिए कि $f : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z} \times \mathbb{Z}$, (जहाँ $\mathbb{Z}$ पूर्णाकों का समुच्चय है) जो $f(x, y) = (2y, 3x)$ द्वारा परिभाषित है एकैक फलन है या नहीं। 2

22. यदि $x = a \sin^3 t$ तथा $y = b \cos^3 t$ है तो $t = \frac{\pi}{4}$ पर $\frac{dy}{dx}$ का मान ज्ञात कीजिए। 2

23. (a) $f(x) = \cos x + \sin^2 x, x \in [0, \pi]$ का निरपेक्ष उच्चतम मान ज्ञात कीजिए। 2

अथवा

(b) यदि एक ठोस अर्धगोले का आयतन एकसमान दर से बढ़ रहा है, तो सिद्ध कीजिए कि इसका पृष्ठीय क्षेत्रफल इसकी त्रिज्या के व्युत्क्रमानुपाती अनुपात से बढ़ता है। 2



19. For two vectors $\vec{a}$ and $\vec{b}$

Assertion (A) : $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$ 1

Reason (R) : $|\vec{a} \times \vec{b}| = (\vec{a} \cdot \vec{b}) \tan \theta, \left(\theta \neq \frac{\pi}{2} \right)$

20. **Assertion (A)** : A line can have direction cosines $< 1, 1, 1 >$ 1

Reason (R) : $\cos \theta = 1$ is possible for $\theta = 0$.

SECTION – B

This section comprises Very Short Answer (VSA) type questions carrying 2 marks each.

21. (a) Check whether $f : \mathbb{R} - \{3\} \rightarrow \mathbb{R}$ defined as $f(x) = \frac{x-2}{x-3}$ is onto or not. 2

OR

- (b) Check whether $f : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z} \times \mathbb{Z}$ (where $\mathbb{Z}$ is the set of integers) defined as $f(x, y) = (2y, 3x)$ is injective or not. 2

22. If $x = a \sin^3 t$, $y = b \cos^3 t$, then find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$. 2

23. (a) Find the absolute maximum value of $f(x) = \cos x + \sin^2 x$, $x \in [0, \pi]$ 2

OR

- (b) If the volume of a solid hemisphere increases at a uniform rate, prove that its surface area varies inversely as its radius. 2



24. यदि $\vec{AB} = \hat{j} + \hat{k}$ तथा $\vec{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$, ΔABC की भुजाओं AB तथा AC की दिशा में सदिशों को निरूपित करते हैं, तो सिद्ध कीजिए कि $\vec{AD} = \frac{\vec{AB} + \vec{AC}}{2}$, जहाँ D भुजा BC का मध्य बिंदु है।

अतः माध्यक AD की लंबाई भी ज्ञात कीजिए।

2

25. रेखा $\vec{r} = -\hat{j} + 3\hat{k} + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ पर एक ऐसे बिंदु के निर्देशांक ज्ञात कीजिए जिसके निर्देशांकों का योग 3 है।

2

खण्ड – ग

इस खण्ड में लघु-उत्तरीय प्रकार के प्रश्न हैं। प्रत्येक प्रश्न के 3 अंक हैं।

26. ज्ञात कीजिए : $\int \frac{x+2}{\sqrt{9x-x^2}} dx$

3

27. (a) मान ज्ञात कीजिए : $\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{dx}{1 + \sqrt{\cot x}}$

3

अथवा

- (b) मान ज्ञात कीजिए : $\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin |x| + \cos |x|) dx$

3

28. यदि $\frac{d}{dx} (F(x)) = \frac{1}{e^x + 1}$ है, तो F(x) ज्ञात कीजिए जबकि दिया है, कि $F(0) = \log \frac{1}{2}$.

3

29. (a) अवकल समीकरण $x \frac{dy}{dx} = y - x \sin^2 \left(\frac{y}{x} \right)$ का हल ज्ञात कीजिए, दिया है कि $y(1) = \frac{\pi}{6}$

3

अथवा

- (b) अवकल समीकरण $y \log y \frac{dx}{dy} + x = \frac{2}{y}$ का व्यापक हल ज्ञात कीजिए।

3



24. If $\vec{AB} = \hat{j} + \hat{k}$ and $\vec{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$ represent the two vectors along the sides AB and AC of $\triangle ABC$, prove that the median $\vec{AD} = \frac{\vec{AB} + \vec{AC}}{2}$, where D is midpoint of BC.

Hence, find the length of median AD.

2

25. Find the co-ordinates of the point on the line $\vec{r} = -\hat{j} + 3\hat{k} + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ such that the sum of co-ordinates is 3.

2

SECTION – C

This section comprises Short Answer (SA) type questions of 3 marks each.

26. Find : $\int \frac{x+2}{\sqrt{9x-x^2}} dx$

3

27. (a) Evaluate : $\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{dx}{1+\sqrt{\cot x}}$

3

OR

- (b) Evaluate : $\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin |x| + \cos |x|) dx$

3

28. If $\frac{d}{dx} (F(x)) = \frac{1}{e^x + 1}$, then find F(x) given that $F(0) = \log \frac{1}{2}$.

3

29. (a) Solve the following differential equation :

3

$$x \frac{dy}{dx} = y - x \sin^2 \left(\frac{y}{x} \right), \text{ given that } y(1) = \frac{\pi}{6}$$

OR

- (b) Find the general solution of the differential equation : $y \log y \frac{dx}{dy} + x = \frac{2}{y}$.

3



30. निम्न रैखिक प्रोग्रामन समस्या को ग्राफ द्वारा हल कीजिए :

3

व्यवरोधों $x + y \leq 50,$

$2x + y \leq 80,$

$x \geq 0, y \geq 0$

$Z = 10500x + 9000y$ के अंतर्गत अधिकतमीकरण कीजिए ।

31. (a) तेज़ हवा की वजह से तूफानी दिन में एक ट्रेन्ड स्नाइपर के टारगेट पर निशाने लगाने की प्रायिकता, टारगेट पर निशाना न लगा पाने की प्रायिकता की तीन गुना है ।

3



एक तूफानी दिन में जब हवा तेज थी स्नाइपर द्वारा दो शाट दागे गए । प्रायिकता ज्ञात कीजिए कि

(i) टारगेट दागा गया ।

(ii) कम से कम एक शॉट ने टारगेट मिस किया ।

अथवा

(b) एक पारिवारिक चित्र में माता, पिता व पुत्र यादृच्छया एक रेखा में खड़े हैं । यदि घटनाएँ E : पुत्र एक सिरे पर खड़ा है, F : पिता मध्य में खड़े हैं, द्वारा प्रदत्त हैं, तो $P(E/F)$ ज्ञात कीजिए ।

3

खण्ड – घ

इस खण्ड में दीर्घ-उत्तरीय प्रकार के प्रश्न हैं । प्रत्येक प्रश्न के 5 अंक हैं ।

32. (a) यदि $P = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ तथा $Q = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ है, तो (QP) ज्ञात कीजिए ।

अतः निम्न समीकरण निकाय को हल कीजिए :

$x - y = 3, 2x + 3y + 4z = 17, y + 2z = 7$

5

अथवा



30. Solve the following linear programming problem graphically : 3

Maximize $Z = 10500x + 9000y$

Subject to constraints

$$x + y \leq 50$$

$$2x + y \leq 80$$

$$x, y \geq 0$$

31. (a) The probability of hitting the target by a trained sniper is three times the probability of not hitting the target on a stormy day due to high wind speed. 3



The sniper fired two shots on the target on a stormy day when wind speed was very high. Find the probability that

- (i) target is hit
(ii) atleast one shot misses the target.

OR

- (b) Mother, Father and Son line up at random for a family picture. Let events E : Son on one end and F : Father in the middle. Find $P(E/F)$. 3

SECTION – D

This section comprises Long Answer (LA) type questions of 5 marks each.

32. (a) If $P = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $Q = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, find (QP) and hence

solve the following system of equations using matrices :

$$x - y = 3, 2x + 3y + 4z = 17, y + 2z = 7$$

5

OR



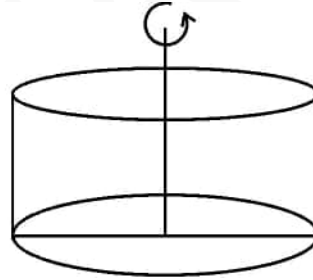
(b) $\Delta = \begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}$ का मान x, y, z के पदों में ज्ञात कीजिए । 5

यदि $\Delta = 0$ है तथा x, y, z शून्येतर वास्तविक संख्याएँ हैं, तो सिद्ध कीजिए कि $x^{-1} + y^{-1} + z^{-1} = -1$.

33. (a) $(0, \pi)$ के वह उपअन्तराल ज्ञात कीजिए जिनमें फलन $f(x) = \cot^{-1}(\sin x + \cos x)$ वर्धमान है अथवा ह्रासमान है । 5

अथवा

- (b) 36 cm परिधि वाले एक आयत को उसकी एक भुजा के चारों ओर घुमाकर अधिकतम आयतन वाला एक बेलन बनाया जाता है । 5



इस आयत की विमाएँ ज्ञात कीजिए ।

34. $g(x) = \cos^{-1}(x^2 - 1)$ का प्रान्त (domain) ज्ञात कीजिए, अतः x का वह मान ज्ञात कीजिए, जिसके लिए $g(x) = \frac{\pi}{3}$ है । 5

$\cos^{-1} x$ की किसी एक शाखा (मुख्य शाखा को छोड़कर) का परिसर भी लिखिए ।

35. बिंदुओं $A(1, 2, 3)$ तथा $B(5, 8, 11)$ से होकर जाने वाली रेखा एक अन्य रेखा

$\vec{r} = 4\hat{i} + \hat{j} + \lambda(5\hat{i} + 2\hat{j} + \hat{k})$ को काटती है । प्रतिच्छेदन बिंदु के निर्देशांक ज्ञात कीजिए ।

अतः एक रेखा का समीकरण लिखिए जो इस प्रतिच्छेदन बिंदु से होकर जाती है तथा दोनों रेखाओं के लंबवत हैं । 5



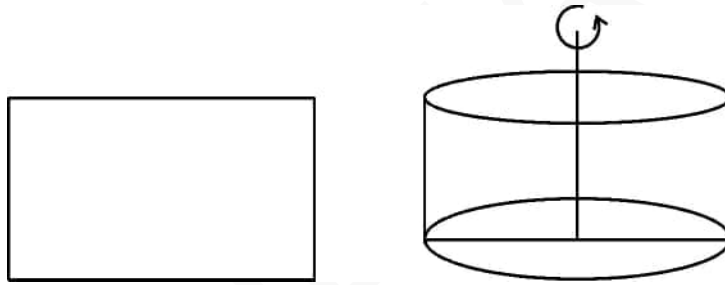
- (b) Obtain the value of $\Delta = \begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}$ in terms of x, y and z . 5

Further, if $\Delta = 0$ and x, y, z are non-zero real numbers, prove that $x^{-1} + y^{-1} + z^{-1} = -1$.

33. (a) Find the sub intervals in which $f(x) = \cot^{-1}(\sin x + \cos x)$, $x \in (0, \pi)$ is increasing and decreasing. 5

OR

- (b) A rectangle of perimeter 36 cm is revolved around one of its sides to sweep out a cylinder of maximum volume. 5



Find the dimensions of the rectangle.

34. Find the domain of $g(x) = \cos^{-1}(x^2 - 1)$. Hence, find the value of x for which $g(x) = \frac{\pi}{3}$. 5

Also, write the range of $\cos^{-1} x$ other than its principal branch.

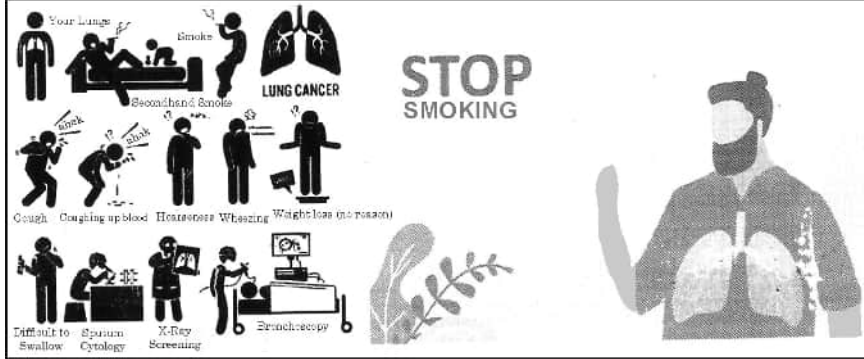
35. A line passing through the points $A(1, 2, 3)$ and $B(5, 8, 11)$ intersects the line $\vec{r} = 4\hat{i} + \hat{j} + \lambda(5\hat{i} + 2\hat{j} + \hat{k})$. Find the co-ordinates of the point of intersection. Hence, write the equation of a line passing through the point of intersection and perpendicular to both the lines. 5



खण्ड – ड

इस खण्ड में 3 प्रकरण अध्ययन आधारित प्रश्न हैं। प्रत्येक प्रश्न के 4 अंक हैं।

36. स्मोकिंग से फेफड़ों की परेशानियों का खतरा बढ़ जाता है।



एक अध्ययन से पता चला कि स्मोकिंग करने वाले 1000 पुरुषों में से 170 और स्मोकिंग करने वाली 1000 महिलाओं में से 120 को फेफड़ों से जुड़ी समस्याएँ होती हैं। एक कॉलोनी में 50 व्यक्ति स्मोकिंग करने वाले पाए गए जिनमें 30 पुरुष हैं।

इन 50 व्यक्तियों में से यादृच्छया एक चुना गया जिसका फेफड़ों संबंधी बीमारी का टेस्ट करवाया गया।

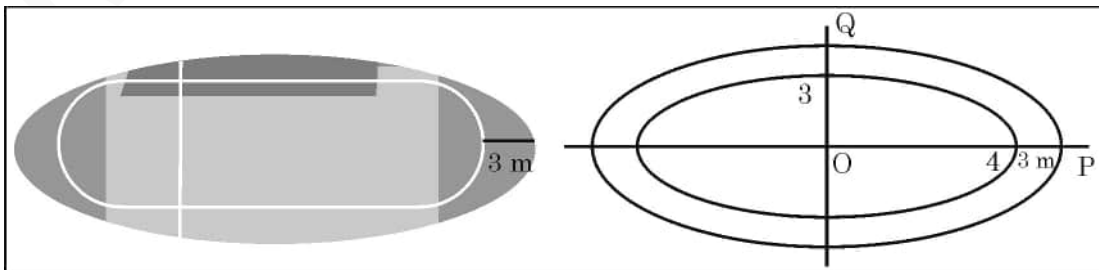
उपरोक्त के आधार पर निम्न प्रश्नों के उत्तर दीजिए :

- (i) क्या प्रायिकता है कि चुना हुआ व्यक्ति एक महिला हो ? 1
- (ii) यदि एक पुरुष चुना गया है तो क्या प्रायिकता है कि उसे फेफड़ों की समस्या नहीं है ? 1
- (iii) (a) यदि यादृच्छया चुने गए व्यक्ति में फेफड़ों की बीमारी पाई गई, तो उस व्यक्ति के महिला होने की प्रायिकता ज्ञात कीजिए। 2

अथवा

- (iii) (b) यादृच्छया चुने गए व्यक्ति में फेफड़ों की समस्या नहीं पाई गई है, तो प्रायिकता ज्ञात कीजिए कि वह एक पुरुष है। 2

37. एक दौड़ने वाला ट्रैक एक दीर्घवृत्तीय मैदान, जिसका समीकरण $9x^2 + 16y^2 = 144$ है, के चारों ओर बनाया गया है। ट्रैक की चौड़ाई 3 m है, जैसा कि आकृति में दर्शाया गया है :

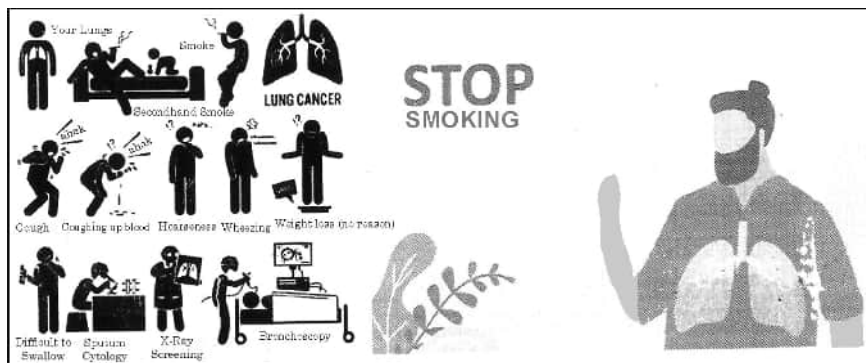




SECTION – E

This section comprises of 3 case study based questions of 4 marks each.

36. Smoking increases the risk of lung problems.



A study revealed that 170 in 1000 males who smoke develop lung complications, while 120 out of 1000 females who smoke develop lung related problems. In a colony, 50 people were found to be smokers of which 30 are males.

A person is selected at random from these 50 people and tested for lung related problems.

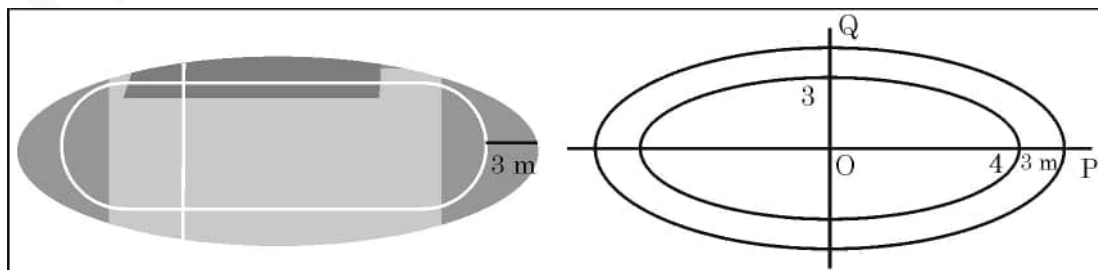
Based on the given information, answer the following questions :

- (i) What is the probability that selected person is a female ? 1
- (ii) If a male person is selected, what is the probability that he will not be suffering from lung problems ? 1
- (iii) (a) A person selected at random is detected with lung complications. Find the probability that selected person is a female. 2

OR

- (iii) (b) A person selected at random is not having lung problems, find the probability that the person is a male. 2

37. A racing track is build around an elliptical ground whose equation is given by $9x^2 + 16y^2 = 144$. The width of the track is 3 m as shown below :





उपरोक्त के आधार पर निम्न प्रश्नों के उत्तर दीजिए :

- (i) दी गई दीर्घ वृत्त की समीकरण से y को x के रूप में व्यक्त कीजिए । 1
- (ii) इस प्रकार (i) में प्राप्त फलन का x के सापेक्ष समाकलन कीजिए । 1
- (iii) (a) समाकलन के प्रयोग से दीर्घ वृत्तीय मैदान का (ट्रैक को छोड़कर) क्षेत्रफल ज्ञात कीजिए । 2

अथवा

- (iii) (b) बिंदुओं P तथा Q जहाँ बाह्य ट्रैक क्रमशः x -अक्ष तथा y -अक्ष को प्रथम चतुर्थांश में काटता है, के निर्देशांक लिखिए तथा समाकलन से P, O, Q से बनने वाली त्रिभुज का क्षेत्रफल ज्ञात कीजिए । 2

38. स्पोर्ट्स कार रेसिंग मोटर स्पोर्ट का एक रूप है जिसमें स्पोर्ट्स कार प्रोटोटाइप का उपयोग किया जाता है । यह प्रतियोगिता अलग-अलग आकार में बनाए गए विशेष ट्रैक पर होता है ।



ऐसे एक ट्रैक का समीकरण नीचे दिया गया है :

$$f(x) = \begin{cases} x^4 - 4x^2 + 4, & 0 \leq x < 3 \\ x^2 + 40, & x \geq 3 \end{cases}$$

उपरोक्त के आधार पर निम्न प्रश्नों के उत्तर दीजिए :

- (i) $0 < x < 3$ के लिए $f'(x)$ ज्ञात कीजिए । 1
- (ii) $f'(4)$ ज्ञात कीजिए । 1
- (iii) (a) $f(x)$ की $x = 3$ पर संतता की जाँच कीजिए । 2

अथवा

- (iii) (b) $x = 3$ पर $f(x)$ की अवकलनीयता की जाँच कीजिए । 2



Based on given information, answer the following questions :

- (i) Express y as a function of x from the given equation of ellipse. 1
- (ii) Integrate the function obtained in (i) with respect to x . 1
- (iii) (a) Find the area of the region enclosed within the elliptical ground excluding the track using integration. 2

OR

- (iii) (b) Write the co-ordinates of the points P and Q where the outer edge of the track cuts x axis and y axis in first quadrant and find the area of the triangle formed by points P, O, Q using integration. 2

38. Sports car racing is a form of motorsport which uses sports car prototypes. The competition is held on special tracks designed in various shapes.



The equation of one such track is given as follows :

$$f(x) = \begin{cases} x^4 - 4x^2 + 4, & 0 \leq x < 3 \\ x^2 + 40, & x \geq 3 \end{cases}$$

Based on given information, answer the following questions :

- (i) Find $f'(x)$ for $0 < x < 3$. 1
- (ii) Find $f'(4)$ 1
- (iii) (a) Test for continuity of $f(x)$ at $x = 3$. 2

OR

- (iii) (b) Test for differentiability of $f(x)$ at $x = 3$. 2



MATHEMATICS

MARKING SCHEME

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Senior Secondary School Examination, 2026

MATHEMATICS (041) (PAPER CODE 65/2/1)

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1.	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2.	“Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its leakage to the public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in Newspaper/Website, etc. may invite action under various rules of the Board and BNS.”
3.	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one’s own interpretation or any other consideration. The Marking Scheme should be strictly adhered to and religiously followed. However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating the competency-based questions, please try to understand the given answer and even if reply is not from a marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.
4.	The Marking Scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5.	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6.	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS ‘X’ be marked. Evaluators will not put right (✓) while evaluating which gives the impression that the answer is correct, and no marks are awarded. This is the most common mistake which evaluators are committing.
7.	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left- hand margin and encircled. This may be followed strictly.
8.	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.

9.	If a student has attempted an extra question, answer to the question deserving more marks should be retained and the other answer scored out with a note “Extra Question” .
10.	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
11.	A full scale of marks (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
12.	Every examiner must necessarily do evaluation work for full working hours, i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
13.	Ensure that you do not make the following common types of errors committed by the Examiner in the past: - • Leaving answer or part thereof unassessed in an answer book. • Giving more marks for an answer than assigned to it. • Wrong totaling of marks awarded on an answer. • Wrong transfer of marks from the inside pages of the answer book to the title page. • Wrong question wise totaling on the title page. • Wrong totaling of marks of the two columns on the title page. • Wrong grand total. • Marks in words and figures not tallying/not same. • Wrong transfer of marks from the answer book to online award list. • Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) Half or a part of the answer marked correct and the rest as wrong, but no marks awarded.
14.	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
15.	Any unassessed portion, non-carrying over of marks to the title page, or total error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
16.	The Examiners should acquaint themselves with the guidelines given in the “Guidelines for Spot Evaluation” before starting the actual evaluation.
17.	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
18.	The candidates are entitled to obtain a photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

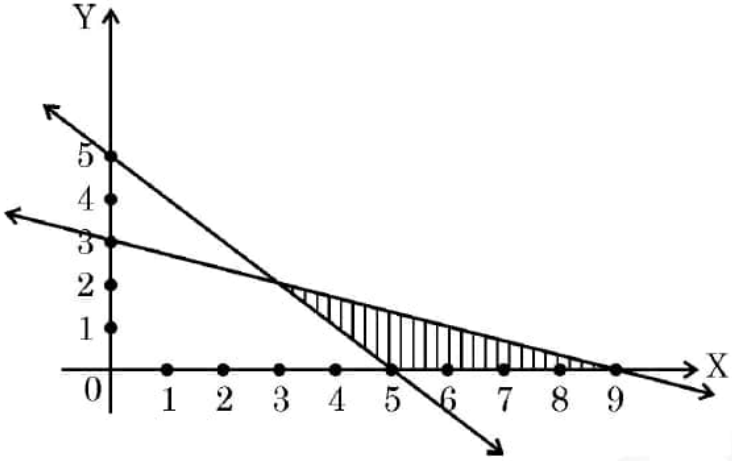
MARKING SCHEME
MATHEMATICS (Subject Code-041)
(PAPER CODE: 65/2/1)

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Steps	Marks
SECTION A			
Q. Number 1 to 20 are multiple choice questions of 1 mark each.			
1.	A relation R on set $A = \{1, 2, 3\}$ defined as $R = \{(1, 1), (2, 2), (1, 2)\}$ is (A) Reflexive only (B) Reflexive and Transitive (C) Symmetric and Transitive (D) Transitive only		
Sol.	(D) Transitive only		1
2.	If A and B are square matrices of same order, then which of the following statements is/are always true ? (i) $(A + B)(A - B) = A^2 - B^2$ (ii) $AB = BA$ (iii) $(A + B)^2 = A^2 + AB + BA + B^2$ (iv) $AB = 0 \Rightarrow A = 0$ or $B = 0$ (A) Only (i) and (iii) (B) Only (ii) and (iii) (C) Only (iii) (D) Only (iii) and (iv)		
Sol.	(C) Only (iii)		1
3.	If $A = \begin{bmatrix} 1 & a & b \\ -1 & 2 & c \\ 0 & 5 & 3 \end{bmatrix}$ is a symmetric matrix, then the value of $3a + b + c$ is (A) 2 (B) 6 (C) 4 (D) 0		
Sol.	(A) 2		1
4.	If $A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$ and $A + A' = I$, then the value of $x \in \left[0, \frac{\pi}{2}\right]$ is (A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$		
Sol.	(C) $\frac{\pi}{3}$		1

5.	For a square matrix A, $(3A)^{-1} =$ (A) $3A^{-1}$ (B) $9A^{-1}$ (C) $\frac{1}{3}A^{-1}$ (D) $\frac{1}{9}A^{-1}$	
Sol.	(C) $\frac{1}{3}A^{-1}$	1
6.	If $\begin{vmatrix} -1 & -2 & 5 \\ -2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = -86$, then the sum of all possible values of a is (A) 4 (B) 5 (C) -4 (D) 9	
Sol.	(C) -4	1
7.	If $e^{-x} + e^{-y} = 2$, then $\frac{dy}{dx}$ is (A) e^{x-y} (B) e^{y-x} (C) $-e^{x-y}$ (D) $-e^{y-x}$	
Sol.	(D) $-e^{y-x}$	1
8.	For $f(x) = x + \frac{1}{x}$ ($x \neq 0$) (A) local maximum value is 2 (B) local minimum value is -2 (C) local maximum value is -2 (D) local minimum value < local maximum value	
Sol.	(C) local maximum value is -2	1

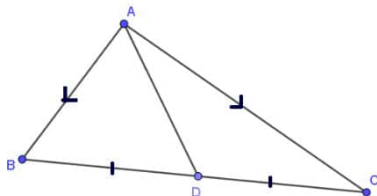
9.	If $\int_0^{2a} \frac{1}{1+4x^2} dx = \frac{\pi}{6}$, then the value of a is (A) $\frac{\sqrt{3}}{4}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\sqrt{3}$ (D) $2\sqrt{3}$	
Sol.	(A) $\frac{\sqrt{3}}{4}$	1
10.	Which of the following expressions will give the area of region bounded by the curve $y = x^2$ and line $y = 16$? (A) $\int_0^4 x^2 dx$ (B) $2 \int_0^4 x^2 dx$ (C) $\int_0^{16} \sqrt{y} dy$ (D) $2 \int_0^{16} \sqrt{y} dy$	
Sol.	(D) $2 \int_0^{16} \sqrt{y} dy$	1
11.	The general solution of the differential equation $\frac{dy}{dx} = \frac{\sqrt{y}}{\sqrt{x}}$ is (A) $\log \sqrt{y} = \log \sqrt{x} + C$ (B) $\sqrt{y} + \sqrt{x} = C$ (C) $\sqrt{y} - \sqrt{x} = C$ (D) $\log \sqrt{y} + \log \sqrt{x} = C$	
Sol.	(C) $\sqrt{y} - \sqrt{x} = C$	1
12.	The integrating factor of the differential equation $2x \frac{dy}{dx} - y = 3$ is (A) $\sqrt{x}$ (B) $\frac{1}{\sqrt{x}}$ (C) e^x (D) e^{-x}	
Sol.	(B) $\frac{1}{\sqrt{x}}$	1

13.	If $\vec{a} = 5$ and $-2 \leq \lambda \leq 1$, then the sum of greatest and the smallest value of $\lambda \vec{a}$ is (A) -5 (B) 5 (C) 10 (D) 15	
Sol.	(C) 10	1
14.	Vector of magnitude 3 making equal angles with x and y axes and perpendicular to z axis is (A) $\hat{i} + 2\sqrt{2}\hat{j}$ (B) $3\hat{k}$ (C) $\frac{3\sqrt{2}}{2}\hat{i} + \frac{3\sqrt{2}}{2}\hat{j}$ (D) $\sqrt{3}\hat{i} + \sqrt{3}\hat{j} + \sqrt{3}\hat{k}$	
Sol.	(C) $\frac{3\sqrt{2}}{2}\hat{i} + \frac{3\sqrt{2}}{2}\hat{j}$	1
15.	Direction cosines of the line given by equations : $\frac{2x-1}{4} = \frac{1-y}{3} = \frac{-z}{6}$ are (A) $2, -3, -6$ (B) $\frac{2}{7}, \frac{-3}{7}, \frac{-6}{7}$ (C) $\frac{2}{7}, \frac{-3}{7}, \frac{6}{7}$ (D) $\frac{4}{\sqrt{61}}, \frac{-3}{\sqrt{61}}, \frac{-6}{\sqrt{61}}$	
Sol.	(B) $\frac{2}{7}, \frac{-3}{7}, \frac{-6}{7}$	1
16.	In a linear programming problem, the linear function which has to be maximized or minimized is called (A) a feasible function (B) an objective function (C) an optimal function (D) a constraint	
Sol.	(B) an objective function	1

17.	For the feasible region shown below, the non-trivial constraints of the linear programming problem are  (A) $x + y \leq 5, x + 3y \leq 9$ (B) $x + y \leq 5, x + 3y \geq 9$ (C) $x + y \geq 5, x + 3y \leq 9$ (D) $x + y \geq 5, 3x + y \leq 9$	
Sol.	(C) $x + y \geq 5, x + 3y \leq 9$	1
18.	For two events A and B such that $P(A) \neq 0$ and $P(B) \neq 1$, $P(A'/B') =$ (A) $1 - P(A/B)$ (B) $1 - P(A'/B)$ (C) $\frac{1 - P(A \cap B)}{P(B')}$ (D) $\frac{1 - P(A \cup B)}{P(B')}$	
Sol.	(D) $\frac{1 - P(A \cup B)}{P(B')}$	1
	Assertion – Reason Based Questions Direction : Question numbers 19 and 20 are Assertion (A) and Reason (R) based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below. (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	

19.	For two vectors $\vec{a}$ and $\vec{b}$ Assertion (A) : $ \vec{a} \times \vec{b} ^2 + (\vec{a} \cdot \vec{b})^2 = \vec{a} ^2 \vec{b} ^2$ Reason (R) : $ \vec{a} \times \vec{b} = (\vec{a} \cdot \vec{b}) \tan \theta, \left(\theta \neq \frac{\pi}{2}\right)$		
Sol.	(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).		1
20.	Assertion (A) : A line can have direction cosines $< 1, 1, 1 >$ Reason (R) : $\cos \theta = 1$ is possible for $\theta = 0$.		
Sol.	(D) Assertion (A) is false, but Reason (R) is true.		1
SECTION B Q. Numbers 21 to 25 are very short answer questions of 2 marks each.			
21.	(a) Check whether $f : \mathbb{R} - \{3\} \rightarrow \mathbb{R}$ defined as $f(x) = \frac{x-2}{x-3}$ is onto or not. OR (b) Check whether $f : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z} \times \mathbb{Z}$ (where $\mathbb{Z}$ is the set of integers) defined as $f(x, y) = (2y, 3x)$ is injective or not.		
Sol. 21(a)	Let $y = \frac{x-2}{x-3}$ i.e. $x = \frac{3y-2}{y-1}$ Here $y \neq 1 \therefore R_f = \mathbb{R} - \{1\} \neq \text{Codomain}$ $\therefore f$ is not onto.	I II	1 1
OR			
Sol. 21(b)	Let $(x_1, y_1), (x_2, y_2) \in \mathbb{Z} \times \mathbb{Z}$ such that $f(x_1, y_1) = f(x_2, y_2)$ $\Rightarrow (2y_1, 3x_1) = (2y_2, 3x_2)$ $\Rightarrow y_1 = y_2, x_1 = x_2$ So, $(x_1, y_1) = (x_2, y_2)$ $\therefore f$ is injective.	I II	1 1
22.	If $x = a \sin^3 t, y = b \cos^3 t$, then find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$.		

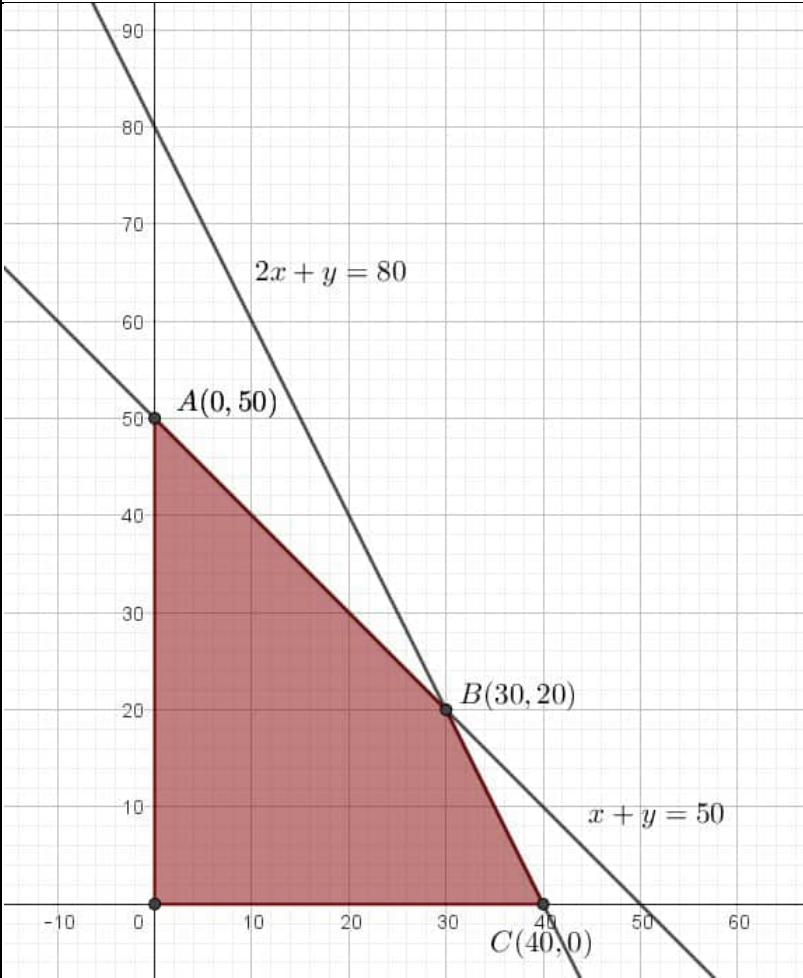
Sol.	$\frac{dx}{dt} = 3a \sin^2 t \cos t, \frac{dy}{dt} = -3b \cos^2 t \sin t$ $\frac{dy}{dx} = -\frac{b}{a} \cot t$ $\left. \frac{dy}{dx} \right _{t=\frac{\pi}{4}} = -\frac{b}{a}$	I	1
		II	$\frac{1}{2}$
		III	$\frac{1}{2}$
23.	(a) Find the absolute maximum value of $f(x) = \cos x + \sin^2 x, x \in [0, \pi]$ OR (b) If the volume of a solid hemisphere increases at a uniform rate, prove that its surface area varies inversely as its radius.		
Sol. 23(a)	$f(x) = \cos x + \sin^2 x, x \in [0, \pi]$ $f'(x) = -\sin x + 2 \sin x \cos x$ $f'(x) = 0 \Rightarrow \sin x (2 \cos x - 1) = 0$ $\Rightarrow \sin x = 0 \text{ or } \cos x = \frac{1}{2}$ $\therefore x = \frac{\pi}{3}$ $\text{Now, } f(0) = 1, f\left(\frac{\pi}{3}\right) = \frac{5}{4}, f(\pi) = -1$ $\therefore \text{Absolute maximum value of } f(x) \text{ is } \frac{5}{4}.$	I	1
		II	$\frac{1}{2}$
		III	$\frac{1}{2}$
OR			
Sol. 23(b)	$V = \frac{2}{3} \pi r^3 \Rightarrow \frac{dV}{dt} = 2\pi r^2 \frac{dr}{dt} = C \text{ (say)}$ $\Rightarrow \frac{dr}{dt} = \frac{C}{2\pi r^2}$ $\text{Now, } S = 3\pi r^2$ $\Rightarrow \frac{dS}{dt} = 6\pi r \frac{dr}{dt} = 6\pi r \times \frac{C}{2\pi r^2} = \frac{3C}{r}$ $\therefore S \text{ varies inversely as its radius.}$	I	1
		II	1

24.	If $\vec{AB} = \hat{j} + \hat{k}$ and $\vec{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$ represent the two vectors along the sides AB and AC of $\triangle ABC$, prove that the median $\vec{AD} = \frac{\vec{AB} + \vec{AC}}{2}$, where D is midpoint of BC. Hence, find the length of median AD.		
Sol.	Considering A as the origin, $\vec{AB}$ and $\vec{AC}$ are p.v. of B and C respectively. Hence p.v. of the mid point D is $\frac{\vec{AB} + \vec{AC}}{2}$  So, $\vec{AD} = \frac{(\hat{j} + \hat{k}) + (3\hat{i} - \hat{j} + 4\hat{k})}{2}$ $\vec{AD} = \frac{3\hat{i} + 5\hat{k}}{2}$ $\vec{AD} = \frac{1}{2} \sqrt{3^2 + 5^2} = \frac{\sqrt{34}}{2}$	I	1
		II	$\frac{1}{2}$
		III	$\frac{1}{2}$
25.	Find the co-ordinates of the point on the line $\vec{r} = -\hat{j} + 3\hat{k} + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ such that the sum of co-ordinates is 3.		
Sol.	$\vec{r} = -\hat{j} + 3\hat{k} + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ i.e. $\vec{r} = 2\lambda\hat{i} + (-2\lambda - 1)\hat{j} + (\lambda + 3)\hat{k}$ According to question, $2\lambda + (-2\lambda - 1) + (\lambda + 3) = 3 \Rightarrow \lambda = 1$ $\therefore \vec{r} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ Co-ordinates of the required point is $(2, -3, 4)$.	I	1
		II	1
SECTION C Q. Numbers 26 to 31 are short answer questions of 3 marks each.			
26.	Find : $\int \frac{x+2}{\sqrt{9x-x^2}} dx$		
Sol.	$I = \int \frac{x+2}{\sqrt{9x-x^2}} dx$ Let $x+2 = A(9-2x) + B$ $\Rightarrow A = -\frac{1}{2}, B = \frac{13}{2}$	I	1

	$I = -\frac{1}{2} \int \frac{9-2x}{\sqrt{9x-x^2}} dx + \frac{13}{2} \int \frac{1}{\sqrt{9x-x^2}} dx$ $= -\frac{1}{2} \left(2\sqrt{9x-x^2} \right) + \frac{13}{2} \int \frac{1}{\sqrt{\left(\frac{9}{2}\right)^2 - \left(x-\frac{9}{2}\right)^2}} dx$ $= -\sqrt{9x-x^2} + \frac{13}{2} \sin^{-1} \left(\frac{2x-9}{9} \right) + C$	II	1
		III	1
27.	(a) Evaluate : $\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{dx}{1+\sqrt{\cot x}}$ OR (b) Evaluate : $\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin x + \cos x) dx$		
Sol. 27(a)	$I = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{dx}{1+\sqrt{\cot x}} = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \quad (i)$ Applying $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$, we get $I = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \quad (ii)$ Adding (i) and (ii) $2I = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} 1 \cdot dx$ $= [x]_{\pi/12}^{5\pi/12} = \frac{\pi}{3}$ $\Rightarrow I = \frac{\pi}{6}$	I	$\frac{1}{2}$
		II	1
		III	1
		IV	$\frac{1}{2}$
OR			

Sol. 27(b)	$I = \int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin x + \cos x) dx$ $= \int_{-\frac{\pi}{6}}^0 (-\sin x + \cos x) dx + \int_0^{\frac{\pi}{2}} (\sin x + \cos x) dx$ $= [\cos x + \sin x]_{-\pi/6}^0 + [-\cos x + \sin x]_0^{\pi/2}$ $= \left(1 - \frac{\sqrt{3}}{2} + \frac{1}{2}\right) + (1 + 1)$ $= \frac{1}{2}(7 - \sqrt{3})$	I II III	1 1 1
28.	If $\frac{d}{dx}(F(x)) = \frac{1}{e^x + 1}$, then find $F(x)$ given that $F(0) = \log \frac{1}{2}$.		
Sol.	$\frac{d}{dx}(F(x)) = \frac{1}{e^x + 1}$ $\Rightarrow F(x) = \int \frac{1}{e^x + 1} dx$ $= \int \frac{e^{-x}}{1 + e^{-x}} dx$ $= -\log(1 + e^{-x}) + C$ Now, $F(0) = -\log 2 + C \Rightarrow \log \frac{1}{2} + \log 2 = C \Rightarrow C = 0$ $\therefore F(x) = -\log(1 + e^{-x})$ Note: The above expression is equivalent to $F(x) = x - \log(e^x + 1)$	I II III IV	1 1 $\frac{1}{2}$ $\frac{1}{2}$
29. (a)	(a) Solve the following differential equation : $x \frac{dy}{dx} = y - x \sin^2 \left(\frac{y}{x} \right), \text{ given that } y(1) = \frac{\pi}{6}$ OR (b) Find the general solution of the differential equation : $y \log y \frac{dx}{dy} + x = \frac{2}{y}$.		
Sol. 29(a)	Here, $\frac{dy}{dx} = \frac{y}{x} - \sin^2 \left(\frac{y}{x} \right)$ Put $\frac{y}{x} = v \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$	I II	$\frac{1}{2}$ $\frac{1}{2}$

	Differential equation reduces to $v + x \frac{dv}{dx} = v - \sin^2 v$ i.e. $x \frac{dv}{dx} = -\sin^2 v$ $\Rightarrow -\int \operatorname{cosec}^2 v \, dv = \int \frac{dx}{x}$ $\Rightarrow \cot v = \log x + C$ $\Rightarrow \cot\left(\frac{y}{x}\right) = \log x + C$ Now $y(1) = \frac{\pi}{6}$ gives $C = \sqrt{3}$ Required solution is : $\cot\left(\frac{y}{x}\right) = \log x + \sqrt{3}$	III	$\frac{1}{2}$
		IV	1
		V	$\frac{1}{2}$
OR			
Sol. 29(b)	Given differential equation can be written as $\frac{dx}{dy} + \frac{1}{y \log y} \cdot x = \frac{2}{y^2 \log y}$ I.F. = $e^{\int \frac{1}{y \log y} dy} = e^{\log(\log y)} = \log y$ Solution is given by $x \times \log y = \int \frac{2}{y^2 \log y} \times \log y \, dy = \int \frac{2}{y^2} dy$ $\Rightarrow x \log y = -\frac{2}{y} + C \quad \text{or} \quad xy \log y = -2 + Cy$	I	1
		II	1
		III	$\frac{1}{2}$
		IV	$\frac{1}{2}$

30.	Solve the following linear programming problem graphically : Maximize $Z = 10500x + 9000y$ Subject to constraints $x + y \leq 50$ $2x + y \leq 80$ $x, y \geq 0$												
Sol.	 <table data-bbox="173 1471 815 1774"><thead><tr><th>Corner Point</th><th>Value of $Z = 10500x + 9000y$</th></tr></thead><tbody><tr><td>$O(0,0)$</td><td>0</td></tr><tr><td>$A(0,50)$</td><td>4,50,000</td></tr><tr><td>$B(30,20)$</td><td>4,95,000</td></tr><tr><td>$C(40,0)$</td><td>4,20,000</td></tr></tbody></table> $Z_{\max} = 4,95,000$ when $x = 30$, $y = 20$	Corner Point	Value of $Z = 10500x + 9000y$	$O(0,0)$	0	$A(0,50)$	4,50,000	$B(30,20)$	4,95,000	$C(40,0)$	4,20,000	I	For correct graph and shading $1\frac{1}{2}$
Corner Point	Value of $Z = 10500x + 9000y$												
$O(0,0)$	0												
$A(0,50)$	4,50,000												
$B(30,20)$	4,95,000												
$C(40,0)$	4,20,000												
		II	For correct table 1										
		III	$\frac{1}{2}$										

31.	(a) The probability of hitting the target by a trained sniper is three times the probability of not hitting the target on a stormy day due to high wind speed.  The sniper fired two shots on the target on a stormy day when wind speed was very high. Find the probability that (i) target is hit (ii) atleast one shot misses the target. OR (b) Mother, Father and Son line up at random for a family picture. Let events E : Son on one end and F : Father in the middle. Find P(E/F).
Sol. 31(a)	(i) Let probability of <i>not</i> hitting = p $\therefore$ Probability of hitting = $3p$ $p + 3p = 1 \Rightarrow p = \frac{1}{4}$ So, Probability of hitting, $P(H) = \frac{3}{4}$, Probability of <i>not</i> hitting, $P(\overline{H}) = \frac{1}{4}$ Now, $P(\text{Target is hit}) = 1 - P(\overline{H} \overline{H}) = 1 - \frac{1}{4} \times \frac{1}{4} = \frac{15}{16}$ (ii) $P(\text{at least one shot misses the target})$ $= 1 - P(HH) = 1 - \frac{3}{4} \times \frac{3}{4} = \frac{7}{16}$ I II III 1 1 1
OR	
Sol. 31(b)	There are 3 people - Mother (M), Father (F), Son (S). Sample Space = $\{MFS, MSF, FMS, FSM, SMF, SFM\}$ Total possible arrangements = 6 Given E: Son on one end, F: Father in the middle $P(E \cap F) = P(\text{Son on one end and Father in the middle}) = \frac{2}{6}$, $P(F) = \frac{2}{6}$ $\therefore P(E F) = \frac{P(E \cap F)}{P(F)} = 1$ I II III 1 1 1

	Alternative Method : For $P(E F)$, as F has already occurred which means that the father is in the middle. So, son has to be on one end. $\therefore P(E F) = 1$ (sure event)	I	1
		II	2

SECTION D

Q. Numbers 32 to 35 are long answer questions of 5 marks each.

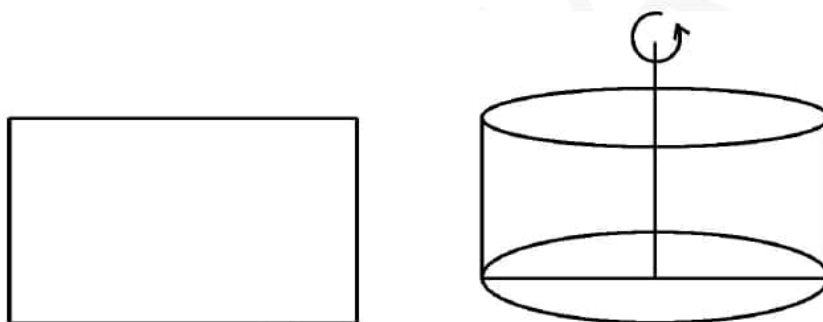
32.	(a) If $P = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $Q = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, find (QP) and hence solve the following system of equations using matrices : $x - y = 3$, $2x + 3y + 4z = 17$, $y + 2z = 7$ OR (b) Obtain the value of $\Delta = \begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}$ in terms of x, y and z. Further, if $\Delta = 0$ and x, y, z are non-zero real numbers, prove that $x^{-1} + y^{-1} + z^{-1} = -1$.
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Sol. 32(a)	$QP = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} \text{ i.e. } QP = 6I$ $\Rightarrow P^{-1} = \frac{1}{6}Q$ Given system of equations is given by $PX = R$ where, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, R = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$ $PX = R \Rightarrow X = P^{-1}R$ $\Rightarrow X = \frac{1}{6}Q.R$ $= \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$ $\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$ $\Rightarrow x=2, y=-1, z=4$	I II III IV V	2 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $1\frac{1}{2}$
OR			
Sol. 32(b)	$\Delta = \begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}$ $= (1+x)[(1+y)(1+z)-1] - 1[(1+z)-1] + 1[1-(1+y)]$ $= (1+x)(1+z+y+yz-1) - z - y$ $= xy + yz + zx + xyz$ $\Delta = 0 \Rightarrow xy + yz + zx + xyz = 0$ $\Rightarrow yz + zx + xy = -xyz$ Dividing both sides by xyz, $x^{-1} + y^{-1} + z^{-1} = -1$ Note: Proportionate marks may be allotted if a student uses properties of determinants to solve this question.	I II III IV	2 1 1 1

33.	(a) Find the sub intervals in which $f(x) = \cot^{-1}(\sin x + \cos x)$, $x \in (0, \pi)$ is increasing and decreasing.
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OR

(b) A rectangle of perimeter 36 cm is revolved around one of its sides to sweep out a cylinder of maximum volume.



Find the dimensions of the rectangle.

Sol.	$f(x) = \cot^{-1}(\sin x + \cos x), x \in (0, \pi)$
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$$f'(x) = \frac{-(\cos x - \sin x)}{1 + (\sin x + \cos x)^2} = \frac{\sin x - \cos x}{1 + (\sin x + \cos x)^2}$$

For critical points of $f(x)$, put $f'(x) = 0$

$$\Rightarrow \sin x = \cos x \Rightarrow x = \frac{\pi}{4}$$


$$f'(x) > 0 \text{ when } x \in \left(\frac{\pi}{4}, \pi\right) \Rightarrow f(x) \text{ is increasing in } \left(\frac{\pi}{4}, \pi\right).$$

Note: Alternatively $f(x)$ is also increasing in the interval $\left[\frac{\pi}{4}, \pi\right)$.

$$f'(x) < 0 \text{ when } x \in \left(0, \frac{\pi}{4}\right) \Rightarrow f(x) \text{ is decreasing in } \left(0, \frac{\pi}{4}\right).$$

Note: Alternatively $f(x)$ is also decreasing in the interval $\left(0, \frac{\pi}{4}\right]$.

I	2
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II	1
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III	1
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IV	1
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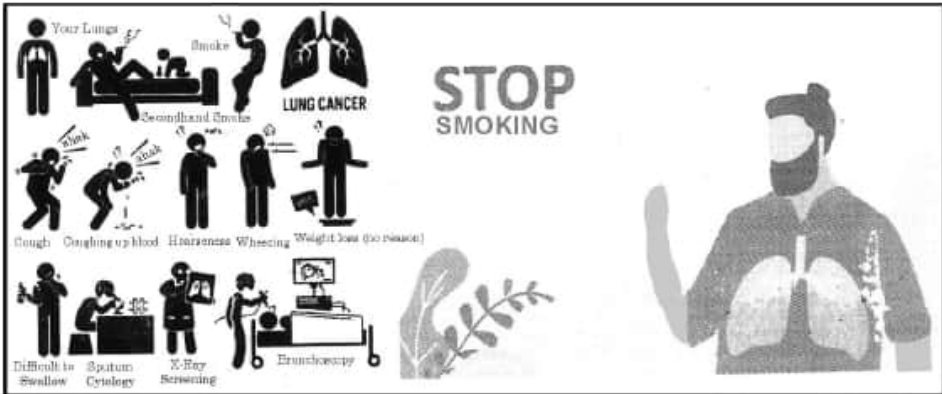
OR

Sol. 33(b)	Let the lengths of the sides of the rectangle be x and $(18 - x)$. Let it be revolved around the side of length $(18 - x)$, so that x becomes the radius of the cylinder. Volume of cylinder, $V = \pi x^2 (18 - x) = \pi (18x^2 - x^3)$ $\frac{dV}{dx} = \pi (36x - 3x^2)$ For critical points, put $\frac{dV}{dx} = 0$ $\Rightarrow x = 12 \text{ cm } (\because x \neq 0)$ Now $\left. \frac{d^2V}{dx^2} = \pi (36 - 6x) ; \frac{d^2V}{dx^2} \right _{x=12 \text{ cm}} = -36\pi < 0$ So, volume is maximum when $x = 12 \text{ cm}$. Hence the dimensions of rectangle are 12 cm and 6 cm.	I II III IV V VI	1 1 $\frac{1}{2}$ 1 1 $\frac{1}{2}$
34.	Find the domain of $g(x) = \cos^{-1} (x^2 - 1)$. Hence, find the value of x for which $g(x) = \frac{\pi}{3}$. Also, write the range of $\cos^{-1} x$ other than its principal branch.		
Sol.	Since domain of $\cos^{-1} x$ is $[-1, 1] \therefore -1 \leq x^2 - 1 \leq 1$ $\Rightarrow 0 \leq x^2 \leq 2 \Rightarrow -\sqrt{2} \leq x \leq \sqrt{2}$ $\therefore$ Domain of $g(x) = [-\sqrt{2}, \sqrt{2}]$ When $g(x) = \frac{\pi}{3}$, we have $\cos^{-1} (x^2 - 1) = \frac{\pi}{3}$ $\Rightarrow x^2 - 1 = \cos \frac{\pi}{3} = \frac{1}{2}$ $\Rightarrow x^2 = \frac{3}{2} \Rightarrow x = \sqrt{\frac{3}{2}}, -\sqrt{\frac{3}{2}}$ Range of $\cos^{-1} x$ other than its principal branch is $[-\pi, 0]$ or $[\pi, 2\pi]$. Note: Any valid branch like $[2\pi, 3\pi], [-2\pi, -\pi], \dots$ etc can be considered.	I II III IV V	1 1 1 1 1
35.	A line passing through the points A(1, 2, 3) and B(5, 8, 11) intersects the line $\vec{r} = 4\hat{i} + \hat{j} + \lambda(5\hat{i} + 2\hat{j} + \hat{k})$. Find the co-ordinates of the point of intersection. Hence, write the equation of a line passing through the point of intersection and perpendicular to both the lines.		

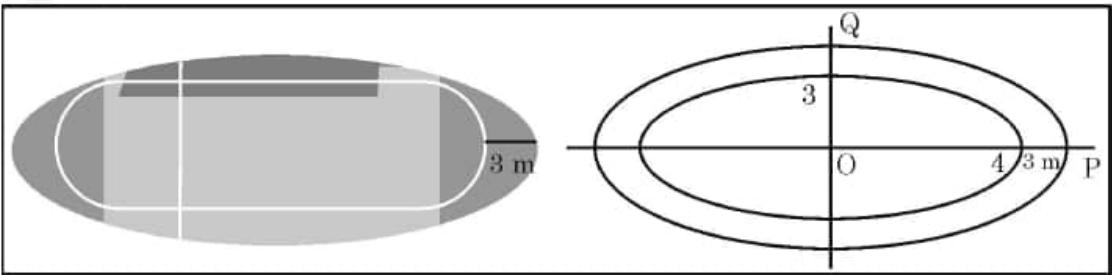
Sol.	The line passing through the points $A(1, 2, 3)$ and $B(5, 8, 11)$ is given by $l_1: \frac{x-1}{5-1} = \frac{y-2}{8-2} = \frac{z-3}{11-3} \text{ i.e. } \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \mu$ Also the given line in its cartesian form is, $l_2: \frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1} = \lambda$ Any point on l_1 is $P(2\mu + 1, 3\mu + 2, 4\mu + 3)$ Any point on l_2 is $Q(5\lambda + 4, 2\lambda + 1, \lambda)$ When the lines intersect, the points P and Q must coincide. $\therefore 2\mu + 1 = 5\lambda + 4, 3\mu + 2 = 2\lambda + 1 \text{ and } 4\mu + 3 = \lambda$ $\Rightarrow 2\mu - 5\lambda = 3 \dots (i), 3\mu - 2\lambda = -1 \dots (ii) \text{ and } 4\mu - \lambda = -3 \dots (iii)$ solving any two above equations, we get $\mu = \lambda = -1$ The point of intersection is $(-1, -1, -1)$. Let the required line(l) passing through $(-1, -1, -1)$ be $\frac{x+1}{a} = \frac{y+1}{b} = \frac{z+1}{c}$ Since $l \perp l_1$ and $l \perp l_2$, $2a + 3b + 4c = 0, 5a + 2b + c = 0$ $\Rightarrow \frac{a}{3-8} = \frac{b}{20-2} = \frac{c}{4-15} \text{ i.e. } \frac{a}{-5} = \frac{b}{18} = \frac{c}{-11}$ Hence, the required line is $\frac{x+1}{-5} = \frac{y+1}{18} = \frac{z+1}{-11}$ or $\frac{x+1}{5} = \frac{y+1}{-18} = \frac{z+1}{11}$ Note: The required line may also be written in the vector form. i.e. $\vec{r} = (-\hat{i} - \hat{j} - \hat{k}) + \lambda(5\hat{i} - 18\hat{j} + 11\hat{k})$ Note: Student can alternatively use point B to write the equation of line l_1.	I	1
		II	$\frac{1}{2}$
		III	1
		IV	$\frac{1}{2}$
		V	1
		VI	1

SECTION E

This section (Q. 36 to 38) has 3 case study-based questions of 4 marks each.

36.	Smoking increases the risk of lung problems. 
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	A study revealed that 170 in 1000 males who smoke develop lung complications, while 120 out of 1000 females who smoke develop lung related problems. In a colony, 50 people were found to be smokers of which 30 are males. A person is selected at random from these 50 people and tested for lung related problems. Based on the given information, answer the following questions : (i) What is the probability that selected person is a female ? 1 (ii) If a male person is selected, what is the probability that he will not be suffering from lung problems ? 1 (iii) (a) A person selected at random is detected with lung complications. Find the probability that selected person is a female. 2 OR (iii) (b) A person selected at random is not having lung problems, find the probability that the person is a male. 2		
Sol.	Total number of smokers in the colony = 50, Number of male smokers = 30, Number of female smokers = 20 Let L represent the event that the person is suffering from the lung complications. (i) $P(F) = \frac{20}{50} = \frac{2}{5}$ (ii) $P(L' M) = 1 - P(L M) = 1 - \frac{170}{1000} = \frac{83}{100}$ (iii) (a) $P(L M) = \frac{170}{1000} = \frac{17}{100}$ and $P(L F) = \frac{120}{1000} = \frac{12}{100}$ Using Baye's Theorem, $P(F L) = \frac{P(F).P(L F)}{P(M).P(L M) + P(F).P(L F)}$ $= \frac{\frac{2}{5} \cdot \frac{12}{100}}{\frac{3}{5} \cdot \frac{17}{100} + \frac{2}{5} \cdot \frac{12}{100}}$ $= \frac{24}{75} \text{ or } \frac{8}{25}$	I I I II	1 1 1½ ½

	OR (iii)(b) Here $P(L' M) = \frac{83}{100}$, $P(L' F) = \frac{88}{100}$ Using Baye's Theorem, $P(M L') = \frac{P(M) \cdot P(L' M)}{P(M) \cdot P(L' M) + P(F) \cdot P(L' F)}$ $= \frac{\frac{3}{5} \cdot \frac{83}{100}}{\frac{3}{5} \cdot \frac{83}{100} + \frac{2}{5} \cdot \frac{88}{100}}$ $= \frac{249}{425}$	I	1½
		II	½
37.	A racing track is build around an elliptical ground whose equation is given by $9x^2 + 16y^2 = 144$. The width of the track is 3 m as shown below :  Based on given information, answer the following questions : (i) Express y as a function of x from the given equation of ellipse. 1 (ii) Integrate the function obtained in (i) with respect to x. 1 (iii) (a) Find the area of the region enclosed within the elliptical ground excluding the track using integration. 2 OR (iii) (b) Write the co-ordinates of the points P and Q where the outer edge of the track cuts x axis and y axis in first quadrant and find the area of the triangle formed by points P, O, Q using integration. 2		
Sol.	(i) $9x^2 + 16y^2 = 144 \Rightarrow y = \frac{3}{4} \sqrt{16 - x^2}$ (ii) $\int y dx = \int \frac{3}{4} \sqrt{16 - x^2} dx$ $= \frac{3}{4} \left[\frac{x}{2} \sqrt{16 - x^2} + 8 \sin^{-1} \left(\frac{x}{4} \right) \right] + C$	I	1
		I	1

	(iii)(a) Required Area $= 4 \times \int_0^4 \frac{3}{4} \sqrt{16 - x^2} dx$ $= 3 \left[\frac{x}{2} \sqrt{16 - x^2} + 8 \sin^{-1} \left(\frac{x}{4} \right) \right]_0^4$ $= 3(8 \sin^{-1} 1 - 0) = 24 \times \frac{\pi}{2} = 12\pi$ OR (iii)(b) The track has a width of 3 m. Clearly $P(7,0)$ and $Q(0,6)$ The equation of line passing through $P(7,0)$ and $Q(0,6)$ is $\frac{x}{7} + \frac{y}{6} = 1$ Here, $y = \frac{6}{7}(7 - x)$ Required area $= \int_0^7 y dx = \int_0^7 \frac{6}{7}(7 - x) dx$ $= \frac{6}{7} \left[\frac{(7 - x)^2}{-2} \right]_0^7$ $= 21$	I	1
		II	1
		I	1
		II	$\frac{1}{2}$
		III	$\frac{1}{2}$
38.	Sports car racing is a form of motorsport which uses sports car prototypes. The competition is held on special tracks designed in various shapes.  The equation of one such track is given as follows : $f(x) = \begin{cases} x^4 - 4x^2 + 4, & 0 \leq x < 3 \\ x^2 + 40, & x \geq 3 \end{cases}$		

	Based on given information, answer the following questions :																																								
	(i) Find $f'(x)$ for $0 < x < 3$.		1																																						
	(ii) Find $f'(4)$		1																																						
	(iii) (a) Test for continuity of $f(x)$ at $x = 3$.		2																																						
	OR																																								
	(iii) (b) Test for differentiability of $f(x)$ at $x = 3$.		2																																						
Sol.	(i) For $0 < x < 3$, $f(x) = x^4 - 4x^2 + 4$ $\therefore f'(x) = 4x^3 - 8x$ (ii) For $x \geq 3$, $f(x) = x^2 + 40$ $\therefore f'(x) = 2x$ Hence, $f'(4) = 2 \times 4 = 8$ (iii)(a) Here $f(3) = 49$ LHL: $\lim_{x \rightarrow 3^-} (x^4 - 4x^2 + 4) = 49$ RHL: $\lim_{x \rightarrow 3^+} (x^2 + 40) = 49$ Since $\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x) = f(3)$, so $f(x)$ is continuous at $x = 3$. OR (iii)(b) Here $f(3) = 49$ getting LHD = 84 getting RHD = 6 Since LHD $\neq$ RHD at $x = 3$, so $f(x)$ is not differentiable at $x = 3$.																																								