Class XII Session 2023-24
Subject - Mathematics
Sample Question Paper - 8

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If A = [a_{ij}] is a scalar matrix of order n $\times$ n such that a_{ij} = k for all i, then trace of A is equal to **[1]**

a) $\frac{n}{k}$ b) none of these

c) nk d) n + k

2. The matrix $\begin{bmatrix} 5 & 10 & 3 \\ -2 & -4 & 6 \\ -1 & -2 & b \end{bmatrix}$ is a singular matrix, if the value of b is **[1]**

a) 3 b) Non-existent

c) -3 d) 0

3. If B is non-singular matrix and A is a square matrix, then det ($B^{-1}AB$) is equal to **[1]**

a) Det (A) b) Det (B)

c) Det (A^{-1}) d) Det (B^{-1})

4. The function $f(x) = \begin{cases} x^2 a & , \quad 0 \leq x < 1 \\ a & , \quad 1 \leq x < \sqrt{2} \\ \frac{2b^2-4b}{x^2} & , \quad \sqrt{2} \leq x < \infty \end{cases}$ is continuous for $0 \leq x < \infty$, then the most suitable values of a and b are **[1]**

a) a = -1, b = 1 b) a = -1, b = 1^+

c) a = - 1, b = -1 d) none of these

5. If O is the origin, OP = 3 with direction ratios proportional to - 1, 2, - 2 then the coordinates of P are **[1]**

a) (3, 6 , - 9) b) (1 , 2 , 2)

c) (- 1 , 2 , - 2)

d) $(\frac{-1}{9}, \frac{2}{9}, \frac{-2}{9})$

6. The order and degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right] = \frac{d^2y}{dx^2}$ respectively, are **[1]**

a) 4, 2

b) 2, 2

c) 1, 2

d) 2, 1

7. The optimal valuie of the objective function is attained at the points **[1]**

a) given by corner points of the feasible region

b) given by intersection of inequations with the axes only

c) None of these

d) given by intersection of inequations with x-axis only

8. if $|\vec{a}| = \sqrt{26}, |\vec{b}| = 7$ and $|\vec{a} \times \vec{b}|$ = 35 then $\vec{a}.\vec{b}$ is **[1]**

a) 13

b) 5

c) 12

d) 7

9. $\int \frac{x^2 \tan^{-1} x^3}{(1+x^6)} dx = ?$ **[1]**

a) $\frac{1}{6}\left(\tan^{-1} x^3\right)^2 + C$

b) $\frac{1}{3}\left(\tan^{-1} x^3\right) + C$

c) None of these

d) log $|\tan^{-1} x^3|$ + C

10. If A and B are two matrices such that AB = A and BA = B, then B^2 is equal to **[1]**

a) 0

b) A

c) B

d) 1

11. The region represented by the inequation system x, y $\geq$ 0, y $\leq$ 6, x + y $\leq$ 3 is **[1]**

a) unbounded in first and second quadrants

b) bounded in first quadrant

c) None of these

d) unbounded in first quadrant

12. If β is perpendicular to both α and γ, where α = $\hat{k}$ and γ = $\gamma = 2\hat{i} + 3\hat{j} + 4\hat{k}$, then what is β equal to? **[1]**

a) $-2\hat{i} + 3\hat{j}$

b) $3\hat{i} + 2\hat{j}$

c) $2\hat{i} - 3\hat{j}$

d) $-3\hat{i} + 2\hat{j}$

13. For what value of λ the following system of equations does not have a solution x + y + z = 6, 4x + λy - λz = 0, 3x + 2y – 4z = - 5 ? **[1]**

a) 1

b) -3

c) 0

d) 3

14. Two men hit at a target with probabilities $\frac{1}{2}$ and $\frac{1}{3}$, respectively. What is the probability that exactly one of them hits the target? **[1]**

a) $\frac{1}{6}$

b) $\frac{1}{2}$

c) $\frac{1}{3}$

d) $\frac{2}{3}$

15. The general solution of the differential equation $\frac{ydx-xdy}{y} = 0$ is **[1]**

a) y = Cx^2

b) x = Cy^2

c) y = Cx

d) xy = C

16. Which one of the following is the unit vector perpendicular to both $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$? **[1]**

a) $\frac{\hat{j}+\hat{k}}{\sqrt{2}}$

b) $\hat{k}$

c) $\pm\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

d) $\frac{\hat{i}-\hat{j}}{\sqrt{2}}$

17. The function f(x) $= \begin{cases} \frac{\sin x}{x} + \cos x, \text{ if } x \neq 0 \\ k \quad , \text{ if } x = 0 \end{cases}$ is continuous at x = 0, then the value of k is **[1]**

a) 1

b) 3

c) 1.5

d) 2

18. The Cartesian equations of a line are $\frac{x-2}{2} = \frac{y+1}{3} = \frac{z-3}{-2}$. What is its vector equation? **[1]**

a) none of these

b) $\vec{r} = (2\hat{i} + 3\hat{j} - 2\hat{k}) + \lambda(2\hat{i} - \hat{j} + 3\hat{k})$

c) $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(2i + 3j - 2k)$

d) $\vec{r} = (2\hat{i} + 3\hat{j} - 2\hat{k})$

19. **Assertion (A):** f(x) = tan x - x always increases. **[1]**

Reason (R): Any function y = f(x) is increasing if $\frac{dy}{dx}$ > 0.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The relation R in the set A = (1, 2, 3, 4) defined as R = {(x, y): y is divisible by x) is an equivalence relation. **[1]**

Reason (R): A relation R on the set A is equivalence if it is reflexive, symmetric and transitive.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Evaluate: $\sin^{-1}(\sin(-600°))$ **[2]**

OR

For the principal value, evaluate $\tan^{-1}\left\{2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right\}$.

22. Separate [0, π/2] into subintervals in which f(x) = sin 3x is increasing or decreasing. **[2]**

23. Show that the function f(x) = x^{100} + sin x - 1 is increasing on the interval $\left(\frac{\pi}{2}, \pi\right)$ **[2]**

OR

Water is running into an inverted cone at the rate of π cubic metres per minute. The height of the cone is 10 metres, and the radius of its base is 5 m. How fast the water level is rising when the water stands 7.5 m below the base.

24. Evaluate: $\int \frac{1}{\sin x \cos^2 x} dx$ **[2]**

25. A man 2 metres high, walks at a uniform speed of 6 metres per minute away from a lamp post, 5 metres high. Find the rate at which the length of his shadow increases. **[2]**

Section C

26. Evaluate: $\int_0^{\pi/4} \frac{\sin 2x}{\cos^4 x + \sin^4 x} dx$ **[3]**

27. Three persons A, B and C apply for a job of Manager in a private company. Chances of their selection (A, B and C are in the ratio 1:2:4. The probabilities that A, B and C can introduce changes to improve profits of the **[3]**

company are 0.8, 0.5 and 0.3, respectively. If the change does not take place, find the probability that it is due to the appointment of C.

28. Evaluate: $\int_{-3}^{3} \frac{x^4}{1+e^x} dx$ **[3]**

OR

Evaluate: $\int_{1/e}^{e} |\log_e x| \, dx$

29. Solve the differential equation: $xy\frac{dy}{dx} = x^2 - y^2$ **[3]**

OR

Solve the initial value problem: $x\frac{dy}{dx} - y = \log x$, $y(1) = 0$

30. Minimise Z = 13x - 15y, subject to the constraints: $x + y \leq 7, 2x - 3y + 6 \geq 0, x \geq 0, y \geq 0$. **[3]**

OR

Maximize Z = 3x + 2y

Subject to constraints

$x + 2y \leq 10$

$3x + y \leq 15$

$x, y \geq 0$

31. If $y = \log\{\sqrt{x-1} - \sqrt{x+1}\}$, show that $\frac{dy}{dx} = \frac{-1}{2\sqrt{x^2-1}}$ **[3]**

Section D

32. Find the area bounded by the curves y = x and $y = x^3$ **[5]**

33. Let A = R - {3} and B = R - {1}. Consider the function f : A $\rightarrow$ B defined by $f(x) = \left(\frac{x-2}{x-3}\right)$ Is f one-one and onto? Justify your answer. **[5]**

OR

Let n be a fixed positive integer. Define a relation R on Z as follows:

(a, b) $\in$ R $\Leftrightarrow$ a - b is divisible by n. Show that R is an equivalence relation on Z.

34. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, verify that $A^3 - 6A^2 + 9A - 4I = 0$ and hence find A^{-1} **[5]**

35. Find the shortest distance between the given lines. $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$, $\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(-2\hat{i} + 3\hat{j} + 8\hat{k})$ **[5]**

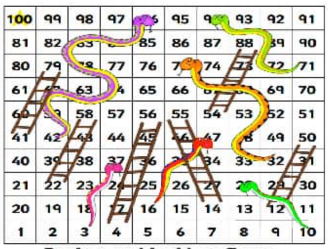
OR

Find the vector equation of the line passing through the point (1, 2, -4) and perpendicular to the two lines: $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Akshat and his friend Aditya were playing the snake and ladder game. They had their own dice to play the game. Akshat was having red dice whereas Aditya had black dice. In the beginning, they were using their own dice to play the game. But then they decided to make it faster and started playing with two dice together.



Aditya rolled down both black and red die together.

First die is black and second is red.

(i) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

(ii) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

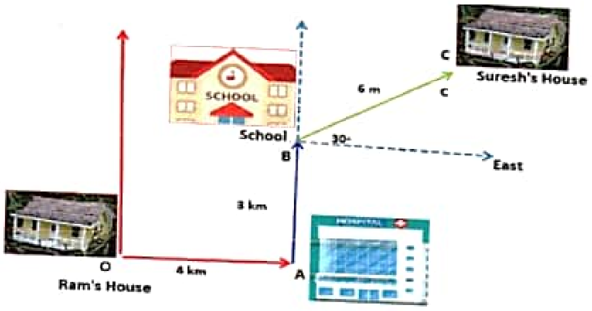
(iii) Find the conditional probability of obtaining the sum 10, given that the black die resulted in even number.

OR

Find the conditional probability of obtaining the doublet, given that the red die resulted in a number more than 4.

37. **Read the text carefully and answer the questions:** **[4]**

Ram's house is situated at Gandhi Nagar at Point O, for going to school he first travels by bus in the east. Here at Point A, a hospital is situated. From Hospital Ram takes an auto and goes 3 km in the north direction, here at point B school is situated. Suresh's house is at 30^o east, 6 km from point B.



(i) Find the displacement between Ram's house and school.

(ii) How many km Ram travels to reach school?

(iii) What is the vector distance from school to Suresh's home?

OR

What is the displacement from Ram's house to Suresh house?

38. **Read the text carefully and answer the questions:** **[4]**

The Government declare that farmers can get ₹300 per quintal for their onions on 1st July and after that, the price will be dropped by ₹3 per quintal per extra day. Govind's father has 80 quintals of onions in the field on 1st July and he estimates that the crop is increasing at the rate of 1 quintal per day.

(i) If x is the number of days after 1^{st} July, then express price and quantity of onion and the revenue as a function of x.

(ii) Find the number of days after 1st July, when Govind's father attains maximum revenue.

Solution

Section A

1.

(c) nk

Explanation: $\because$ A = $[a_{ij}]_{n \times n}$

Trace of A, i.e., tr (A) = $\sum a_{ij}^{n} i$ = 1 = a_{11} + a_{22} + ... + a_{nm}

= k + k + k + k + k + ...(n times)

= k(n)

= nk

2.

(b) Non-existent

Explanation: $\begin{bmatrix} 5 & 10 & 3 \\ -2 & -4 & 6 \\ -1 & -2 & b \end{bmatrix}$ is singular matrix. So its determinant value of this matrix is zero.

i.e, $\begin{vmatrix} 5 & 10 & 3 \\ -2 & -4 & 6 \\ -1 & -2 & b \end{vmatrix} = 0$

$\Rightarrow$ 5(-4b + 12) - 10(-2b + 6) + 3(4 - 4) = 0

$\Rightarrow$ -20b + 60 + 20b - 60 = 0

b does not exist

3. **(a)**

Det (A)

Explanation: $|B^{-1}AB| = |B^{-1}| \times |A| \times |B|$

$\frac{1}{B} \times |A| \times |B| = |A|$

4.

(c) a = - 1, b = -1

Explanation: f(x) is continuous at x = 1

$\Rightarrow \lim\limits_{x \to 1} f(x) = f(1)$

$\Rightarrow \lim\limits_{h \to 0} f(1+h) = a$

$\Rightarrow \lim\limits_{h \to 0} \frac{(1+h)^2}{a} = a$

$\Rightarrow \frac{1}{a} = a$

$\Rightarrow a^2 = 1$

$\Rightarrow a = \pm 1$

Consider,

$\lim\limits_{x \to \sqrt{2}} \frac{2b^2 - 4b}{x^2} = a$

$b^2 - 2b = \pm 1$

for a = 1

using formulas for quadratic equation

$b^2 - 2b - 1 = 0 \Rightarrow b = 1 \pm \sqrt{2}$

for a = -1

$b^2 - 2b = -1$

$b^2 - 2b + 1 = 0$

$(b - 1)^2 = 0$

b = 1
a = -1, b = 1

5.

(c) (- 1 , 2 , - 2)
Explanation: (- 1 , 2 , - 2)
Direction ratios of OP = Coordinates of P - Coordinates of O
-1, 2, 2 = (x - 0), (y - 0), (z - 0)
Thus, coordinates of P are (-1,2,-2)

6.

(d) 2, 1
Explanation: We have $\left[1+\left(\frac{dy}{dx}\right)^2\right]=\frac{d^2y}{dx^2}$
$\therefore$ Order = 2 and degree = 1

7. **(a)** given by corner points of the feasible region
Explanation: It is known that the optimal value of the objective function is attained at any of the corner point. Thus, the optimal value of the objective function is attained at the points given by corner points of the feasible region.

8.

(d) 7
Explanation: 7
Hint
$|\vec{a}\times\vec{b}|=35\Rightarrow|\vec{a}||\vec{b}|\ \sin\theta=35\Rightarrow\sin\theta=\frac{35}{\sqrt{26}\times7}=\frac{5}{\sqrt{26}}$
$\cos\theta=\sqrt{1-\sin^2\theta}=\sqrt{1-\frac{25}{26}}=\frac{1}{\sqrt{26}}$
$\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta=\sqrt{26}\times7\times\frac{1}{\sqrt{26}}=7$

9. **(a)** $\frac{1}{6}\left(\tan^{-1}x^3\right)^2+C$
Explanation: Given integral is $\int\frac{x^2\tan^{-1}x^3}{\left(1+x^6\right)}dx$
Let, $\tan^{-1}x^3$ = z
$\Rightarrow\frac{1}{1+x^6}\times3x^2dx=dz$
$\Rightarrow\frac{x^2}{1+x^6}dx=\frac{dz}{3}$
So,
$\frac{1}{3}\int z\,dz$
$=\frac{1}{3}\frac{z^2}{2}+c$
$=\frac{z^2}{6}+c$
$=\frac{\left(\tan^{-1}x^3\right)^2}{6}+c$
where c is the integrating constant.

10.

(c) B
Explanation: AB = A ...(i)
BA = B ...(ii)
From equation (ii)
B $\times$ (AB) = B
B^2A= B
From equation (ii)
B^2A = BA
B^2 = B.
Which is the required solution.

11.

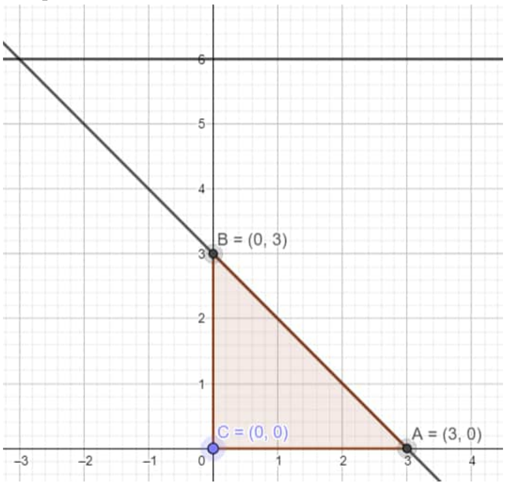
(b) bounded in first quadrant

Explanation: Converting the given inequations into equations, we obtain
y = 6, x + y = 3, x = 0 and y = 0, y = 6 is the line passing through (0, 6) and parallel to the X axis.The region below the line y = 6 will satisfy the given inequation.

The line x + y = 3 meets the coordinate axis at A(3, 0) and B(0, 3). Join these points to obtain the line x + y = 3 Clearly, (0, 0) satisfies the inequation x + y $\leq$3 . So, the region in x y -plane that contains the origin represents the solution set of the given equation.

The region represented by x $\geq$ 0 and y $\geq$ 0 :

Since every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations.



12.

(d) $-3\hat{i}+2\hat{j}$

Explanation: Given that, $\alpha=\hat{k}$

and $\gamma=2\hat{i}+3\hat{j}+4\hat{k}$

Since, β is perpendicular to both α and γ.

i.e., $\beta=\pm(\alpha\times\gamma)=\pm\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 0 & 1\\ 2 & 3 & 4\end{vmatrix}$

= $\pm\hat{i}(0-3)-\hat{j}(0-2)+\hat{k}(0-0)$

= $\pm(-3\hat{i}+2\hat{j})$

13.

(d) 3

Explanation: The given system of equations does not have solution if $\begin{vmatrix}1 & 1 & 1\\ 4 & \lambda & -\lambda\\ 3 & 2 & -4\end{vmatrix}=0\Rightarrow\begin{vmatrix}0 & 0 & 1\\ 4+\lambda & 2\lambda & -\lambda\\ 7 & 6 & 4\end{vmatrix}=0$

$\Rightarrow(24+6\lambda-14\lambda)=0\Rightarrow\lambda=3$

14.

(b) $\frac{1}{2}$

Explanation: Let A be the event that Mr. A hit the target and B be the event that Mr. B hit the target

$\therefore$ P(A) = $\frac{1}{2}$ and P(B) = $\frac{1}{3}$

Now, P (exactly one o f them hits the target)

= P(A $\cap\bar{B}$ or $\bar{A}\cap$ B)

= P(A $\cap\bar{B}$) + P($\bar{A}\cap$ B)

= P(A) $\cdot$ P$(\bar{B})$ + P$(\bar{A})$ $\cdot$ P(B)

= $\frac{1}{2}\cdot\frac{2}{3}+\frac{1}{2}\cdot\frac{1}{3}=\frac{3}{6}=\frac{1}{2}$

15.

(c) y = Cx

Explanation: It is given that $\frac{\mathrm{ydx-xdy}}{\mathrm{y}}=0$

$\Rightarrow\frac{\mathrm{ydx-xdy}}{\mathrm{xy}}=0$

$\Rightarrow\frac{1}{x}dx-\frac{1}{y}dy=0$

Integrating both sides, we get,

log|x| - log|y| = log k

$\Rightarrow \log\left|\frac{x}{y}\right| = \log k$

$\Rightarrow \frac{x}{y} = k$

$\Rightarrow$ y $= \frac{1}{\mathrm{k}}$x

$\Rightarrow$ y = Cx where C = $\frac{1}{k}$

16.

(c) $\pm\frac{\hat{\mathrm{i}}+\hat{\mathrm{j}}}{\sqrt{2}}$

Explanation: Since, unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ = $\pm\frac{a\times b}{|a\times b|}$

$\therefore \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$

$= \hat{i}[1+1] - \hat{j}[-1-1] + q\hat{k}[1-1]$

$= 2\hat{i} + 2\hat{j} + 0\hat{k} = 2(\hat{i}+\hat{j})$

and $|\vec{a}\times\vec{b}| = \sqrt{4+4} = 2\sqrt{2}$

$\therefore$ Required unit vector

$= \pm\frac{2(\hat{\mathbf{i}}+\hat{\mathbf{j}})}{2\sqrt{2}} = \pm\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}}$

17.

(d) 2

Explanation: Since the given function is continuous,

$\therefore$ k = $\lim\limits_{x\to 0}\frac{Sinx}{x} + Cosx$

$\Rightarrow$ k = 1 + 1 = 2

18.

(c) $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(2i + 3j - 2k)$

Explanation: Fixed point is $2\hat{\imath} - \hat{\jmath} + 3\hat{\mathrm{k}}$ and the vector is $2\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$

Equation $(2\hat{\imath} - \hat{\jmath} + 3\hat{k}) + \lambda(2\hat{\imath} + 3\hat{\jmath} - 2\hat{k})$

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

20.

(d) A is false but R is true.

Explanation: The assertion is false because relation R is not symmetric, (1, 2) $\in$ R but (2, 1) $\notin$ R

The reason is true because for a relationship to be equivalence it must be reflexive, symmetric, and transitive.

Section B

21. $\sin^{-1}(\sin(-600^\circ)) = \sin^{-1}\left\{\sin\left(-600\times\frac{\pi}{180}\right)\right\}$

$= \sin^{-1}\left\{\sin\left(-\frac{10\pi}{3}\right)\right\} = \sin^{-1}\left(-\sin\frac{10\pi}{3}\right)$

$= \sin^{-1}\left\{-\sin\left(3\pi+\frac{\pi}{3}\right)\right\} = \sin^{-1}\left\{-\left(-\sin\frac{\pi}{3}\right)\right\}$

$= \sin^{-1}\left(\sin\frac{\pi}{3}\right) = \frac{\pi}{3}$

OR

We know that $\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$.

$\therefore \quad \tan^{-1}\left\{2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right\}$

$= \tan^{-1}\left\{2\cos\left(2\times\frac{\pi}{6}\right)\right\}$

$= \tan^{-1}\left(2\cos\frac{\pi}{3}\right) = \tan^{-1}\left(2\times\frac{1}{2}\right) = \tan^{-1}1 = \frac{\pi}{4}$

22. Given: f(x) = sin 3x

f'(x) = 3 cos 3x

Now, $0 < x < \pi/2 \Rightarrow 0 < 3x < 3\pi/2$

Since cosine function is positive in first quadrant and negative in the second and third quadrants. Therefore, we consider the following cases.

Case1: When $0 < 3x < \pi/2$ i.e. $0 < x < \pi/6$

In this case, we have

$0 < 3x < \pi/2 \Rightarrow \cos 3x > 0 \Rightarrow 3\cos 3x > 0 \Rightarrow f'(x) > 0$

$f'(x) > 0,$ for $0 < 3x < \pi/2$ i.e. $0 < x < \pi/6$

So, f(x) is increasing function on $(0, \pi/6)$

Case 2: When $\frac{\pi}{2} < 3x < \frac{3\pi}{2}$ i.e. $\frac{\pi}{6} < x < \frac{\pi}{2}$

in this case, we have,

$\pi/2 < 3x < 3\pi/2 \Rightarrow \cos 3x < 0 \Rightarrow 3\cos 3x < 0 \Rightarrow f'(x) < 0$

Thus, $f'(x) < 0$ for $\pi/2 < 3x < 3\pi/2$ i.e. $\pi/6 < x < \pi/2$

Hence, f(x) is decreasing on $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$

23. Given interval: $x \in (\pi/2, \pi)$

$\Rightarrow \quad \pi/2 < x < \pi$

$x^{99} > 1$

$100x^{99} > 100$

Again, $x \in (\pi/2, \pi) \Rightarrow -1 < \cos x < 0 \Rightarrow 0 > \cos x > -1$

$100x^{99} > 100 \text{ and } \cos x > -1$

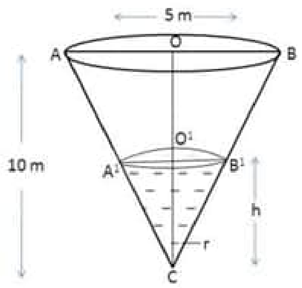
$100x^{99} + \cos x > 100 - 1 = 99$

$100x^{99} + \cos x > 0$

$f'(x) > 0$

Thus f(x) is increasing on $(\pi/2, \pi)$

OR



Let α be the semi vertical angle of the cone whose height CO = 10 m and radius OB = 5 m.

Now, $\tan\alpha = \frac{OB}{CO} = \frac{5}{10}$

$\tan\alpha = \frac{1}{2}$

Let V be the volume of water in the cone, then

$v = \frac{1}{3}\pi(0'B')^2\,(CO')$

$v = \frac{1}{3}\pi h^3 \tan^2\alpha$

$v = \frac{\pi}{12}h^3$

$\frac{dv}{dt} = \frac{3\pi}{12}h^2\frac{dh}{dt}$

$\pi = \frac{\pi}{4}h^2\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{4}{h^2}$

$\left(\frac{dh}{dt}\right)_{h\,=\,2.5} = \frac{4}{(2.5)^2}$

$= \frac{4}{6.25}$

= 0.64m/min.

So, the water level is rising at the rate of 0.64 m/min.

24. Let I = $\int \frac{1}{\sin x\cos^2 x}dx$, then we have

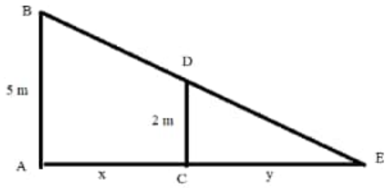
$I = \int \frac{\sin^2 x + \cos^2 x}{\sin x \cos^2 x}dx$

$= \int \frac{\sin^2 x}{\sin x\cos^2 x}dx + \int \frac{\cos^2 x}{\sin x\cos^2 x}dx$

= $\int$ sec x tan x dx + $\int$ cosec x dx

= sec x + log $\left|\tan\frac{x}{2}\right| + c$

$\therefore$ I = sec x + log $\left|\tan\frac{x}{2}\right| + c$

25. 

Let AB be the lamp-post. Let at any time t, the man CD be at a distance x metres from the lamp-post and let the length of his shadow be y metres. Then,

$\frac{dx}{dt} = 6 \text{ metres/minute}$

Clearly, triangles ABE and CDE are similar.

$\therefore \quad \frac{AB}{CD} = \frac{AE}{CE}$

$\Rightarrow \quad \frac{5}{2} = \frac{x+y}{y}$

$\Rightarrow \quad 5y = 2x + 2y$

$\Rightarrow \quad 3y = 2x$

$\Rightarrow \quad 3\frac{dy}{dt} = 2\frac{dx}{dt}$

$\Rightarrow \quad 3\frac{dy}{dt} = 2(6)$

$\Rightarrow \quad \frac{dy}{dt} = 4$ Thus, the shadow increases at a rate of 4 metres/minute.

Section C

26. Let $I = \int_0^{\pi/4} \frac{\sin 2x}{\cos^4 x + \sin^4 x} dx$. Then, $I = \int_0^{\pi/4} \frac{2 \sin x \cos x}{\cos^4 x + \sin^4 x} dx$

Dividing numerator and denominator by $\cos^4 x$,we get

$I = \int_0^{\pi/4} \frac{2 \tan x \sec^2 x}{1 + \tan^4 x} dx$

Let $\tan^2 x = t$.

Then, $d\left(\tan^2 x\right) = dt$

$\Rightarrow 2 \tan x \sec^2 x dx = dt$

Also, $x = 0 \Rightarrow t = \tan^2 0 = 0$ and $x = \frac{\pi}{4} \Rightarrow t = \tan^2 \frac{\pi}{4} = 1$

[Substituting t = $\tan^2$ x and 2 tan x $\sec^2$ x dx = dt, we get]

$I = \int_0^1 \frac{1}{1+t^2} dt$

$= \left[\tan^{-1} t\right]_0^1$

$= \left(\frac{\pi}{4} - 0\right)$

$= \frac{\pi}{4}$

Hence the result.

27. Let us define the following events

A = selecting person A

B = selecting person B

C = selecting person C

P(A) = $\frac{1}{1+2+4}$, P(B) = $\frac{2}{1+2+4}$

and P(C) = $\frac{4}{1+2+4}$

P(A) = $\frac{1}{7}$, P(B) = $\frac{2}{7}$

and P(C) = $\frac{4}{7}$

Let E = Event to introduce the changes in their profit.

Also given $P\left(\frac{E}{A}\right)$ = 0.8, $P\left(\frac{E}{B}\right)$ = 0.5 and $P\left(\frac{E}{C}\right)$ = 0.3

$\Rightarrow P\left(\frac{\bar{E}}{A}\right)$ = 1 - 0.8 = 0.2, $P\left(\frac{\bar{E}}{B}\right)$ = 1 - 0.5 = 0.5

and $P\left(\frac{\bar{E}}{C}\right)$ = 1 - 0.3 = 0.7

The probability that change does not take place by the appointment of C,

$$P\left(\frac{C}{E}\right) = \frac{P(C) \cdot P\left(\frac{\bar{E}}{C}\right)}{P(A) \times P\left(\frac{\bar{E}}{A}\right) + P(B) \times P\left(\frac{\bar{E}}{B}\right) + P(C) \times P\left(\frac{\bar{E}}{C}\right)}$$

$= \frac{\frac{4}{7}\times 0.7}{\frac{1}{7}\times 0.2+\frac{2}{7}\times 0.5+\frac{4}{7}\times 0.7}$

$= \frac{2.8}{0.2+1.0+2.8} = \frac{2.8}{4}$ = 0.7

28. $I = \int_{-3}^{3} \frac{x^4}{1+e^x} dx$

Using $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$I = \int_{-3}^{3} \frac{(-x)^4}{(1+e^{-x})} dx$

$I = \int_{-3}^{3} \frac{e^x.(x)^4}{(1+e^x)} dx = \int_{-3}^{3} \frac{(e^x+1-1)x^4}{e^x+1} dx$

$I = \int_{-3}^{3} \frac{(e^x+1).(x)^4}{(1+e^x)} dx - \int_{-3}^{3} \frac{x^4}{e^x+1} dx$

$I = \int_{-3}^{3} x^4 dx - I$

$2I = \left[\frac{x^5}{5}\right]_{-3}^{3}$

$2I = \frac{1}{5}\left[(3)^5 - (-3)^5\right]$

$I = \frac{243}{5}$

OR

We know that $\log_e$ x < 0 for x ∈ (0,1) and $\log_e$ x, $x \geq 0$ for $x \geq 1$

Therefore Given function can be made as,

$|log_e x| = \begin{cases} -\log_e x & \text{if } 1/e < x < 1 \\ \log_e x & \text{if } 1 < x < e \end{cases}$

Let $I = \int_{1/e}^{e} |\log_e x|\, dx$. Then

$I = \int_{1/e}^{1} |\log_e x|\, dx + \int_1^e |\log_e x|\, dx$ [Using additive property]

$\Rightarrow \quad I = \int_{1/e}^{1} -\log_e x dx + \int_1^e \log_e x dx$

$\Rightarrow \quad I = -\int_{1/e}^{1} \log_e x dx + \int_1^e \log_e x dx$

$\Rightarrow \quad I = -[x(\log_e x - 1)]_{1/e}^{1} + [x(\log_e x - 1)]_1^e$ $[\because \int \log_e x dx = x(\log_e x - 1)]$

$\Rightarrow \quad I = -\left[1(0-1) - \frac{1}{e}(-1-1)\right] + [e(1-1) - 1(0-1)]$

$\Rightarrow I = -\left[-1 + \frac{2}{e}\right] + (0+1) = 2 - \frac{2}{e}$

29. The given differential equation is,

$xy\frac{dy}{dx}$ = x^2 - y^2

$\Rightarrow \frac{dy}{dx} = \frac{x^2-y^2}{xy}$

This is a homogeneous differential equation

Putting y = vx and $\frac{dy}{dx}$ = v + x$\frac{dv}{dx}$, we have,

$v + x\frac{dv}{dx} = \frac{x^2-v^2x^2}{vx^2}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{1-v^2}{v}$

$\Rightarrow x\frac{dv}{dx} = \frac{1-v^2}{v} - v$

$\Rightarrow x\frac{dv}{dx} = \frac{1-2v^2}{v}$

$\Rightarrow \frac{v}{1-2v^2}dv = \frac{1}{x}dx$

Integrating both sides, we have,

$\int \frac{v}{1-2v^2}dv = \int \frac{1}{x}dx$

$\Rightarrow \frac{-1}{4}$ log |1 - 2v^2| = log |x| + log C

$\Rightarrow$ log |1 - 2v^2| = -4 log |x| -4 log C

$\Rightarrow$ log |(1 - 2v^2) (x^4)| = log $\frac{1}{C^4}$

Putting v = $\frac{y}{x}$, we have,

$\Rightarrow$ log |x^2 (x^2 - 2y^2)| = log $\frac{1}{C^4}$

$\Rightarrow x^2 (x^2 - 2y^2) = C_1$

Where, $C_1 = \frac{1}{C^4}$

Hence, $x^2 (x^2 - 2y^2) = C_1$ is the required solution.

OR

The given differential equation is,

$x\frac{dy}{dx}$ - y = log x

$\frac{dy}{dx} - \frac{y}{x} = \frac{\log x}{x}$

It is a linear differential equation. Comparing it with,

$\frac{dy}{dx}$ + Py = Q

P = $-\frac{1}{x}$, Q = $\frac{\log x}{x}$

I.F. $= e^{\int pdx}$

$= e^{-\int \frac{1}{x}dx}$

$= e^{-[\log|x|]}$, x > 0

$= e^{\log\left(\frac{1}{x}\right)}$, x > 0

$= \frac{1}{x}$, x > 0

Solution of the equation is given by,

y × (I.F.) = $\int$ Q × (I.F.) dx + c

y × (I.F.) = $\int \frac{\log x}{x}\left(\frac{1}{x}\right)$ dx + c

$y\left(\frac{1}{x}\right) = \int \frac{\log x}{x^2}$ dx + c

$= \log x \times \left(-\frac{1}{x}\right) - \int\left(\frac{1}{x} \times \left(-\frac{1}{x}\right)\right)dx + c$

$y\left(\frac{1}{x}\right) = -\frac{1}{x}\log x + \left(-\frac{1}{x}\right) + c$

y = - log x - 1 + cx ...(i)

Put y = 0, x = 1

0 = -log 1 - 1 + c

0 = 0 - 1 + c

c = 0

Put c = 1 in equation (i),

y = -log x - 1 + x

y = x - 1 - log x

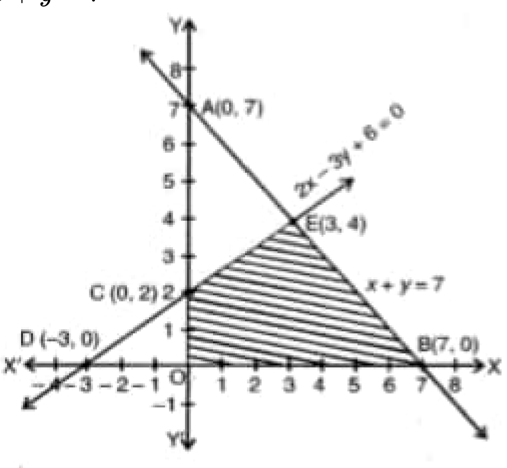
30. Consider$x + y = 7$

When $x = 0,\ then\ y = 7$ and

when $y = 0,\ then\ x = 7$

So, A(0, 7) and B(7, 0) are the points on line

$x + y = 7$



Consider $2x - 3y + 6 = 0$

When x = 0, then y = 2 and when y = 0, then x = - 3, So C(0, 2) and D(-3, 0) are the points on line $2x - 3y + 6 = 0$

Also, we have x > 0 and v > 0.

The feasible region OBEC is bounded, so, minimum value will obtain at a comer point of this feasible region.

Corner points are O(0, 0), B(7, 0), E(3, 4) and C(0, 2)

$Z = 13x - 15y$
At $O(0,0), Z = 0$
At $B(7,0), Z = 13(7) - 15(0) = 91$
At $E(3,4), Z = 13(3) - 15(4) = -21$
At $C(0,2), Z = 13(0) - 15(2)$
= -30 (minimum)
Hence , the minimum value is -30 at the point (0, 2).

OR

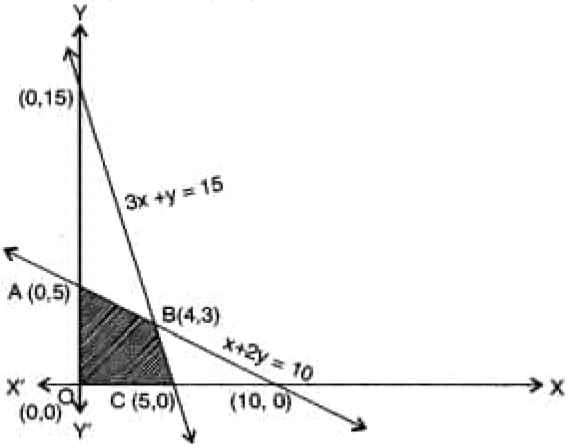
Linear constraints
$x + 2y \leq 10$
$3x + y \leq 15$
$x, y \geq 0$
and objective function is max $(Z) = 3x + 2y$.
Reducing the above inequations into equations and finding their point of intersections, i.e.,
$x + 2y = 10$ ... (i)
$3x + y = 15$ ... (ii)
$x = 0, y = 0$ ... (iii)

Equations	Point of intersection
(i) and (ii)	$x = 4\ and\ y = 3 \Rightarrow (4,3)$
(i) and (iii)	at $x = 0 \Rightarrow y = 5 \Rightarrow (0,5)$
	at y = 0 ⇒ x = 10 ⇒ (10, 0)
(ii) and (iii)	at x = 0 ⇒ y = 15 ⇒ (0, 15)
	at y = 0 ⇒ x = 5 ⇒ (5, 0)

Now for feasible region, using origin testing method for each constraint
$x + 2y \leq 10$, let x = 0, y = 0
$\Rightarrow 0 \leq 10$ i.e., true
$\Rightarrow$ The shaded region will be toward the origin.
Non negative restrictions $x \geq 0, y \geq 0$ indicates that the feasible region will exist in first quadrant.
Now, corner points are $A(0,5), B(4,3) C(5,0)\ and\ D(0,0)$.
For optimal solution substituting the value of all corner points in $Z = 3x + 2y$,

Corner points	Z
$A(0,5)$	10
$B(4,3)$	18 maximum
$C(5,0)$	15
$D(0,0)$	0

Hence, the maximum value of Z exist when x = 4 and y = 3
$\Rightarrow$ max (Z) = 18 at (4, 3)



31. Here, $y=\log(\sqrt{x-1}-\sqrt{x+1})$

Differentiate it with respect to x we get,

$\frac{dy}{dx}=\frac{d}{dx}\log(\sqrt{x-1}-\sqrt{x+1})$

$=\frac{1}{(\sqrt{x-1}-\sqrt{x+1})}\frac{d}{dx}(\sqrt{x-1}-\sqrt{x+1})$ [Using chain rule]

$=\frac{1}{(\sqrt{x-1}-\sqrt{x+1})}\left[\frac{d}{dx}\sqrt{x-1}-\frac{d}{dx}\sqrt{x+1}\right]$

$=\frac{1}{(\sqrt{x-1}-\sqrt{x+1})}\left[\frac{1}{2}(x-1)^{\frac{-1}{2}}-\frac{1}{2}(x+1)^{\frac{-1}{2}}\right]$

$=\frac{1}{2}\frac{1}{(\sqrt{x-1}-\sqrt{x+1})}\left(\frac{1}{\sqrt{x-1}}-\frac{1}{\sqrt{x+1}}\right)$

$=\frac{1}{2}\frac{1}{(\sqrt{x-1}-\sqrt{x+1})}\left\{\frac{-(\sqrt{x-1}-\sqrt{x+1})}{(\sqrt{x-1})(\sqrt{x+1})}\right\}$

$=\frac{-1}{2}\left(\frac{1}{(\sqrt{x-1})(\sqrt{x+1})}\right)$

$=\frac{-1}{2\sqrt{x^2-1}}$

So, $\frac{dy}{dx}=\frac{-1}{2\sqrt{x^2-1}}$

Section D

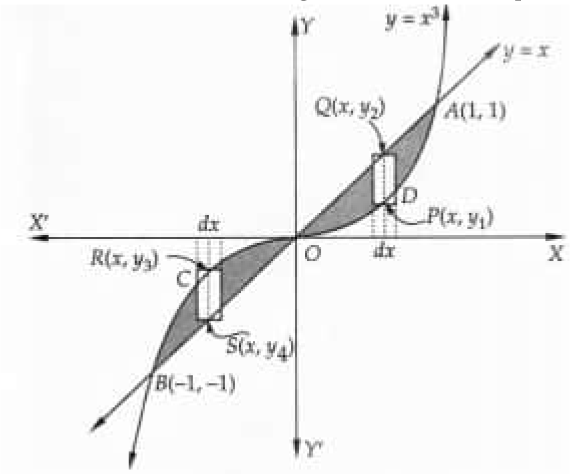
32. The given curves are ,

y = x ...(i)

and y = x^3 ...(ii)

The sketch of the curve y = x^3 is shown in Fig. Clearly, y =x is a line passing through the origin and making an angle of 45° with x-axis. The shaded portion shown in Fig. is the region bounded by the curves y = x and y = x^3. Solving y = x and y = x^3 simultaneously, we find that the two curves intersect at O (0, 0), A (1,1) and B (- 1, - 1).

When we slice the shaded region into vertical strips, we observe that the vertical strips change their character at O.



Therefore, the required area is given by,

Required area = Area BCOB + Area ODAO

Area BCOB: Each vertical strip in this region has its lower end on y = x and the upper end on y = x^3. Therefore, the approximating rectangle shown in this region has length $=|y_4-y_3|$, width = dx and area $=|y_4-y_3|$ dx. Since the approximating rectangle can move from x = - 1 to x = 0.

$\therefore$ Area BCOB $=\int_0^0|y_4-y_3|\,dx=\int_{-1}^0-(y_4-y_3)\,dx$ [$\because$ $y_4 < y_3$ $\therefore$ $y_4 - y_3 < 0$]

$=\int_{-1}^0-\left(x-x^3\right)dx$ [$\because$ R (x, y_3) and S (x, y_4) lie on (ii) and (i) respectively $\therefore$ $y_3 = x_3$ and $y_4 = x$]

$=\int_{-1}^0\left(x^3-x\right)dx=\left[\frac{x^4}{4}-\frac{x^2}{2}\right]_{-1}^0=0-\left(\frac{1}{4}-\frac{1}{2}\right)=\frac{1}{4}$ sq. units

Area ODAO: Each vertical strip in this region has its two ends on (ii) and (i) respectively. So, the approximating rectangle shown in this region has length $=|y_2-y_1|$, width = dx and therefore,we have,

Area ODAO $=\int_0^1|y_2-y_1|\,dx=\int_0^1(y_2-y_1)\,dx$ [$\because$ $y_2 > y_1$ $\therefore$ $y_2 - y_1 > 0$]

$=\int_0^1\left(x-x^3\right)dx$ [$\because$ P (x, y_1) and Q (x, y_2) lie on (ii) and (i) respectively $\therefore$ $y_1 = x^3$ and $y_2 = x$]

$=\left[\frac{x^2}{2}-\frac{x^4}{4}\right]_0^1=\frac{1}{2}-\frac{1}{4}=\frac{1}{4}$ sq. units

$\therefore$ Required area = Area BCOB + Area ODAO $=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$ sq. units

33. A = R - {3} and B = R - {1} and $f(x) = \frac{x-2}{x-3}$

Let $x_1, x_2 \in$ A, then $f(x_1) = \frac{x_1-2}{x_1-3}$ and $f(x_2) = \frac{x_2-2}{x_2-3}$

Now, for $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$

$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow -3x_1 - 2x_2 = -2x_1 - 3x_2$

$\Rightarrow x_1 = x_2 \therefore$ f is one-one function.

Now, $y = \frac{x-2}{x-3}$

$\Rightarrow y(x - 3) = x - 2$

$\Rightarrow xy - 3y = x - 2$

$\Rightarrow x(y - 1) = 3y - 2$

$\Rightarrow x = \frac{3y-2}{y-1}$

$\therefore f\left(\frac{3y-2}{y-1}\right) = \frac{\frac{3y-2}{y-1}-2}{\frac{3y-2}{y-1}-3} = \frac{3y-2-2y+2}{3y-2-3y+3} = y$

$\Rightarrow f(x) = y$

Therefore, f is an onto function.

OR

R = {(a, b): a - b is divisible by n} on Z.

Now,

Reflexivity: Let a $\in$ Z

$\Rightarrow$ a - a = 0 $\times$ n

$\Rightarrow$ a - a is divisible by n

$\Rightarrow$ (a, a) $\in$ R

$\Rightarrow$ R is reflexive

Symmetric: Let (a, b) $\in$ R

$\Rightarrow$ a - b = np for some p $\in$ Z

$\Rightarrow$ b - a = n(-p)

$\Rightarrow$ b - a is divisible by n

$\Rightarrow$ (b, a) $\in$ R

$\Rightarrow$ R is symmetric

Transitive: Let (a, b) $\in$ R and (b, c) $\in$ R

$\Rightarrow$ a - b = np and b - c = nq for some p, q $\in$ Z

$\Rightarrow$ a - c = n(p + q)

$\Rightarrow$ a - c is divisible by n

$\Rightarrow$ (a, c) $\in$ R

$\Rightarrow$ R is transitive

Thus, R being reflexive, symmetric and transitive on Z.

Hence, R is an equivalence relation on Z

34. Given: $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$\therefore A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 4+1+1 & -2-2-1 & 2+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$

Now $A^3 = A^2.A = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 12+5+5 & -6-10-5 & 6+5+10 \\ -10-6-5 & 5+12+5 & -5-6-10 \\ 10+5+6 & -5-10-6 & 5+5+12 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

L.H.S. $= A^3 - 6A^2 + 9A - 4I$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6 \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9 \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix} + \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 22-36 & -21+30 & 21-30 \\ -21+30 & 22-36 & -21+30 \\ 21-30 & -21+30 & 22-36 \end{bmatrix} + \begin{bmatrix} 18-4 & -9-0 & 9-0 \\ -9-0 & 18-4 & -9-0 \\ 9-0 & -9-0 & 18-4 \end{bmatrix}$$

$$= \begin{bmatrix} -14 & 9 & -9 \\ 9 & -14 & 9 \\ -9 & 9 & 14 \end{bmatrix} + \begin{bmatrix} -14 & 9 & -9 \\ 9 & -14 & 9 \\ -9 & 9 & 14 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0 = \text{R.H.S.}$$

Now, to find A^{-1}, multiplying $A^3 - 6A^2 + 9A - 4I^{-1} = 0.A^{-1}$ by A^{-1}

$\Rightarrow A^3A^{-1} - 6A^2A^{-1} + 9AA^{-1} - 4I.\,A^{-1} = 0A^{-1}$

$\Rightarrow A^2 - 6A + 9I - 4A^{-1} = 0$

$\Rightarrow 4A^{-1} = A^2 - 6A + 9I$

$$\Rightarrow 4A^{-1} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6 \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow 4A^{-1} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - \begin{bmatrix} 12 & -6 & 6 \\ -6 & 12 & -6 \\ 6 & -6 & 12 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\Rightarrow 4A^{-1} = \begin{bmatrix} 6-12+9 & -5+6+0 & 5-6+0 \\ -5+6+0 & 6-12+9 & -5+6+0 \\ 5-6+0 & -5+6+0 & 6-12+9 \end{bmatrix} = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{4} \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

35. Given

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(-2\hat{i} + 3\hat{j} + 8\hat{k})$

Here, we have

$\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}$

$\overrightarrow{\mathrm{b}_1} = 2\hat{i} + 3\hat{j} + 6\hat{\mathrm{k}}$

$\overrightarrow{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$

$\overrightarrow{\mathrm{b}_2} = -2\hat{i} + 3\hat{j} + 8\hat{\mathrm{k}}$

Thus,

$$\overrightarrow{\mathrm{b}_1} \times \overrightarrow{\mathrm{b}_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ -2 & 3 & 8 \end{vmatrix}$$

$= \hat{i}(24 - 18) - \hat{j}(16+12) + \hat{k}(6 - 6)$

$\overrightarrow{\mathrm{b}_1} \times \overrightarrow{\mathrm{b}_2} = 6\hat{\mathrm{i}} - 28\hat{\mathrm{j}} + 0\hat{\mathrm{k}}$

$\therefore |\overrightarrow{\mathrm{b}_1} \times \overrightarrow{\mathrm{b}_2}| = \sqrt{6^2 + (-28)^2 + 0^2}$

$= \sqrt{36 + 784 + 9}$

$= \sqrt{820}$

$\overrightarrow{a_2} - \overrightarrow{a_1} = (3-1)\hat{i} + (3-2)\hat{j} + (-5+4)\hat{\mathrm{k}}$

$\therefore \overrightarrow{a_2} - \overrightarrow{a_1} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

Now, we have

$(\overrightarrow{\mathrm{b}_1} \times \overrightarrow{\mathrm{b}_2}) \cdot (\overrightarrow{\mathrm{a}_2} - \overrightarrow{\mathrm{a}_1}) = (6\hat{\imath} - 28\hat{\jmath} + 0\hat{\mathrm{k}}) \cdot (2\hat{\imath} + \hat{\jmath} - \hat{\mathrm{k}})$

$= (6 \times 2) + ((-28) \times 1) + (0 \times (-1))$

= 12 - 28 + 0

= -16

Thus,the shortest distance between the given lines is

$\mathrm{d} = \left| \frac{(\overrightarrow{\mathrm{b}_1} \times \overrightarrow{\mathrm{b}_2}) \cdot (\overrightarrow{\mathrm{a}_2} - \overrightarrow{\mathrm{a}_1})}{|\overrightarrow{\mathrm{b}_1} \times \overrightarrow{\mathrm{b}_2}|} \right|$

$\Rightarrow \mathrm{d} = \left| \frac{-16}{\sqrt{820}} \right|$

$\therefore d = \frac{16}{\sqrt{820}}$ units

OR

We have to find the vector and cartesian equations of a line passing through (1, 2,-4) and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

Now, we know that any line through (1, 2, -4) can be written as

$\frac{x-1}{a} = \frac{y-2}{b} = \frac{z+4}{c}$

where a,b,c are the direction ratios of line (i)

Now, the line (i) is perpendicular to the lines

$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$

and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

The direction ratios of the above lines are (3,-16,7) and (3,8,-5), respectively which are perpendicular to the Equation (i).

3a - 16b + 7c = 0 ...(ii)

and 3a + 8b - 5c = 0 ...(iii)

By cross-multiplication, we get

$\frac{a}{80-56} = \frac{b}{21+15} = \frac{c}{24+48}$

$\Rightarrow \frac{a}{24} = \frac{b}{36} = \frac{c}{72}$

$\Rightarrow \frac{a}{2} = \frac{b}{3} = \frac{c}{6} = \lambda(\text{ say })$

$\Rightarrow a = 2\lambda, b = 3\lambda, c = 6\lambda$

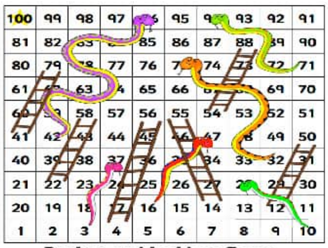
The equation of the required line in cartesian form is

$\frac{x-1}{2\lambda} = \frac{y-2}{3\lambda} = \frac{z+4}{6\lambda}$ or $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$ and in vector form is $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$.

Section E

36. **Read the text carefully and answer the questions:**

Akshat and his friend Aditya were playing the snake and ladder game. They had their own dice to play the game. Akshat was having red dice whereas Aditya had black dice. In the beginning, they were using their own dice to play the game. But then they decided to make it faster and started playing with two dice together.



Aditya rolled down both black and red die together.

First die is black and second is red.

(i) Let A represents obtaining a sum greater than 9 and B represents black die resulted in a 5.

n(S) = 36

n(A) = {(4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)} = 6

n(B) = {(5, 1), (5, 2), (5, 3), (5, 4), (5, 5)} = 6

$n(A \cap B) = \{(5, 5), (5, 6)\} = 2$

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{\frac{2}{36}}{\frac{6}{36}} = \frac{1}{3}$

(ii) Let A represents obtaining a sum 8 and B represents red die resulted in number less than 4.

$n(S) = 36$

$n(A) = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2) = 5$

$n(B) = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1), (6, 2), (6, 3)\} = 18$

$n(A \cap B) = \{(5, 3), (6, 2)\} = 2$

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{\frac{2}{36}}{\frac{18}{36}} = \frac{1}{9}$

(iii) Let A represents obtaining a sum 10 and B represents black die resulted in even number.

$n(S) = 36$

$n(A) = \{(4, 6), (6, 4), (5, 5)\} = 3$

$n(B) = \{(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\} = 18$

$n(A \cap B) = \{(4, 6), (6, 4)\} = 2$

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{\frac{2}{36}}{\frac{18}{36}} = \frac{1}{9}$

OR

Let A represents getting doublet and B represents red die resulted in number greater than 4.

$n(S) = 36$

$n(A) = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\} = 6$

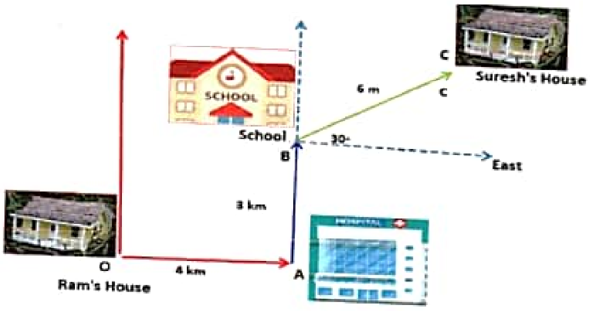
$n(B) = \{(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6), (5, 5), (5, 6), (6, 5), (6, 6)\} = 12$

$n(A \cap B) = \{(4, 4), (5, 5), (6, 6)\} = 3$

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{\frac{3}{36}}{\frac{12}{36}} = \frac{1}{4}$

37. **Read the text carefully and answer the questions:**

Ram's house is situated at Gandhi Nagar at Point O, for going to school he first travels by bus in the east. Here at Point A, a hospital is situated. From Hospital Ram takes an auto and goes 3 km in the north direction, here at point B school is situated. Suresh's house is at 30^{o} east, 6 km from point B.



(i) Displacement between Ram's house and school = $\sqrt{4^2 + 3^2}$

$= \sqrt{25}$

$= 5$ km

(ii) Distance travelled to reach school by Ram = 4 + 3

= 7 km

(iii) Position vector of school = $4\hat{i} + 3\hat{j}$

Position vector of Suresh's

$= (4 + 6 \cos 30)\hat{i} + (3 + 6 \sin 30)\hat{j}$

$= (4 + \frac{6\sqrt{3}}{2})\hat{i} + (3 + \frac{6 \times 1}{2})\hat{j}$

$= (4 + 3\sqrt{3})\hat{i} + 6\hat{j}$

Vector distance from school to Suresh's home

[(4 + $3\sqrt{3}$)$\hat{i}$ + 6$\hat{j}$] - [(4$\hat{i}$ + 3$\hat{j}$)]

$3\sqrt{3}\hat{i} + 3\hat{j}$

OR

Position vector of Ram's house = 0$\hat{i}$ + 0$\hat{j}$

Position vector of Suresh's house = (4 + $3\sqrt{3}$)$\hat{i}$ + 6$\hat{j}$

$\therefore$ Displacement from Ram's house to suresh's house

= (4 + $3\sqrt{3}$)$\hat{i}$ + 6$\hat{j}$ - (0$\hat{i}$ + 0$\hat{j}$)

$(4 + 3\sqrt{3})\hat{i} + 6\hat{j}$

38. **Read the text carefully and answer the questions:**

The Government declare that farmers can get ₹300 per quintal for their onions on 1st July and after that, the price will be dropped by ₹3 per quintal per extra day. Govind's father has 80 quintals of onions in the field on 1st July and he estimates that the crop is increasing at the rate of 1 quintal per day.

(i) Let x be the number of extra days after 1st July.

$\therefore$ Price = ₹(300 - 3 $\times$ x) = ₹(300 - 3x)

Quantity = 80 quintals + x(1 quintal per day) = (80 + x) quintals

Revenue = R(x) = Quantity $\times$ Price = (80 + x) (300 - 3x) = 24000 - 240x + 300x - $3x^2$

R(x) = 24000 + 60x - $3x^2$

(ii) We have, R(x) - 24000 + 60x - $3x^2$

$\Rightarrow$ R'(x) = 60 - 6x $\Rightarrow$ R''(x) = -6

For R(x) to be maximum, R'(x) = 0 and R''(x) < 0

$\Rightarrow$ 60 - 6x = 0 $\Rightarrow$ x = 10