Class X Session 2024-25
Subject - Mathematics (Standard)
Sample Question Paper - 11

Time: 3 Hours **Total Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A - E.
2. Section A has 18 multiple choice questions and 2 Assertion-Reason based questions carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take π = 22/7 wherever required if not stated.

Section A

Section A consists of 20 questions of 1 mark each.

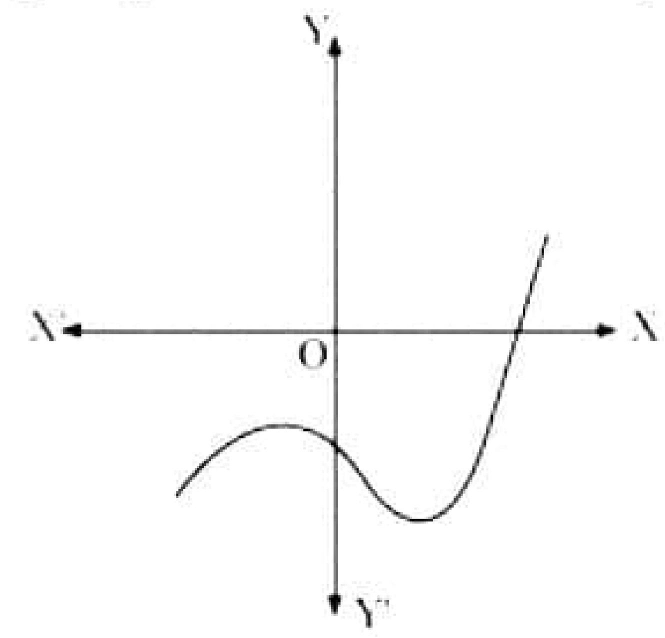
Choose the correct answers to the questions from the given options. **[20]**

1. If 'a' and 'b' are two prime numbers, what is their HCF?

   A. a
   B. b
   C. 1
   D. Ab

2. Find the value of k for which x = 1 is a root of the equation $x^2 + kx + 3 = 0$.
   A. 4
   B. 3
   C. -4
   D. -3

3. The graph of y = p(x) is given in the following figure for some polynomial p(x). Find the number of zeroes of p(x).

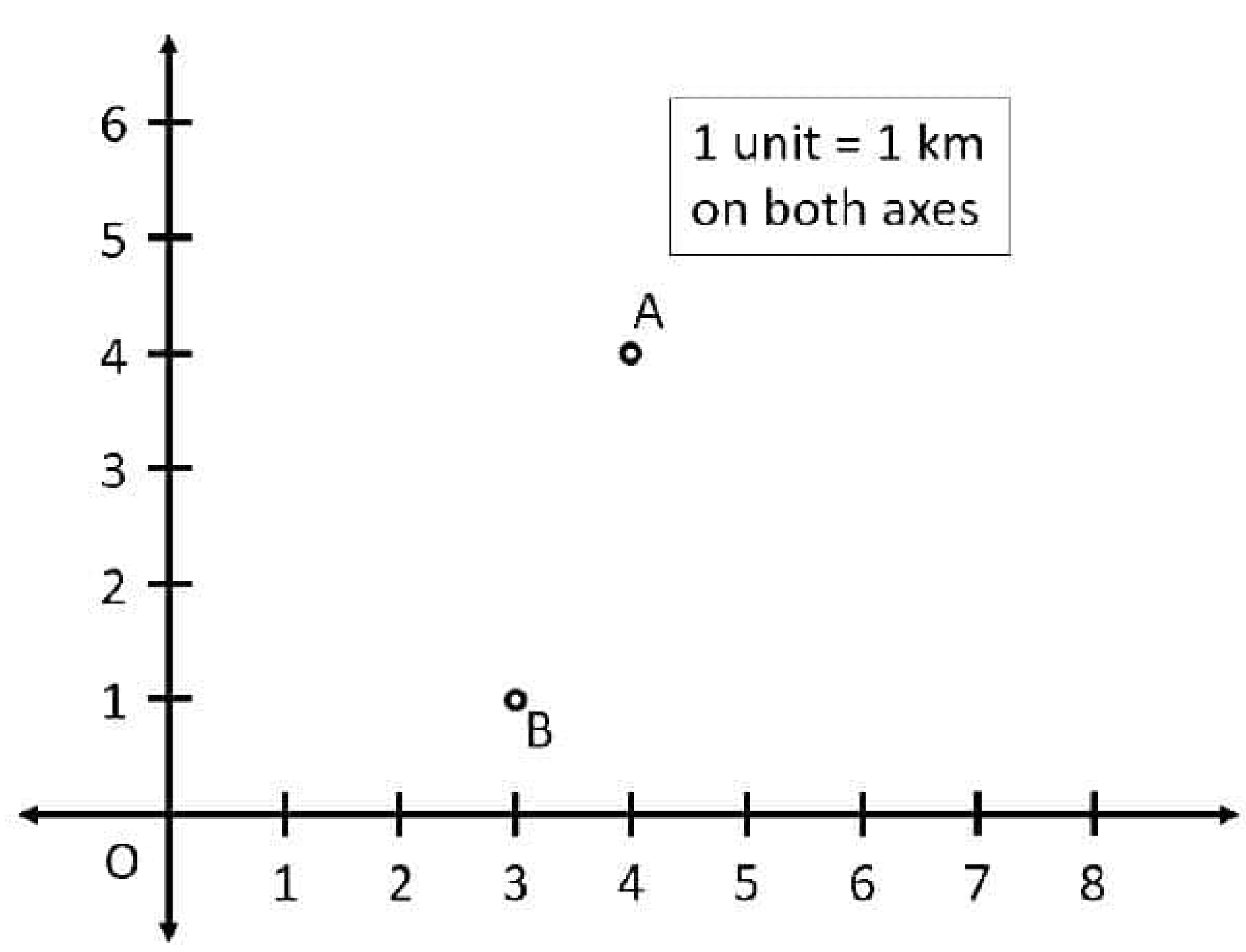


A. 2
B. 1
C. 0
D. 3

4. A two-digit number has the sum of digits 10. If the difference between digits is 2, then the number is
A. 91
B. 82
C. 73
D. 64

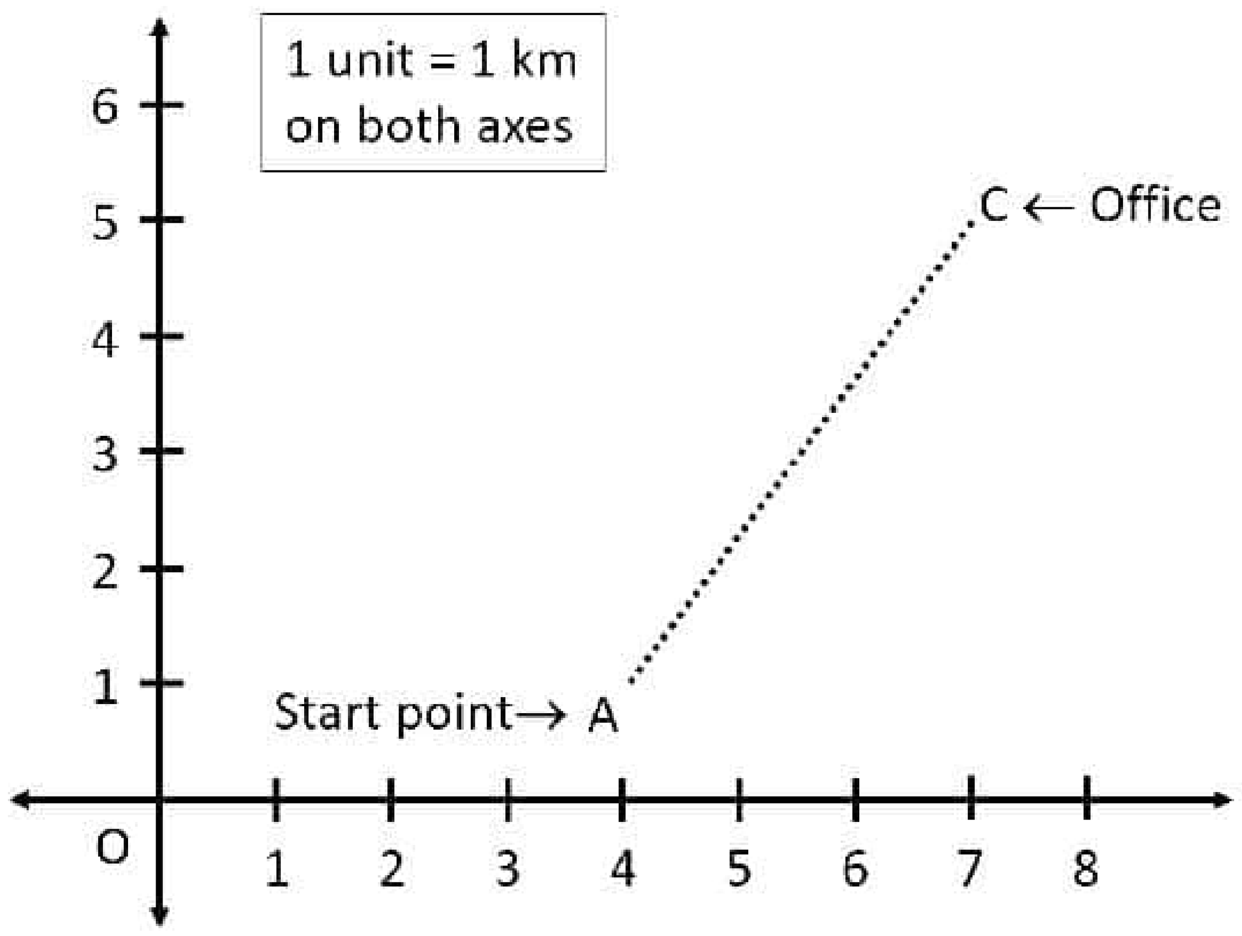
5. Find the value of a so that the point (3, a) lies on the line represented by 2x – 3y = 5.
A. -3
B. -1/3
C. 3
D. 1/3

6. The position of Atul (A) and Bipin (B) is shown by the graph below.



Find the distance between their positions.
A. $5\sqrt{2}$ km
B. $2\sqrt{5}$ km
C. $\sqrt{10}$ km
D. 10 km

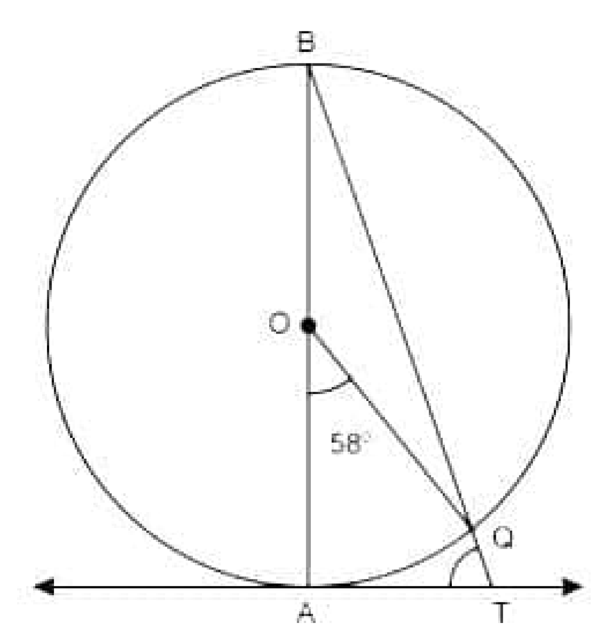
7. Amey travels to office daily (refer to the graph). Half way along the path lies the house of his friend Raju. Find the co-ordinates of Raju's house.



A. (4.5, 2)
B. (2, 4.5)
C. (3, 5.5)
D. (5.5, 3)

8. In the given figure, AB is a diameter of a circle with centre O and AT is a tangent. If ∠AOQ = 58°, find ∠ATQ.

A. 29°
B. 58°
C. 61°
D. 90°



9. ΔABC ~ ΔDEF, such that AB = 3 cm, BC = 2 cm, CA = 2.5 cm, EF = 4 cm. The perimeter of ΔABC is

A. 15 cm
B. 20 cm
C. 12 cm
D. 18 cm

10. All equilateral triangles are _______ to each other

A. equal
B. congruent
C. similar
D. None of above

11. Arjun was writing down the values of cot θ, for the different angles. He knew that cot θ was not defined for a particular angle, but couldn't remember it. So what is this particular angle?

A. 0°
B. 30°
C. 60°
D. 90°

12. 2 sin 2A = $\sqrt{3}$, then the value of A is

A. 30°
B. 45°
C. 60°
D. 90°

13. $\cos^4 \theta - \sin^4 \theta =$

A. $2\cos^2 \theta - 1$
B. $1 - 2\sin^2 \theta$
C. Both A and B
D. None of these

14. Find the area of a sector with radius 7 cm and central angle 90°

A. 38 cm^2
B. 39 cm^2
C. 38.5 cm^2
D. 37.5 cm2

15. Find the volume of a right circular cone of height 70 cm and radius 9 cm.

A. 5490 cm^3
B. 5940 cm^3
C. 5740 cm^3
D. 5690 cm^3

16. Find the mode

Marks obtained	Frequency
11	11
20	15
23	20
25	30
29	14
30	10

A. 20
B. 23
C. 25
D. 29

17. A letter is chosen at random from the letters P, R, O, G, R, E, S, I, O and N. Find the probability that the chosen letter is a vowel.

A. 5/2
B. 2/5
C. 1/5
D. 3/5

18. If a point (c, d) lies in the 3rd quadrant, which of the following is true?

A. c is positive and d is negative
B. both c and d are positive
C. both c and d are negative
D. c is negative and d is positive

DIRECTION: In the question number 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option

19. **Statement A (Assertion):** The length of a chain used as the boundary of a semi-circular park is 90 m. Hence the area of the park will be 481.25 m^2.

Statement R (Reason): If the radius of the circle is 'r', then, $\pi r + 2r = 90$.

A. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
B. Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
C. Assertion (A) is true but reason (R) is false.
D. Assertion (A) is false but reason (R) is true.

20. **Statement A (Assertion):** If (p − q), p and (p + q) are the zeros of the polynomial $x^3 - 12x^2 + 19x - 28$, then the value of p is −4.

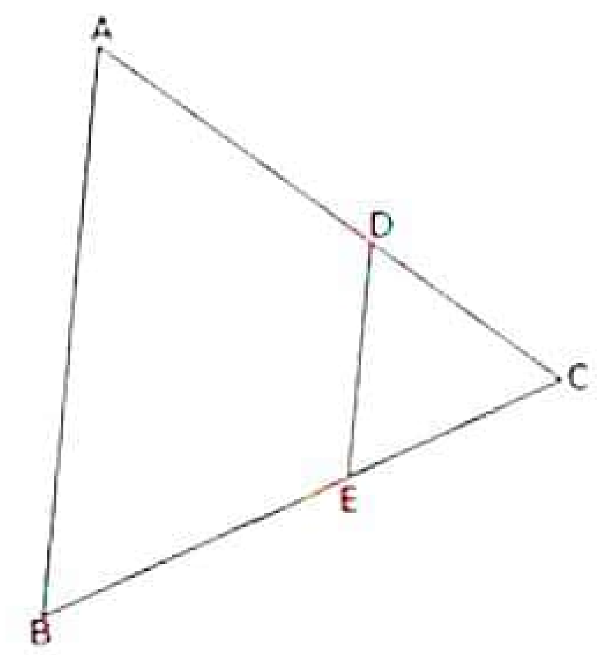
Statement R (Reason): Sum of the zeros $= -\dfrac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

A. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
B. Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).
C. Assertion (A) is true but reason (R) is false.
D. Assertion (A) is false but reason (R) is true.

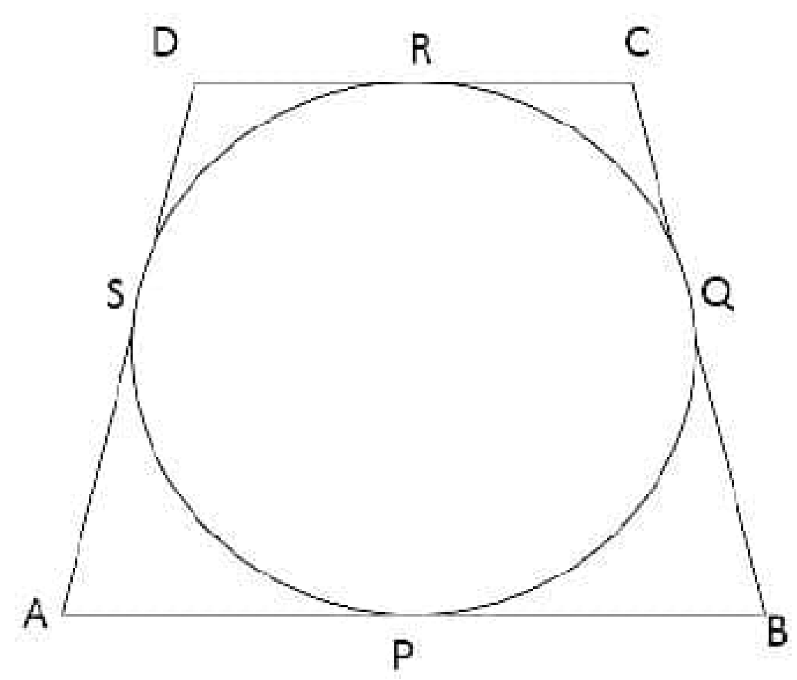
Section B

21. Mr. Shastri's cell phone PIN is dbac such that $42000 = a^4 \times b \times c^3 \times d$. Find the PIN. [2]

22. In figure, If $\angle A = \angle B$ and AD = BE show that DE || AB in ΔABC. [2]



23. In the given figure, a circle touches all the four sides of a quadrilateral ABCD whose three sides are AB = 6 cm, BC = 7 cm and CD = 4 cm. Find AD. [2]



24. Prove that: $(\sin\theta + \cos\theta)(\tan\theta + \cot\theta) = \sec\theta + \operatorname{cosec}\theta$ [2]

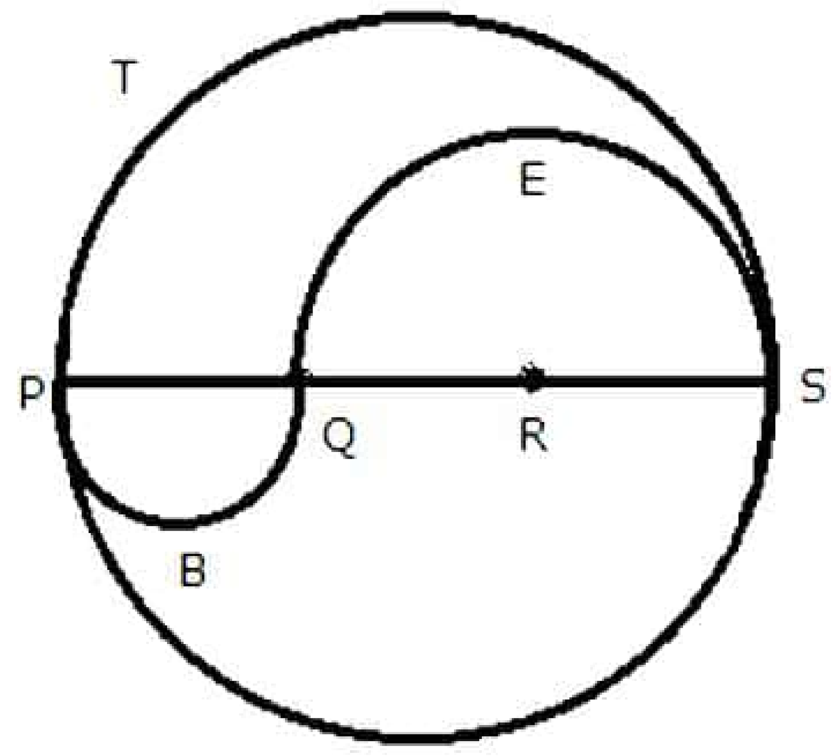
OR

In ΔPQR, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

25. Find the area of a sector of a circle with radius 6 cm, if angle of the sector is 60°. [2]

OR

PQRS is a diameter of a circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semicircles are drawn with PQ and QS as diameters, as shown in the given figure. If PS = 12 cm, find the perimeter and area of the shaded region. (Take π = 3.14)



Section C

Section C consists of 6 questions of 3 marks each.

26. Find the LCM and HCF of the following integers by applying the prime factorisation method. [3]

(i) 12, 15 and 21

(ii) 17, 23 and 29

27. John and Jayanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. Find out how many marbles they had. [3]

28. A fraction becomes $\frac{1}{3}$, if 2 is added to both its numerator and denominator. If 3 is added to both its numerator and denominator, then it becomes $\frac{2}{5}$. Find the fraction. [3]

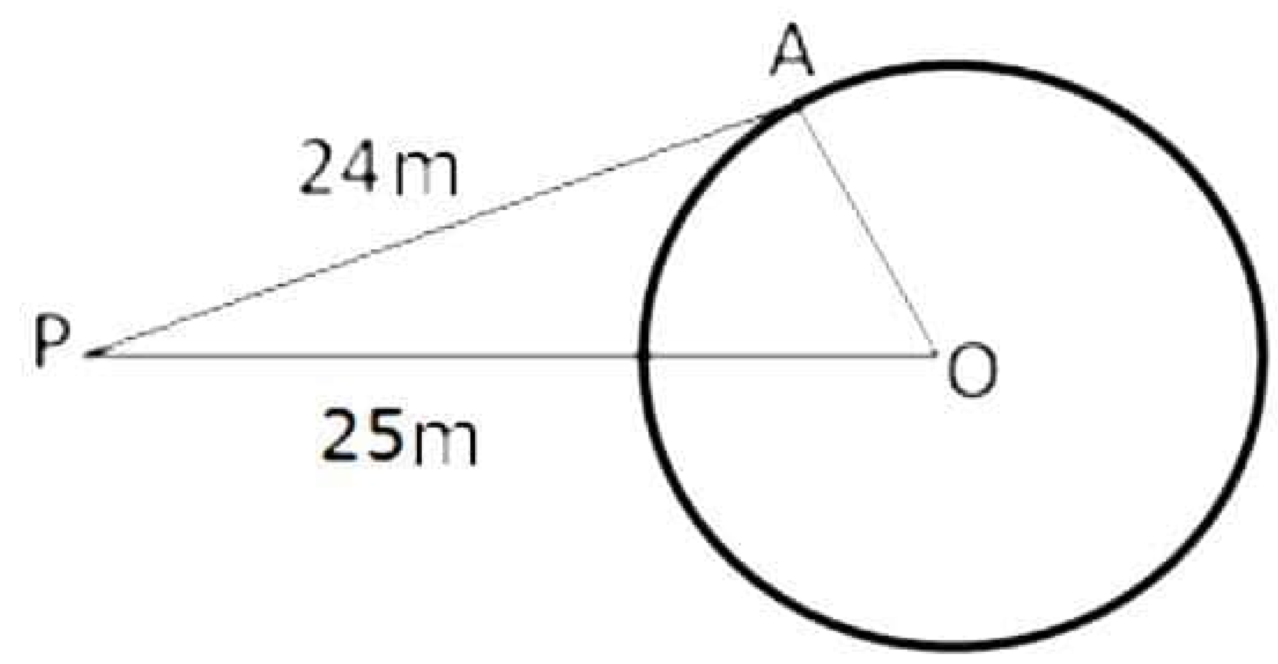
OR

Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

29. If two tangents inclined at an angle of 60° are drawn to a circle of radius 3 cm, then find the length of each tangent. [3]

OR

Arjun is standing at a point P, which is 25 m away from the centre (O) of a circular park, and the length of a road from the point P to the gate of the park (A) is 24 m.



Find the distance of the centre of park to the gate.

30. If x = cot A + cos A and y = cot A – cos A, then show that

$$\left(\frac{x-y}{x+y}\right)^2 + \left(\frac{x-y}{2}\right)^2 = 1$$ [3]

31. 2 Red kings, 2 red queens and 2 red jacks are removed from a deck of 52 playing cards and then well shuffled. A card is drawn from the remaining cards. Find the probability of getting (i) a king, (ii) a red card, (iii) a spade. [3]

Section D

Section D consists of 4 questions of 5 marks each.

32. A sailor can row a boat 8 km downstream and return to the start point in 1 hour 40 minutes. If the speed of the stream is 2 km/hr, then find the speed of the boat in still water. [5]

OR

Two pipes running together can fill a cistern in $3\frac{1}{13}$ minutes. If one pipe takes 3 minutes more than the other to fill it, then find the time in which each pipe would fill the cistern.

33. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{OC}{OD}$ [5]

34. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2. [5]

OR

A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs. 500 per m^2.

35. The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is Rs.18. Find the missing frequency. [5]

Daily pocket allowance (in Rs)	11–13	13–15	15–17	17–19	19–21	21–23	23–25
Number of workers	7	6	9	13	f	5	4

Section E

Case study based questions are compulsory.

36. Your elder brother wants to buy a car and plans to take loan from a bank for his car. He repays his total loan of Rs. 118000 by paying every month starting with the first instalment of Rs. 1000. If he increases the instalment by Rs. 100 every month, then answer the following questions.

i. Find the amount paid by him in 30^{th} instalment. [1]

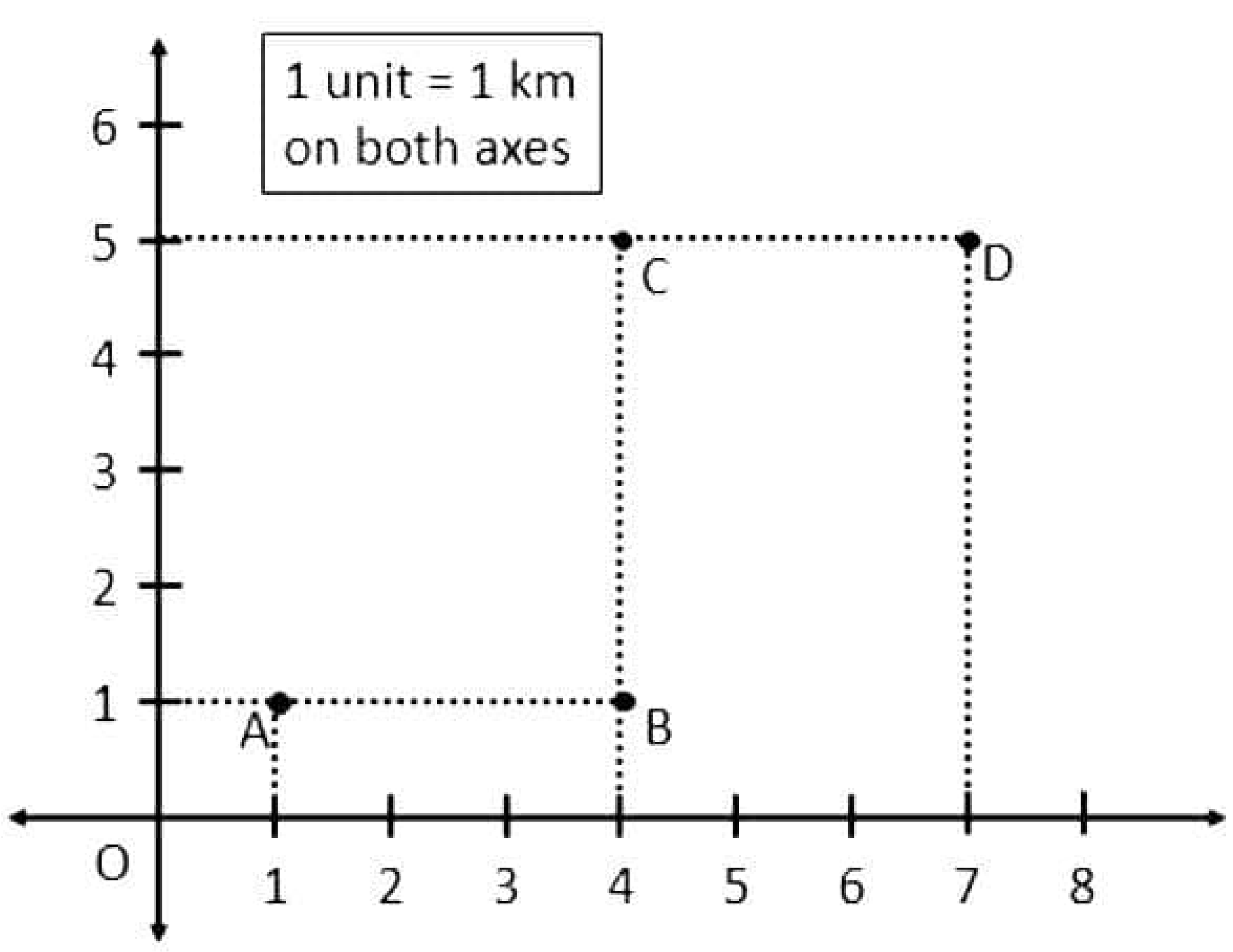
ii. Find the amount to be paid in the 40^{th} instalment. [1]

iii. Find the ratio of the 19^{th} instalment to the 28^{th} instalment. [2]

OR

Find the total amount paid by him in 30 instalments.

37. Amey runs a grocery store that offers home delivery of fresh groceries to its customers. His store is located at location A as indicated in the graph below. Now, he receives regular orders from the families living in the colonies located at B, C and D. Now, using the data given, answer the following questions.



i. Find the shortest distance between locations A and C. [2]

OR

Find the shortest distance between locations B and D.

ii. Find the shortest distance between locations B and A. [1]

iii. Find the shortest distance between locations C and B. [1]

38. There are two poles of equal height on either side of the road. Each pole has one hoarding on it. A car is standing on the road at point A. From A, the angle of elevation of the top of the poles are 60° and 30° respectively.

If height of each pole is 30 m, then answer the following questions.

i. Find the distance between the left pole and point A. [2]

OR

Find the distance between the right pole and point A.

ii. Find the width of the road. [1]

iii. Name the angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level. [1]

Solution

Section A

1. Correct option: C
   Explanation:
   'a' and 'b' are two prime numbers. Thus, the only factors of a are 1 and a and the only factors of b are 1 and b.
   Hence, the only common factor of a and b is 1.

2. Correct option: C
   Explanation:
   Since x = 1 is a solution of $x^2 + kx + 3 = 0$, it must satisfy the equation.
   $\therefore (1)^2 + k(1) + 3 = 0 \Rightarrow k = -4$
   Hence, the required value of k = −4.

3. Correct option: B
   Explanation:
   The graph of P(x) intersects the x-axis at only 1 point.
   So, the number of zeroes is 1.

4. Correct option: D
   Explanation:
   Let the digits be x and y.
   x + y = 10 ....(i)
   x − y = 2 ....(ii)
   Adding (i) and (ii), we get 2x = 12 ⇒ x = 6
   From (i), we get y = 10 − 6 = 4
   Two-digit number = 10x + y = 10(6) + 4 = 64

5. Correct option: D
   Explanation:
   The point (3, a) lies on the line 2x − 3y = 5.
   Substituting the values of x and y in the given equation:
   2 × 3 − 3 × a = 5
   ∴ 6 − 3a = 5
   $\therefore 3a = 1 \Rightarrow a = \frac{1}{3}$

6. Correct option: C
   Explanation:
   Using the graph, we get the coordinates of A and B as A(4,4) and B(3,1).
   Thus, $d(AB) = \sqrt{(4-3)^2 + (4-1)^2} = \sqrt{(1)^2 + (3)^2} = \sqrt{10}$ km
   Hence, the distance between their positions is $\sqrt{10}$ km.

7. Correct option: D
Explanation:
Using the graph, we get the coordinates of A and C as A(4,1) and C(7, 5).
Let B(x,y) be the co-ordinates of Raju's house, which is the mid-point of AC.
Hence, by using midpoint formula

$$B(x, y) = \left(\frac{4+7}{2}, \frac{1+5}{2}\right) = (5.5,\ 3)$$

Hence, the co-ordinates of Raju's house is (5.5, 3).

8. Correct option: C
Explanation:
$\angle AOQ = 58°$ (given)

$$\angle ABQ = \frac{1}{2}\angle AOQ = \frac{1}{2} \times 58° = 29°$$

In right-angled ΔBAT, $\angle ABT + \angle BAT + \angle ATB = 180°$
$29° + 90° + \angle ATB = 180°$
$\angle ATB = 61°$
That is, $\angle ATQ = 61°$

9. Correct option: A
Explanation:
$\Delta ABC \sim \Delta DEF$

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$$

$$\Rightarrow \frac{3}{DE} = \frac{2.5}{DF} = \frac{2}{4}$$

$\Rightarrow DE = 6\text{ cm},\ DF = 5\text{ cm}$

$\text{Perimeter} = 4 + 5 + 6 = 15\text{ cm}$

10. Correct option: C
Explanation:
All equilateral triangles are similar to each other.

11. Correct option: A
Explanation:
The value of cot θ is not defined for θ = 0°.
Hence the angle in question here is 0°.

12. Correct Option: A
Explanation:

$2\sin 2A = \sqrt{3}$

$\therefore \sin 2A = \frac{\sqrt{3}}{2}$

$\therefore \sin 2A = \sin 60°$

$\therefore 2A = 60°$

$\therefore A = 30°$

13. Correct Option: C
Explanation:
$\cos^4\theta - \sin^4\theta = (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$
$= \cos^2\theta - \sin^2\theta$
$= 1 - \sin^2\theta - \sin^2\theta$
$=1 - 2\sin^2\theta$
Also, $\cos^2\theta - \sin^2\theta = \cos^2\theta - (1 - \cos^2\theta) = 2\cos^2\theta - 1$

14. Correct Option: C
Explanation:
Area of sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi r^2 = \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (7)^2 = 38.5 \text{ cm}^2$

15. Correct option: B
Explanation:
For a cone, height, h = 70 cm and radius, r = 9 cm
Volume of the circular cone $= \frac{1}{3}(\pi \times 9^2 \times 70) = 5940 \text{ cm}^3$

16. Correct Option: C
Explanation:
The marks with the highest frequency will be the mode, hence mode is 25.

17. Correct Option: B
Explanation:
Total number of letters = 10
The vowels involved are E, I and O where O appears twice.
⇒ Number of vowels = 4
Therefore, probability of getting a vowel $= \frac{4}{10} = \frac{2}{5}$

18. Correct option: C
Explanation:
If a point lies in the 3rd quadrant, then its x-coordinate as well as its y-coordinate will be negative.

19. Correct Option: A
Explanation:
Let the radius of the park be r metres.
Thus, $\pi r + 2r = 90 \Rightarrow \frac{22r}{7} + 2r = 90$
Hence, the reason (R) is true.
$\Rightarrow \frac{36r}{7} = 90 \Rightarrow r = \frac{90 \times 7}{36}$
$\Rightarrow r = 17.5 \text{ m}$
Area of semicircular park $= \frac{1}{2}\pi r^2 = \left(\frac{1}{2} \times \frac{22}{7} \times 17.5 \times 17.5\right) \text{m}^2 = 481.25 \text{ m}^2$
Hence, both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

20. Correct Option: D

Explanation:

The equation given in reason is correct and hence, reason is true.

Given polynomial = $x^3 - 12x^2 + 19x - 28$

Sum of the zeros $= -\dfrac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

$(p - q) + p + (p + q) = -\dfrac{-12}{1}$

$3p = 12 \Rightarrow p = 4$

Hence, assertion is false.

Thus, assertion (A) is false but reason (R) is true.

Section B

21. 42000 = 2 × 2 × 2 × 2 × 3 × 5 × 5 × 5 × 7

$= 2^4 \times 3 \times 5^3 \times 7$

$= a^4 \times b \times c^3 \times d$

Then, PIN = dbac = 7325

22. Since ∠A = ∠B, AC = BC ... (1)

Also, AD = BE (given) ... (2)

Subtracting (2) from (1),

AC − AD = BC − BE

⇒DC = EC

From (2) and (3), we have

$$\frac{CD}{AD} = \frac{CE}{BE}$$

Therefore, DE ∥ AB

(By converse of Basic Proportionality theorem)

23. Let the circle touch the sides AB, BC, CD and DA at P, Q, R and S, respectively.

We know that the length of tangents drawn from an external point to a circle are equal.

AP = AS ... (1) {tangents from A}

BP = BQ ... (2) {tangents from B}

CR = CQ ... (3) {tangents from C}

DR = DS ... (4) {tangents from D}

Adding (1), (2), (3) and (4), we get

∴ AP + BP + CR + DR = AS + BQ + CQ + DS

⇒ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

⇒ AB + CD = AD + BC

⇒ AD = (AB + CD) − BC = {(6 + 4) − 7} cm = 3 cm

Hence, AD = 3 cm.

24.

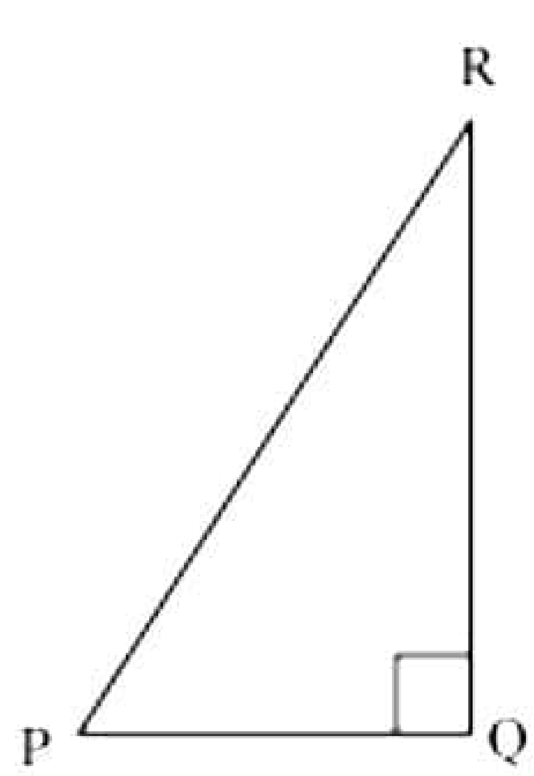
$$\begin{aligned}
\text{L.H.S.} &= (\sin\theta + \cos\theta)(\tan\theta + \cot\theta)\\
&= (\sin\theta + \cos\theta)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)\\
&= (\sin\theta + \cos\theta)\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)\\
&= (\sin\theta + \cos\theta)\left(\frac{1}{\sin\theta\cos\theta}\right)\\
&= \frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}\\
&= \frac{\sin\theta}{\sin\theta\cos\theta} + \frac{\cos\theta}{\sin\theta\cos\theta}\\
&= \frac{1}{\cos\theta} + \frac{1}{\sin\theta}\\
&= \sec\theta + \operatorname{cosec}\theta\\
&= \text{R.H.S.}
\end{aligned}$$

OR

Given that PR + QR = 25 and PQ = 5

Let PR be x.

So, QR = 25 − x



Now applying Pythagoras theorem in ΔPQR, We have

$PR^2 = PQ^2 + QR^2$

$x^2 = (5)^2 + (25 - x)^2$

$x^2 = 25 + 625 + x^2 - 50x$

$50x = 650$

$x = 13$

So, PR = 13 cm

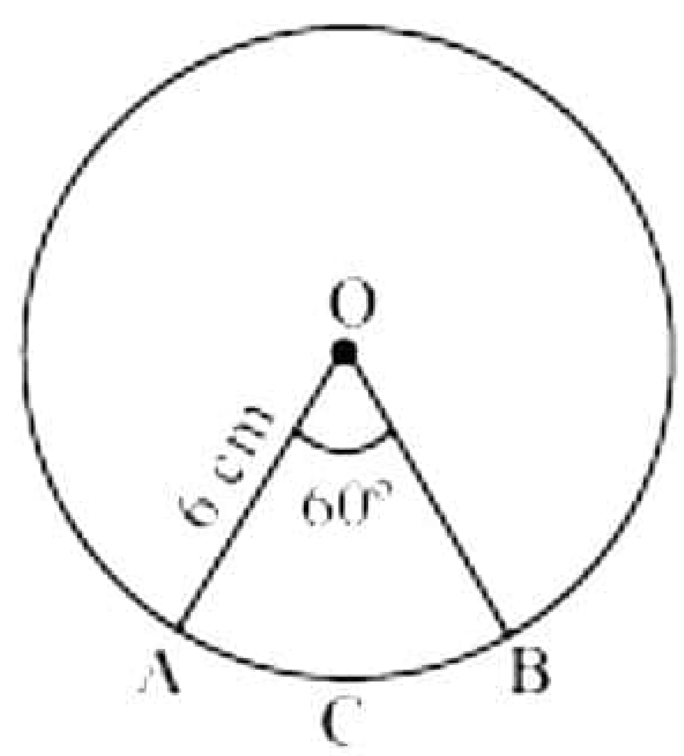
QR = 25 − 13 = 12 cm

$\sin P = \frac{\text{Side opposite to } \angle P}{\text{hypotenuse}} = \frac{QR}{PR} = \frac{12}{13}$

$\cos P = \frac{\text{side adjacent to } \angle P}{\text{hypotenuse}} = \frac{PQ}{PR} = \frac{5}{13}$

$\tan P = \frac{\text{Side opposite to } \angle P}{\text{side adjacent to } \angle P} = \frac{QR}{PQ} = \frac{12}{5}$

25.



Let OACB be a sector of circle making 60° angle at centre O of the circle.

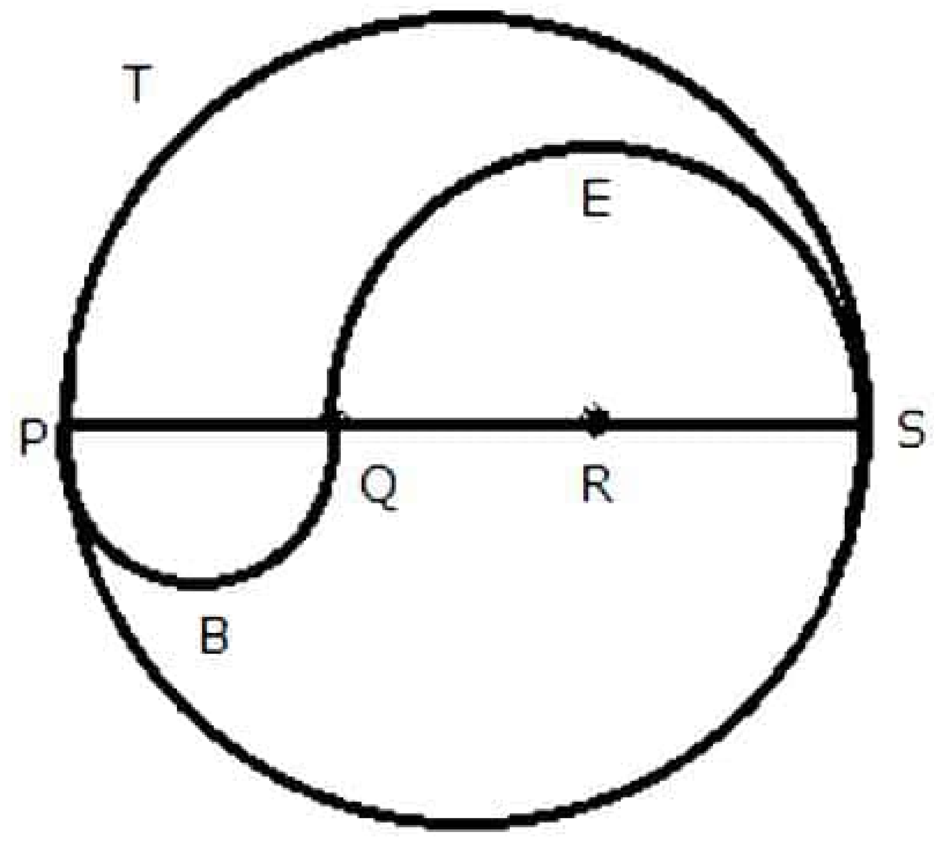
Area of sector of angel $\theta = \frac{\theta}{360°} \times \pi r^2$

So, area of sector OACB = $\frac{60°}{360°} \times \frac{22}{7} \times (6)^2$

$= \frac{1}{6} \times \frac{22}{7} \times 6 \times 6 = \frac{132}{7}$ cm^2

So, the area of sector of circle making 60° at the centre of a circle is $\frac{132}{7}$ cm^2.

OR



PS = 12 cm

PQ = QR = RS = 4 cm, QS = 8 cm

Perimeter of the shaded region = arc PTS + arc PBQ + arc QES

$= (\pi \times 6 + \pi \times 2 + \pi \times 4)$ cm

$= 12\pi$ cm

$= 12 \times 3.14$ cm

$= 37.68$ cm

Area of the shaded region = (area of semi-circle PBQ) + (area of semi-circle PTS) – (area of semi-circle QES)

$= \left[\frac{1}{2}\pi \times (2)^2 + \frac{1}{2} \times \pi \times (6)^2 - \frac{1}{2} \times \pi \times (4)^2\right] cm^2$

$= [2\pi + 18\pi - 8\pi] = 12\pi\ cm^2 = (12 \times 3.14)\ cm^2$

$= 37.68\ cm^2$

Section C

26.

(i) 12, 15 and 21

$12 = 2^2 \times 3$

$15 = 3 \times 5$

$21 = 3 \times 7$

$\text{HCF} = 3$

$\text{LCM} = 2^2 \times 3 \times 5 \times 7 = 420$

(ii) 17, 23 and 29

$17 = 1 \times 17$

$23 = 1 \times 23$

$29 = 1 \times 29$

$\text{HCF} = 1$

$\text{LCM} = 17 \times 23 \times 29 = 11339$

27. Let the number of John's marbles be *x*.

Therefore, number of Jayanti's marbles = 45 − x

After losing 5 marbles,

Number of John's marbles = x − 5

Number of Jivanti's marbles = 45 − x − 5 = 40 − x

It is given that the product of their marbles is 124.

$\therefore (x-5)(40-x) = 124$

$\Rightarrow x^2 - 45x + 324 = 0$

$\Rightarrow x^2 - 36x - 9x + 324 = 0$

$\Rightarrow x(x - 36) - 9(x - 36) = 0$

$\Rightarrow (x - 36)(x - 9) = 0$

Either x = 36 = 0 or x – 9 = 0
i.e., x = 36 or x = 9
If the number of John's marbles = 36,
Then, number of Jivanti's marbles = 45 – 36 = 9
If number of John's marbles = 9,
Then, number of Jivanti's marbles = 45 – 9 = 36

28. Let the fraction be $\frac{x}{y}$.

When 2 is added to both numerator and denominator, the fraction becomes

$\frac{x+2}{y+2} = \frac{1}{3}$ or $3x + 6 = y + 2$

$\Rightarrow 3x - y = -4$ ....(1)

When 3 is added both to numerator and denominator, the fractions becomes

$\frac{x+3}{y+3} = \frac{2}{5}$ or $5x + 15 = 2y + 6$

$\Rightarrow 5x - 2y = -9$ ....(2)

Multiplying (1) by 2, we get

$6x - 2y = -8$ ....(3)

Subtracting (2) from (3), we get x = 1

From (1),

$3 - y = -4 \Rightarrow y = 7$

$\therefore$ Required fraction is $\frac{1}{7}$

OR

Let the age of Jacob be x and the age of his son be y.
According to the given information,

$(x + 5) = 3(y + 5)$

$x - 3y = 10$ ... (1)

$(x - 5) = 7(y - 5)$

$x - 7y = -30$ ... (2)

From (1), we obtain

$x = 3y + 10$ ... (3)

Substituting this value in equation (2), we obtain

$3y + 10 - 7y = -30$

$-4y = -40$

$y = 10$ ... (4)

Substituting this value in equation (3), we obtain

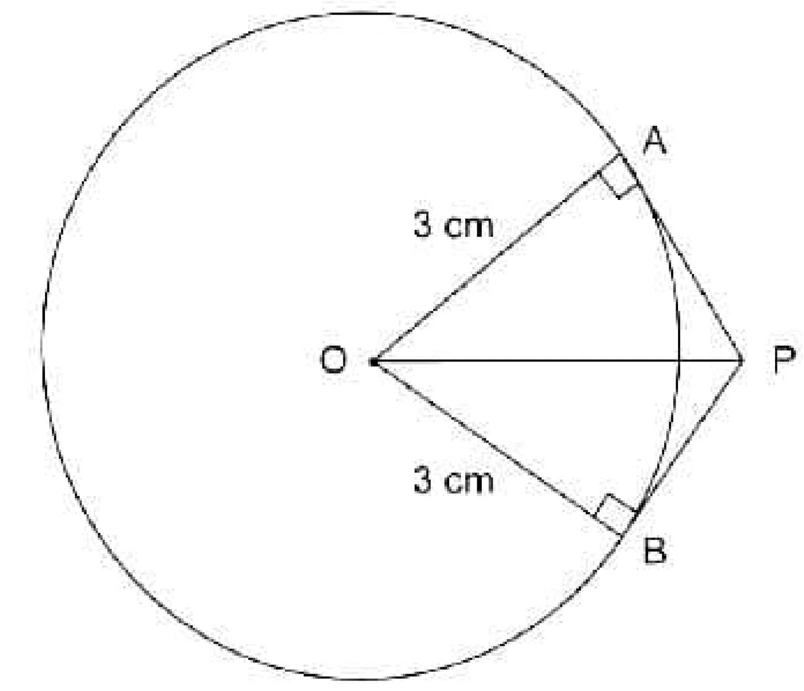
$x = 3\times10 + 10 = 40$

Hence, the present age of Jacob is 40 years whereas the present age of his son is 10 years.

29. Let P be the external point and PA and PB be the tangents such that, $\angle APB = 60^\circ$.

Now OA and OB are the radii of the circle.

$\therefore$ OA = OB = 3 cm



Also we know that the tangents drawn from an external point are equally inclined to the line joining the point to the centre.

$$\Rightarrow \angle OPA = \angle BPO = \frac{\angle APB}{2} = \frac{60^\circ}{2} = 30^\circ$$

Now, in $\triangle OAP$

$\angle OPA = 30^\circ$

$$\Rightarrow \tan 30^\circ = \frac{OA}{AP}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{3}{AP}$$

$$\Rightarrow AP = 3\sqrt{3} \text{ cm} = BP$$

Hence, the length of each tangent is $3\sqrt{3}$ cm.

OR

PA is the tangent to the circle with centre O, such that PO = 25 m, PA = 24 m.

In $\triangle PAO$, $\angle A = 90^\circ$ (since tangent $\perp$ radius)

By Pythagoras' theorem,

$PO^2 = PA^2 + OA^2$

$OA^2 = PO^2 - PA^2 = 25^2 - 24^2 = (25 - 24)(25 + 24) = 49$ m

So, OA = 7 m

Hence, the distance from the centre of the park to the gate is 7 m.

30. X = cot A + cos A and y = cot A − cos A

Thus, we have

$$x + y = (\cot A + \cos A) + (\cot A - \cos A) = 2\cot A$$

$$x - y = (\cot A + \cos A) - (\cot A - \cos A) = 2\cos A$$

$$\text{L.H.S.} = \left(\frac{x-y}{x+y}\right)^2 + \left(\frac{x-y}{2}\right)^2$$

$$= \left(\frac{2\cos A}{2\cot A}\right)^2 + \left(\frac{2\cos A}{2}\right)^2$$

$$= \left(\frac{\cos A}{\cot A}\right)^2 + (\cos A)^2$$

$$= \left(\frac{\cos A}{\cos A / \sin A}\right)^2 + (\cos A)^2$$

$= (\sin A)^2 + (\cos A)^2$
$= \sin^2 A + \cos^2 A$
$= 1$
$=$ R.H.S.

31. 2 red kings, 2 red queens, 2 red jacks are removed.
Remaining number of cards = 52 − 6 = 46

i. As 2 red kings are removed, only 2 black king cards are left.

$\therefore$ Probability of getting a king card $= \frac{2}{46} = \frac{1}{23}$

ii. 6 red cards are removed.

$\therefore$ 20 red cards are left.

$\therefore$ Probability of getting a red card $= \frac{20}{46} = \frac{10}{23}$

iii. There are 13 spade cards.

$\therefore$ Probability of getting a spade card $= \frac{13}{46}$

Section D

32. Let the speed of the boat in still water be x km/hr

Then, the speed of the boat downstream = (x + 2) km/hr

And the speed of the boat upstream = (x − 2) km/hr

Time taken to cover 8 km downstream = $\frac{8}{(x+2)}$ hrs

Time taken to cover 8 km upstream = $\frac{8}{(x-2)}$ hrs

Total time taken $= 1\frac{40}{60} = \frac{5}{3}$ hrs

$$\frac{8}{(x+2)} + \frac{8}{(x-2)} = \frac{5}{3}$$

$$\Rightarrow \frac{1}{x+2} + \frac{1}{x-2} = \frac{5}{24}$$

$$\Rightarrow \frac{x-2+x+2}{(x+2)(x-2)} = \frac{5}{24}$$

$$\Rightarrow \frac{2x}{x^2-4} = \frac{5}{24}$$

$$\Rightarrow 5x^2 - 48x - 20 = 0$$

$$\Rightarrow 5x^2 - 50x + 2x - 20 = 0$$

$$\Rightarrow 5x(x-10) + 2(x-10) = 0$$

$$\Rightarrow (x-10)(5x+2) = 0$$

$$\Rightarrow x = 10 \text{ or } x = \frac{-2}{5}$$

$\Rightarrow x = 10$ (speed cannot be negative)

Then the speed of the boat in still water is 10 km/hr.

OR

Let the faster pipe take x minutes to fill the cistern.
Then the other pipe takes (x + 3) minutes.

$$\frac{1}{x}+\frac{1}{(x+3)}=\frac{13}{40}$$

$$\Rightarrow \frac{(x+3)+x}{x(x+3)}=\frac{13}{40}$$

$$\Rightarrow 40(2x+3)=13\left(x^2+3x\right)$$

$$\Rightarrow 80x+120=13x^2+39x$$

$$\Rightarrow 13x^2-41x-120=0$$

$$\Rightarrow 13x^2-65x+24x-120=0$$

$$\Rightarrow 13x(x-5)+24(x-5)=0$$

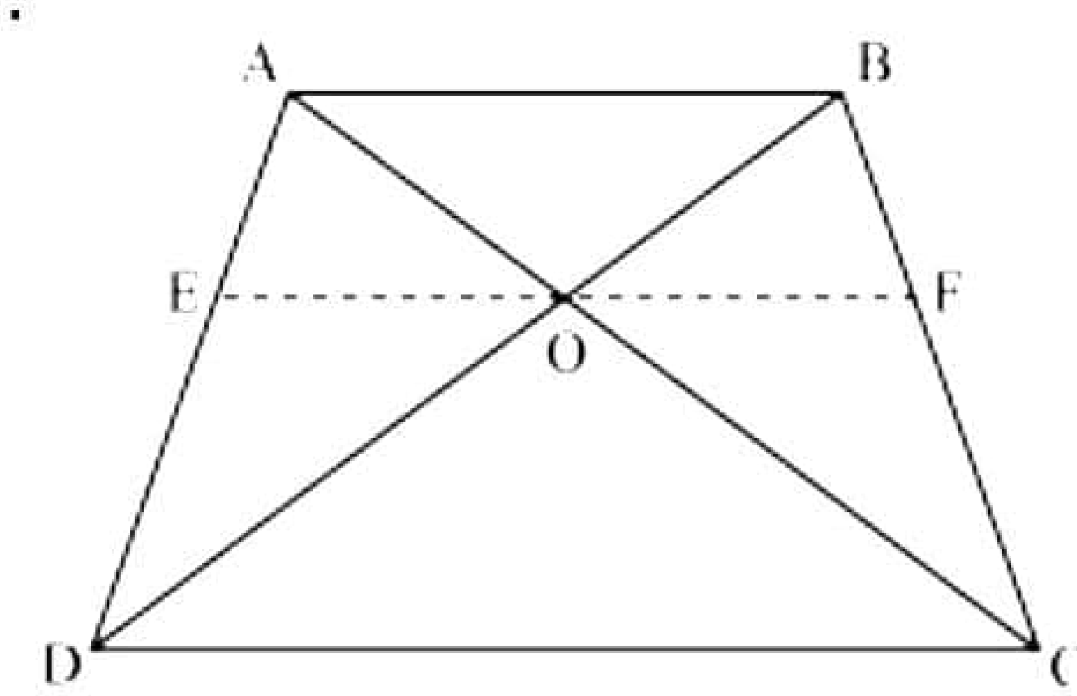
$$\Rightarrow (x-5)(13x+24)=0$$

$$\Rightarrow x=5 \quad \text{or} \quad x=\frac{-24}{13}$$

$$\Rightarrow x=5 \ \ (\text{Time cannot be negative})$$

If the faster pipe takes 5 minutes to fill the cistern, then the other pipe takes (5 + 3) minutes = 8 minutes

33. Draw a line EF through point O such that EF||CD.



In ΔADC, EO || CD

So, by basic proportionality theorem

$$\frac{AE}{ED}=\frac{AO}{OC} \quad ...(1)$$

Similarly, in ΔBDC, FO || CD

So, by basic proportionality theorem

$$\frac{BF}{FC}=\frac{BO}{OD} \quad ...(2)$$

Now consider trapezium ABCD

As FE||CD

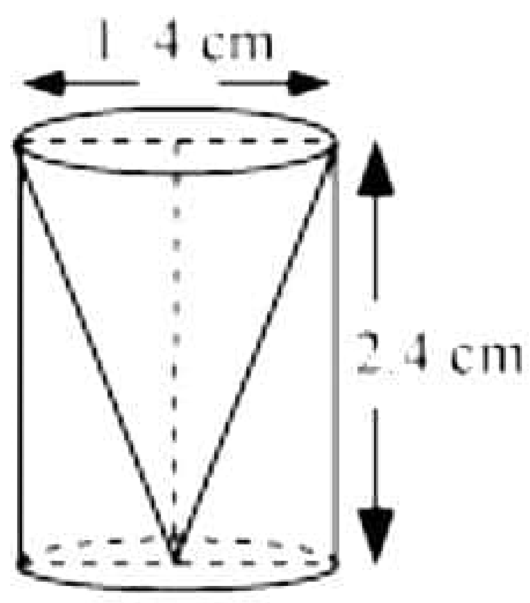
So, $\frac{AE}{ED}=\frac{BF}{FC}$ ... (3)

Now from equations (1), (2), (3)

$$\frac{AO}{OC}=\frac{BO}{OD}$$

Or, $\frac{AO}{BO}=\frac{OC}{OD}$

34.



Given that

Height (h) of the conical part = Height (h) of the cylindrical part = 2.4 cm

Diameter of the cylindrical part = 1.4 cm

So, radius (r) of the cylindrical part = 0.7 cm

Slant height (l) of conical part = $\sqrt{r^2 + h^2}$

$$= \sqrt{(0.7)^2 + (2.4)^2} = \sqrt{0.49 + 5.76}$$

$$= \sqrt{6.25} = 2.5$$

Total surface area of remaining solid

= CSA of cylindrical part + CSA of conical part + Area of cylindrical base

$= 2\pi rh + \pi rl + \pi r^2$

$= 2 \times \frac{22}{7} \times 0.7 \times 2.4 + \frac{22}{7} \times 0.7 \times 2.5 + \frac{22}{7} \times 0.7 \times 0.7$

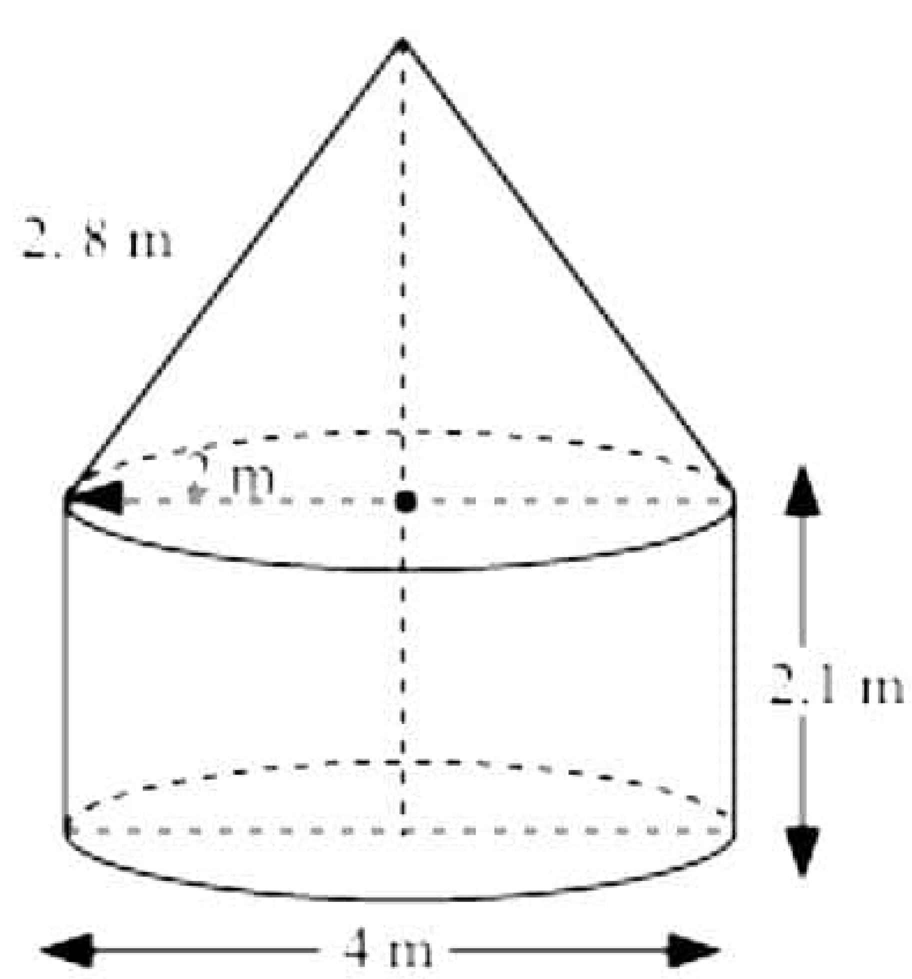
$= 4.4 \times 2.4 + 2.2 \times 2.5 + 2.2 \times 0.7$

$= 10.56 + 5.50 + 1.54$

$= 17.60 \text{ cm}^2$

Clearly total surface area of the remaining solid to the nearest cm^2 is 18 cm^2.

OR



Given that

Height (h) of the cylindrical part = 2.1 m

Diameter of the cylindrical part = 4 m

So, radius of the cylindrical part = 2 m

Slant height (l) of conical part = 2.8 m

Area of canvas used = CSA of conical part + CSA of cylindrical part

$= \pi rl + 2\pi rh$

$= \pi \times 2 \times 2.8 + 2\pi \times 2 \times 2.1$

$= 2\pi[2.8 + 4.2]$

$= 2 \times \frac{22}{7} \times 7$

$= 44\ m^2$

Cost of 1 m^2 canvas = Rs.500

Cost of 44 m^2 canvas = 44 × 500 = Rs. 22000

So, it will cost Rs.22000 for making such a tent.

35. Given that mean pocket allowance $\overline{x} = \text{Rs.}18$

Now taking 18 as assured mean (a) we may calculate d_i and f_id_i as following.

Daily pocket allowance (in Rs.)	Number of children (f_i)	Class-mark (x_i)	$d_i = x_i - 18$	f_id_i
11–13	7	12	–6	–42
13–15	6	14	–4	–24
15–17	9	16	–2	–18
17–19	13	18	0	0
19–21	f	20	2	2f
21–23	5	22	4	20
23–25	4	24	6	24
Total	$\sum f_i = 44 + f$			2f – 40

Here,

$\sum f_i = 44 + f$ and $\sum f_id_i = 2f - 40$

$$\overline{x} = a + \frac{\sum f_id_i}{\sum f_i}$$

$$18 = 18 + \left(\frac{2f - 40}{44 + f}\right)$$

$$0 = \left(\frac{2f - 40}{44 + f}\right)$$

$2f - 40 = 0$

$2f = 40$

$f = 20$

Hence the messing frequency is 20.

Section E

36.

i. Here, a = 1000 and d = 100
This is an A.P.
Therefore, a_{30} = a + (30 – 1)d
⇒ a_{30} = 1000 + 29(100) = 1000 + 2900 = Rs. 3900

ii. a_{40} = a + 39d = 1000 + 3900 = Rs. 4900

iii. Here, a_{19} = 1000 + 1800 = 2800
and a_{28} = 1000 + 2700= 3700
2800:3700 = 28:37

OR

Here, a = 1000 and d = 100

$$S_{30} = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{30}{2}\left[2(1000) + 29 \times 100\right] = Rs. 73500$$

37.

i. From the graph, the coordinates of points A and C are (1, 1) and (4, 5) respectively.

$$\therefore d(AC) = \sqrt{(4-1)^2 + (5-1)^2} = 5 \text{ km}$$

OR

From the graph, the coordinates of points B and D are (4, 1) and (7, 5) respectively.

$$\therefore d(BD) = \sqrt{(7-4)^2 + (5-1)^2} = 5 \text{ km}$$

ii. From the graph, the coordinates of points B and A are (4, 1) and (1, 1) respectively.

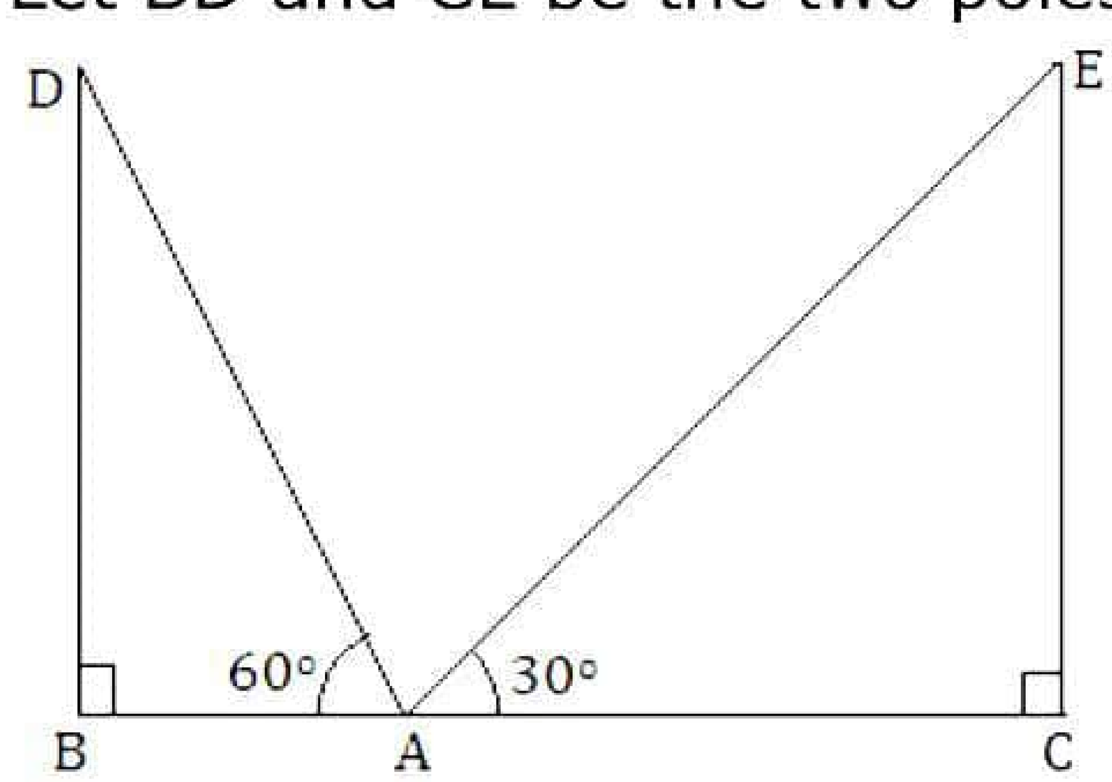
$$\therefore d(BA) = \sqrt{(1-4)^2 + (1-1)^2} = 3 \text{ km}$$

iii. From the graph, the coordinates of points B and C are (4, 1) and (4, 5) respectively.

$$\therefore d(BC) = \sqrt{(4-4)^2 + (5-1)^2} = 4 \text{ km}$$

38.

i. Let BD and CE be the two poles.



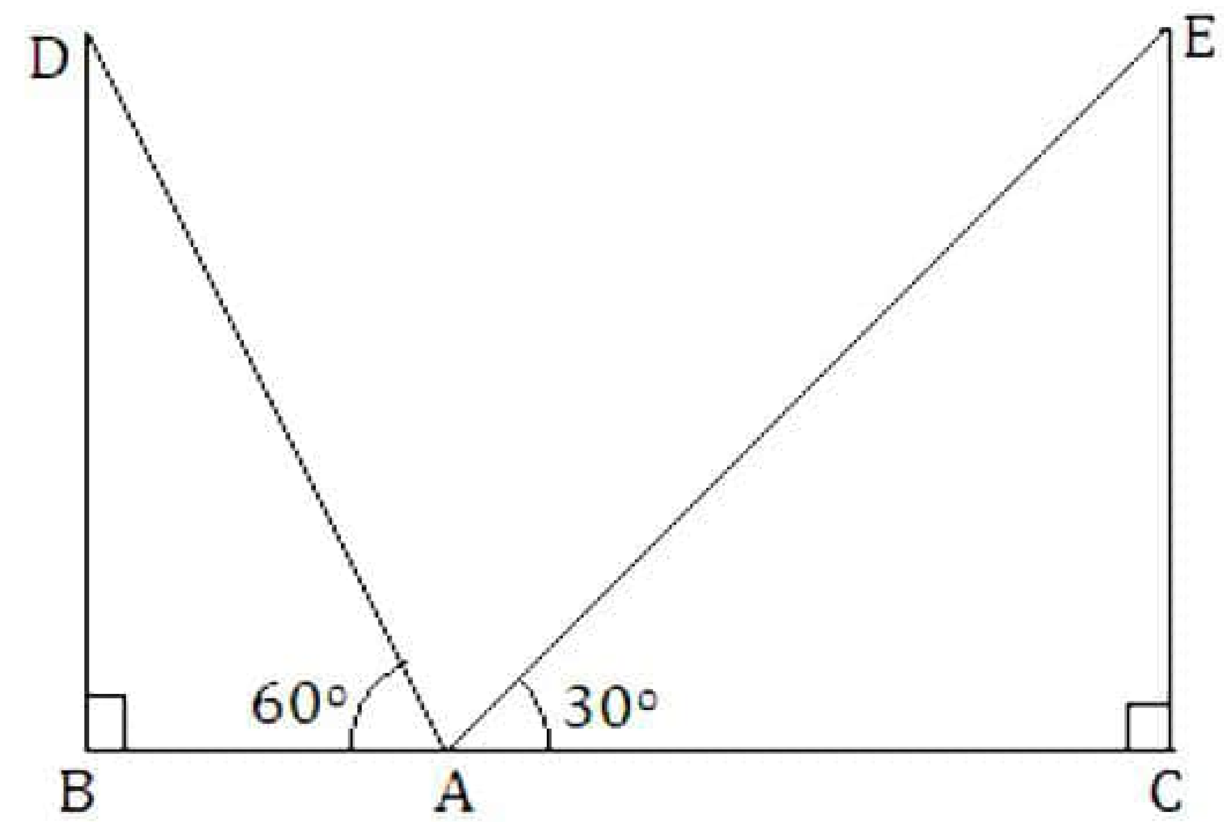
In ΔABD, we have

$$\tan 60^o = \frac{BD}{AB}$$

$$\Rightarrow \sqrt{3} = \frac{30}{AB}$$

$$\Rightarrow AB = 10\sqrt{3} \text{ m}$$

OR



In ΔACE, we have

$$\tan 30^o = \frac{EC}{AC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{30}{AC}$$

$$\Rightarrow AC = 30\sqrt{3} \text{ m}$$

ii. Width of the road = BC = BA + AC = $10\sqrt{3} + 30\sqrt{3} = 40\sqrt{3}$ m

iii. Angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level is known as the angle of elevation.