



Series EF1GH/2



SET~3

रोल नं. Roll No.					

प्रश्न-पत्र कोड
Q.P. Code **65/2/3**

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।

Candidates must write the Q.P. Code on the title page of the answer-book.

गणित

MATHEMATICS

निर्धारित समय : 3 घण्टे

Time allowed : 3 hours

अधिकतम अंक : 80

Maximum Marks : 80

नोट / NOTE :

- (i) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं।
Please check that this question paper contains 23 printed pages.
- (ii) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- (iii) कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं।
Please check that this question paper contains 38 questions.
- (iv) कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें।
Please write down the serial number of the question in the answer-book before attempting it.
- (v) इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।
15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.





सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका पालन कीजिए :

- (i) इस प्रश्न-पत्र में 38 प्रश्न हैं। सभी प्रश्न अनिवार्य हैं।
- (ii) प्रश्न-पत्र पाँच खण्डों में विभाजित है – खण्ड क, ख, ग, घ तथा ङ।
- (iii) खण्ड-क में प्रश्न संख्या 1 से 18 तक बहुविकल्पीय तथा प्रश्न संख्या 19 एवं 20 अभिकथन एवं कारण आधारित एक-एक अंक के प्रश्न हैं।
- (iv) खण्ड-ख में प्रश्न संख्या 21 से 25 तक अति लघु-उत्तरीय (VSA) प्रकार के दो-दो अंकों के प्रश्न हैं।
- (v) खण्ड-ग में प्रश्न संख्या 26 से 31 तक लघु-उत्तरीय (SA) प्रकार के तीन-तीन अंकों के प्रश्न हैं।
- (vi) खण्ड-घ में प्रश्न संख्या 32 से 35 तक दीर्घ-उत्तरीय (LA) प्रकार के पाँच-पाँच अंकों के प्रश्न हैं।
- (vii) खण्ड-ङ में प्रश्न संख्या 36 से 38 तक अध्ययन/केस/अनुच्छेद/एकीकृत इकाइयाँ मूल्यांकन आधारित चार-चार अंकों के प्रश्न हैं।
- (viii) प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड-ख के 2 प्रश्नों में, खण्ड-ग के 3 प्रश्नों में, खण्ड-घ के 2 प्रश्नों में तथा खण्ड-ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।
- (ix) कैल्कुलेटर का उपयोग वर्जित है।

खण्ड – क

(बहुविकल्पीय प्रश्न)

प्रत्येक प्रश्न का 1 अंक है।

1. यदि \vec{a} तथा \vec{b} के बीच का कोण $\frac{\pi}{3}$ है तथा $|\vec{a} \times \vec{b}| = 3\sqrt{3}$ है, तो $\vec{a} \cdot \vec{b}$ का मान है : 1

(A) 9 (B) 3

(C) $\frac{1}{9}$ (D) $\frac{1}{3}$



General Instructions :

Read the following instructions very carefully and follow them :

- (i) This Question Paper contains 38 questions. All questions are compulsory.
- (ii) Question paper is divided into FIVE Sections – Section A, B, C, D and E.
- (iii) In Section A – Question Nos. 1 to 18 are Multiple Choice Questions (MCQs) and Question Nos. 19 & 20 are Assertion-Reason based questions of 1 mark each.
- (iv) In Section B – Question Nos. 21 to 25 are Very Short Answer (VSA) type questions of 2 marks each.
- (v) In Section C – Question Nos. 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.
- (vi) In Section D – Question Nos. 32 to 35 are Long Answer (LA) type questions carrying 5 marks each.
- (vii) In Section E – Question Nos. 36 to 38 are source based/case based/passage based/integrated units of assessment questions carrying 4 marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
- (ix) Use of calculators is NOT allowed.

SECTION – A
(Multiple Choice Questions)
Each question carries 1 mark.

1. If the angle between \vec{a} and \vec{b} is $\frac{\pi}{3}$ and $|\vec{a} \times \vec{b}| = 3\sqrt{3}$, then the value of $\vec{a} \cdot \vec{b}$ is **1**

(A) 9 (B) 3
(C) $\frac{1}{9}$ (D) $\frac{1}{3}$



2. एक समान्तर चतुर्भुज ABCD के तीन क्रमागत शीर्षों के स्थिति सदिश $A(4\hat{i} + 2\hat{j} - 6\hat{k})$, $B(5\hat{i} - 3\hat{j} + \hat{k})$ तथा $C(12\hat{i} + 4\hat{j} + 5\hat{k})$ हैं तो बिंदु D का स्थिति सदिश है : 1

(A) $-3\hat{i} - 5\hat{j} - 10\hat{k}$ (B) $21\hat{i} + 3\hat{j}$
(C) $11\hat{i} + 9\hat{j} - 2\hat{k}$ (D) $-11\hat{i} - 9\hat{j} + 2\hat{k}$

3. यदि दो घटनाओं A तथा B के लिए $P(A - B) = \frac{1}{5}$ तथा $P(A) = \frac{3}{5}$ है, तो $P\left(\frac{B}{A}\right)$ बराबर है : 1

(A) $\frac{1}{2}$ (B) $\frac{3}{5}$
(C) $\frac{2}{5}$ (D) $\frac{2}{3}$

4. यदि $\int_0^{2\pi} \cos^2 x \, dx = k \int_0^{\pi/2} \cos^2 x \, dx$ है, तो k का मान है : 1

(A) 4 (B) 2
(C) 1 (D) 0

5. यदि एक $\triangle ABC$ के शीर्ष (a, b), (c, d) तथा (e, f) हैं तथा $\triangle ABC$ का क्षेत्रफल Δ द्वारा निरूपित किया जाता है, तो $\begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}^2$ बराबर है : 1

(A) $2\Delta^2$ (B) $4\Delta^2$
(C) 2Δ (D) 4Δ

6. फलन $f(x) = x|x|$, $x = 0$ पर 1

(A) संतत तथा अवकलनीय है। (B) संतत है, परन्तु अवकलनीय नहीं है।
(C) अवकलनीय है, परन्तु संतत नहीं है। (D) न संतत और न ही अवकलनीय है।



2. The position vectors of three consecutive vertices of a parallelogram ABCD are $A(4\hat{i} + 2\hat{j} - 6\hat{k})$, $B(5\hat{i} - 3\hat{j} + \hat{k})$ and $C(12\hat{i} + 4\hat{j} + 5\hat{k})$. The position vector of D is given by 1

(A) $-3\hat{i} - 5\hat{j} - 10\hat{k}$ (B) $21\hat{i} + 3\hat{j}$
(C) $11\hat{i} + 9\hat{j} - 2\hat{k}$ (D) $-11\hat{i} - 9\hat{j} + 2\hat{k}$

3. If for two events A and B, $P(A - B) = \frac{1}{5}$ and $P(A) = \frac{3}{5}$, then $P\left(\frac{B}{A}\right)$ is equal to 1

(A) $\frac{1}{2}$ (B) $\frac{3}{5}$
(C) $\frac{2}{5}$ (D) $\frac{2}{3}$

4. If $\int_0^{2\pi} \cos^2 x \, dx = k \int_0^{\pi/2} \cos^2 x \, dx$, then the value of k is 1

(A) 4 (B) 2
(C) 1 (D) 0

5. If (a, b), (c, d) and (e, f) are the vertices of ΔABC and Δ denotes the area of ΔABC , then $\begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}^2$ is equal to 1

(A) $2\Delta^2$ (B) $4\Delta^2$
(C) 2Δ (D) 4Δ

6. The function $f(x) = x|x|$ is 1

(A) continuous and differentiable at $x = 0$.
(B) continuous but not differentiable at $x = 0$.
(C) differentiable but not continuous at $x = 0$.
(D) neither differentiable nor continuous at $x = 0$.



7. यदि $\tan\left(\frac{x+y}{x-y}\right) = k$ है, तो $\frac{dy}{dx}$ बराबर है : 1

(A) $-\frac{y}{x}$ (B) $\frac{y}{x}$
 (C) $\sec^2\left(\frac{y}{x}\right)$ (D) $-\sec^2\left(\frac{y}{x}\right)$

8. एक रैखिक प्रोग्रामन समस्या के उद्देश्य फलन $Z = ax + by$ का (4, 6) पर अधिकतम मान 42 है तथा (3, 2) पर न्यूनतम मान 19 है, तो निम्न में से कौन सा सत्य है ? 1

(A) $a = 9, b = 1$ (B) $a = 5, b = 2$
 (C) $a = 3, b = 5$ (D) $a = 5, b = 3$

9. एक रैखिक प्रोग्रामन समस्या के सुसंगत क्षेत्र के शीर्ष बिंदु (0, 4), (8, 0) तथा $\left(\frac{20}{3}, \frac{4}{3}\right)$ हैं। यदि इसका उद्देश्य फलन $Z = 30x + 24y$ है, तो (Z का अधिकतम मान – Z का न्यूनतम मान) बराबर है : 1

(A) 40 (B) 96
 (C) 120 (D) 136

10. कोटि 3×3 के सममित आव्यूहों, जिनका प्रत्येक अवयव 1 या –1 हो, की संख्या है : 1

(A) 512 (B) 64
 (C) 8 (D) 4

11. यदि $\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} = P + Q$ है, जहाँ P एक सममित आव्यूह है तथा Q एक विषम सममित आव्यूह है, तो Q बराबर है : 1

(A) $\begin{bmatrix} 2 & 5/2 \\ 5/2 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$
 (C) $\begin{bmatrix} 0 & 5/2 \\ -5/2 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & -5/2 \\ 5/2 & 4 \end{bmatrix}$



7. If $\tan\left(\frac{x+y}{x-y}\right) = k$, then $\frac{dy}{dx}$ is equal to 1

(A) $\frac{-y}{x}$ (B) $\frac{y}{x}$
(C) $\sec^2\left(\frac{y}{x}\right)$ (D) $-\sec^2\left(\frac{y}{x}\right)$

8. The objective function $Z = ax + by$ of an LPP has maximum value 42 at (4, 6) and minimum value 19 at (3, 2). Which of the following is true? 1

(A) $a = 9, b = 1$ (B) $a = 5, b = 2$
(C) $a = 3, b = 5$ (D) $a = 5, b = 3$

9. The corner points of the feasible region of a linear programming problem are $(0, 4)$, $(8, 0)$ and $\left(\frac{20}{3}, \frac{4}{3}\right)$. If $Z = 30x + 24y$ is the objective function, then (maximum value of Z – minimum value of Z) is equal to 1

(A) 40 (B) 96
(C) 120 (D) 136

10. Number of symmetric matrices of order 3×3 with each entry 1 or -1 is 1

(A) 512 (B) 64
(C) 8 (D) 4

11. If $\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} = P + Q$, where P is a symmetric and Q is a skew symmetric matrix, then Q is equal to 1

(A) $\begin{bmatrix} 2 & 5/2 \\ 5/2 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$
(C) $\begin{bmatrix} 0 & 5/2 \\ -5/2 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & -5/2 \\ 5/2 & 4 \end{bmatrix}$



12. यदि $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{bmatrix}$ एक व्युत्क्रमणीय आव्यूह है तथा $a \in A$ है, तो समुच्चय A बराबर है : 1

(A) \mathbb{R} (B) $\{0\}$
(C) $\{4\}$ (D) $\mathbb{R} - \{4\}$

13. यदि $|A| = |kA|$ है, जहाँ A कोटि 2 का वर्ग आव्यूह है, तो k के सभी संभव मानों का योगफल है : 1

(A) 1 (B) -1
(C) 2 (D) 0

14. यदि $\frac{d}{dx} [f(x)] = ax + b$ है तथा $f(0) = 0$ है, तो $f(x)$ बराबर है : 1

(A) $a + b$ (B) $\frac{ax^2}{2} + bx$
(C) $\frac{ax^2}{2} + bx + c$ (D) b

15. अवकल समीकरण $\sin x + \cos \left(\frac{dy}{dx} \right) = y^2$ की घात है : 1

(A) 2 (B) 1
(C) परिभाषित नहीं है। (D) 0

16. अवकल समीकरण $(1 - y^2) \frac{dx}{dy} + yx = ay$, $(-1 < y < 1)$ का समाकलन गुणक है : 1

(A) $\frac{1}{y^2 - 1}$ (B) $\frac{1}{\sqrt{y^2 - 1}}$
(C) $\frac{1}{1 - y^2}$ (D) $\frac{1}{\sqrt{1 - y^2}}$



12. If $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{bmatrix}$ is non-singular matrix and $a \in A$, then the set A is 1

(A) \mathbb{R} (B) $\{0\}$
(C) $\{4\}$ (D) $\mathbb{R} - \{4\}$

13. If $|A| = |kA|$, where A is a square matrix of order 2, then sum of all possible values of k is 1

(A) 1 (B) -1
(C) 2 (D) 0

14. If $\frac{d}{dx}[f(x)] = ax + b$ and $f(0) = 0$, then $f(x)$ is equal to 1

(A) $a + b$ (B) $\frac{ax^2}{2} + bx$
(C) $\frac{ax^2}{2} + bx + c$ (D) b

15. Degree of the differential equation $\sin x + \cos \left(\frac{dy}{dx} \right) = y^2$ is 1

(A) 2 (B) 1
(C) not defined (D) 0

16. The integrating factor of the differential equation $(1 - y^2) \frac{dx}{dy} + yx = ay$, $(-1 < y < 1)$ is 1

(A) $\frac{1}{y^2 - 1}$ (B) $\frac{1}{\sqrt{y^2 - 1}}$
(C) $\frac{1}{1 - y^2}$ (D) $\frac{1}{\sqrt{1 - y^2}}$



17. यदि P तथा Q के निर्देशांक क्रमशः $(2, 1, -1)$ तथा $(4, 4, -7)$ हैं, तो \vec{PQ} के अनुदिश मात्रक सदिश है : 1

(A) $2\hat{i} + 3\hat{j} - 6\hat{k}$ (B) $-2\hat{i} - 3\hat{j} + 6\hat{k}$
 (C) $\frac{-2\hat{i}}{7} - \frac{3\hat{j}}{7} + \frac{6\hat{k}}{7}$ (D) $\frac{2\hat{i}}{7} + \frac{3\hat{j}}{7} - \frac{6\hat{k}}{7}$

18. बिंदु $(1, 2, 3)$ से होकर जाने वाली वह रेखा जो निर्देशांक अक्षों से समान कोण बनाती है, का समीकरण है : 1

(A) $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ (B) $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$
 (C) $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{3}$ (D) $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1}$

अभिकथन – तर्क आधारित प्रश्न

प्रश्न संख्या 19 एवं 20 में एक अभिकथन (A) के बाद एक तर्क आधारित कथन (R) दिया है। प्रत्येक में निम्न चारों में से कोई एक सही विकल्प चुनिए :

(A) (A) तथा (R) दोनों सत्य हैं और कथन (R), कथन (A) की सही व्याख्या करता है ।

(B) (A) तथा (R) दोनों सत्य हैं, परन्तु कथन (R), कथन (A) की सही व्याख्या नहीं करता है ।

(C) (A) सत्य है, परन्तु (R) सत्य नहीं है ।

(D) (A) असत्य है तथा (R) सत्य है ।

19. अभिकथन (A) : $(\cos^{-1} x)^2$ का अधिकतम मान π^2 है । 1

तर्क (R) : $\cos^{-1} x$ की मुख्य मान शाखा का परिसर $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ है ।

20. अभिकथन (A) : यदि एक रेखा निर्देशांक अक्षों की धनात्मक दिशाओं से α, β, γ के कोण बनाती है, तो $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$ है । 1

तर्क (R) : एक रेखा के दिक कोसाइनों के वर्गों का योग 1 होता है ।



17. Unit vector along \vec{PQ} , where coordinates of P and Q respectively are (2, 1, -1) and (4, 4, -7), is 1

(A) $2\hat{i} + 3\hat{j} - 6\hat{k}$ (B) $-2\hat{i} - 3\hat{j} + 6\hat{k}$
(C) $\frac{-2\hat{i}}{7} - \frac{3\hat{j}}{7} + \frac{6\hat{k}}{7}$ (D) $\frac{2\hat{i}}{7} + \frac{3\hat{j}}{7} - \frac{6\hat{k}}{7}$

18. Equation of a line passing through point (1, 2, 3) and equally inclined to the coordinate axis, is 1

(A) $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ (B) $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$
(C) $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{3}$ (D) $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1}$

ASSERTION-REASON BASED QUESTIONS

In the following questions 19 & 20, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices :

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).
(B) Both (A) and (R) are true, but (R) is not the correct explanation of (A).
(C) (A) is true, but (R) is false.
(D) (A) is false, but (R) is true.

19. **Assertion (A) :** Maximum value of $(\cos^{-1} x)^2$ is π^2 . 1

Reason (R) : Range of the principal value branch of $\cos^{-1}x$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

20. **Assertion (A) :** If a line makes angles α, β, γ with positive direction of the coordinate axes, then $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$. 1

Reason (R) : The sum of squares of the direction cosines of a line is 1.



खण्ड – ख

इस खण्ड में अति लघु-उत्तरीय (VSA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 2 अंक हैं।

21. यदि बिंदुओं A, B तथा C के स्थिति सदिश क्रमशः $2\hat{i} + \hat{j}$ तथा $2\hat{k}$ हैं, तो दर्शाइए कि ΔABC एक समद्विबाहु त्रिभुज है। 2

22. (a) मान ज्ञात कीजिए : $\sin^{-1}\left(\sin \frac{3\pi}{4}\right) + \cos^{-1}(\cos \pi) + \tan^{-1}(1)$ 2

अथवा

(b) $\cos^{-1} x$ का आलेख खर्चिए जहाँ $x \in [-1, 0]$ तथा इसका परिसर भी लिखिए। 2

23. यदि स्थिर आधार 10 cm वाले समद्विबाहु त्रिभुज की समान भुजाएँ 4 cm/s की दर से बढ़ रही हैं, तो उस समय जब त्रिभुज की तीनों भुजाएँ समान हो जाएँ, त्रिभुज के क्षेत्रफल के बढ़ने की दर क्या होगी ? 2

24. रेखा $\frac{x}{1} = \frac{y-1}{2} = \frac{z+1}{2}$ पर स्थित ऐसे बिंदुओं के निर्देशांक ज्ञात कीजिए, जिनकी मूल बिंदु से दूरी $\sqrt{11}$ इकाई हो। 2

25. (a) यदि $y = \sqrt{ax + b}$ है, तो सिद्ध कीजिए कि $y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 = 0$ 2

अथवा

(b) यदि $f(x) = \begin{cases} ax + b & ; \quad 0 < x \leq 1 \\ 2x^2 - x & ; \quad 1 < x < 2 \end{cases}$ द्वारा दिया गया फलन $(0, 2)$ पर अवकलनीय है, तो a तथा b के मान ज्ञात कीजिए। 2

खण्ड – ग

इस खण्ड में लघु-उत्तरीय (SA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 3 अंक हैं।

26. निम्न रैखिक प्रोग्रामन समस्या को आलेख द्वारा हल कीजिए :

व्यवरोधों : $x + 2y \geq 240$,

$$3x + 1.5y \geq 270,$$

$$1.5x + 2y \leq 310,$$

$$x \geq 0, y \geq 0$$

के अंतर्गत $Z = 3x + 3.5y$ का अधिकतम मान ज्ञात कीजिए। 3



SECTION – B

This section comprises of Very Short Answer Type (VSA) questions, each of **2** marks.

21. If points A, B and C have position vectors $2\hat{i}$, \hat{j} and $2\hat{k}$ respectively, then show that ΔABC is an isosceles triangle. **2**

22. (a) Evaluate $\sin^{-1}\left(\sin \frac{3\pi}{4}\right) + \cos^{-1}(\cos \pi) + \tan^{-1}(1)$. **2**

OR

(b) Draw the graph of $\cos^{-1} x$, where $x \in [-1, 0]$. Also, write its range. **2**

23. If equal sides of an isosceles triangle with fixed base 10 cm are increasing at the rate of 4 cm/sec, how fast is the area of triangle increasing at an instant when all sides become equal ? **2**

24. Find the coordinates of points on line $\frac{x}{1} = \frac{y-1}{2} = \frac{z+1}{2}$ which are at a distance of $\sqrt{11}$ units from origin. **2**

25. (a) If $y = \sqrt{ax + b}$, then prove that $y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 = 0$. **2**

OR

(b) If $f(x) = \begin{cases} ax + b & ; \quad 0 < x \leq 1 \\ 2x^2 - x & ; \quad 1 < x < 2 \end{cases}$ is a differentiable function in $(0, 2)$, then find the values of a and b. **2**

SECTION – C

The section comprises Short Answer (SA) type questions of **3** marks each.

26. Solve the following Linear Programming problem graphically :

Maximize : $Z = 3x + 3.5y$

subject to constraints : $x + 2y \geq 240$,

$$3x + 1.5y \geq 270,$$

$$1.5x + 2y \leq 310,$$

$$x \geq 0, y \geq 0.$$

3



27. (a) ज्ञात कीजिए : $\int \frac{x+2}{\sqrt{x^2-4x-5}} dx$ 3

अथवा

(b) मान ज्ञात कीजिए : $\int_{-a}^a f(x) dx$, जहाँ $f(x) = \frac{9^x}{1+9^x}$ है । 3

28. ज्ञात कीजिए : $\int e^{\cot^{-1} x} \left(\frac{1-x+x^2}{1+x^2} \right) dx$ 3

29. मान ज्ञात कीजिए : $\int_{\log \sqrt{2}}^{\log \sqrt{3}} \frac{1}{(e^x + e^{-x})(e^x - e^{-x})} dx$ 3

30. (a) अवकल समीकरण $(xy - x^2) dy = y^2 dx$ का व्यापक हल ज्ञात कीजिए । 3

अथवा

(b) अवकल समीकरण $(x^2 + 1) \frac{dy}{dx} + 2xy = \sqrt{x^2 + 4}$ का व्यापक हल ज्ञात कीजिए । 3

31. (a) पहली छः सम प्राकृत संख्याओं में से यादृच्छया, बिना प्रतिस्थापना के, दो संख्याएँ चुनी गईं । यदि X, चुनी गई संख्याओं में से बड़ी संख्या को निरूपित करता है, तो X का प्रायिकता बंटन ज्ञात कीजिए । 3

अथवा

(b) एक न्याय्य सिक्के तथा एक अनभिनत पासे को उछाला गया । मान लें A घटना 'सिक्के पर चित्त आता है' और B घटना 'पासे पर संख्या 3 प्रकट होती है' को निरूपित करते हैं । जाँच कीजिए कि घटनाएँ A और B स्वतंत्र हैं या नहीं । 3



27. (a) Find $\int \frac{x+2}{\sqrt{x^2-4x-5}} dx.$ 3

OR

(b) Evaluate $\int_{-a}^a f(x) dx$, where $f(x) = \frac{9^x}{1+9^x}.$ 3

28. Find $\int e^{\cot^{-1} x} \left(\frac{1-x+x^2}{1+x^2} \right) dx.$ 3

29. Evaluate $\int_{\log \sqrt{2}}^{\log \sqrt{3}} \frac{1}{(e^x + e^{-x})(e^x - e^{-x})} dx$ 3

30. (a) Find the general solution of the differential equation :

$$(xy - x^2) dy = y^2 dx. \quad \text{3}$$

OR

(b) Find the general solution of the differential equation :

$$(x^2 + 1) \frac{dy}{dx} + 2xy = \sqrt{x^2 + 4} \quad \text{3}$$

31. (a) Two numbers are selected from first six even natural numbers at random without replacement. If X denotes the greater of two numbers selected, find the probability distribution of $X.$ 3

OR

(b) A fair coin and an unbiased die are tossed. Let A be the event, "Head appears on the coin" and B be the event, "3 comes on the die". Find whether A and B are independent events or not. 3



खण्ड – घ

इस खण्ड में दीर्घ-उत्तरीय (LA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 5 अंक हैं।

32. एक फलन $f: [-4, 4] \rightarrow [0, 4]$, $f(x) = \sqrt{16 - x^2}$ द्वारा प्रदत्त है। दर्शाइए कि f एक आच्छादक फलन है, परन्तु ऐकैकी फलन नहीं है। इसके अतिरिक्त 'a' के वह सभी मान ज्ञात कीजिए जिनके लिए $f(a) = \sqrt{7}$ है।

5

33. (a) यदि $A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ तथा $B = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$ हैं, तो AB ज्ञात कीजिए तथा इसके प्रयोग से निम्न समीकरण निकाय को हल कीजिए :

$$x - 2y = 3$$

$$2x - y - z = 2$$

$$-2y + z = 3$$

5

अथवा

(b) यदि $f(\alpha) = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ है, तो सिद्ध कीजिए कि $f(\alpha) \cdot f(-\beta) = f(\alpha - \beta)$

5

34. (a) समांतर चतुर्भुज PQRS जिसके शीर्ष $P(4, 2, -6)$, $Q(5, -3, 1)$, $R(12, 4, 5)$ तथा $S(11, 9, -2)$ हैं, के विकर्णों के समीकरण ज्ञात कीजिए तथा इनके प्रयोग से विकर्णों का प्रतिच्छेदन बिंदु ज्ञात कीजिए।

5

अथवा

(b) बिंदु $(-1, 3, -2)$ से होकर जाने वाली दोनों रेखाओं $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ तथा $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$ के लंबवत, एक रेखा l का सदिश समीकरण ज्ञात कीजिए। अतः इस रेखा की मूल बिंदु से दूरी ज्ञात कीजिए।

5

35. समाकलन के प्रयोग से वक्रों $\frac{x^2}{25} + \frac{y^2}{16} = 1$ तथा $\frac{x}{5} + \frac{y}{4} = 1$ द्वारा घिरे लघु क्षेत्र का क्षेत्रफल ज्ञात कीजिए।

5



SECTION – D

This section comprises Long Answer type (LA) questions of **5** marks each.

32. A function $f: [-4, 4] \rightarrow [0, 4]$ is given by $f(x) = \sqrt{16 - x^2}$. Show that f is an onto function but not a one-one function. Further, find all possible values of 'a' for which $f(a) = \sqrt{7}$. 5

33. (a) If $A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find AB and use it to solve the following system of equations :

$$\begin{aligned} x - 2y &= 3 \\ 2x - y - z &= 2 \\ -2y + z &= 3 \end{aligned}$$

5

OR

(b) If $f(\alpha) = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, prove that $f(\alpha) \cdot f(-\beta) = f(\alpha - \beta)$ 5

34. (a) Find the equations of the diagonals of the parallelogram PQRS whose vertices are $P(4, 2, -6)$, $Q(5, -3, 1)$, $R(12, 4, 5)$ and $S(11, 9, -2)$. Use these equations to find the point of intersection of diagonals. 5

OR

(b) A line l passes through point $(-1, 3, -2)$ and is perpendicular to both the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$. Find the vector equation of the line l . Hence, obtain its distance from origin. 5

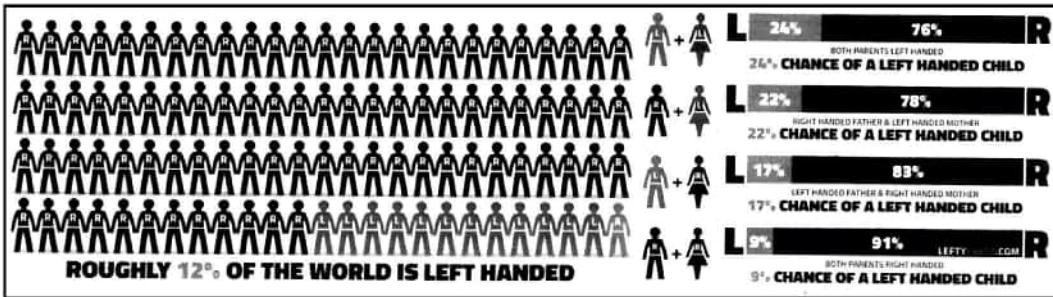
35. Find the area of the smaller region bounded by the curves $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and $\frac{x}{5} + \frac{y}{4} = 1$, using integration. 5



खण्ड – ड

इस खण्ड में 3 प्रकरण अध्ययन/केस/अनुच्छेद/एकीकृत इकाइयाँ मूल्यांकन आधारित प्रश्न हैं जिनमें प्रत्येक के 4 अंक हैं।

36. आधुनिक अध्ययन यह सुझाते हैं कि दुनिया की आबादी में लगभग 12% लोग वामहस्तिक हैं।



माता-पिता पर निर्भर करते हुए, बच्चे के वामहस्तिक होने के संयोग निम्न हैं :

A : जब पिता तथा माता दोनों वामहस्तिक हों :
वामहस्तिक बच्चे के संयोग 24% हैं।

B : जब पिता दक्षिणहस्तिक तथा माता वामहस्तिक हो :
वामहस्तिक बच्चे के संयोग 22% हैं।

C : जब पिता वामहस्तिक तथा माता दक्षिणहस्तिक हो :
वामहस्तिक बच्चे के संयोग 17% हैं।

D : जब पिता व माता दोनों दक्षिणहस्तिक हों :
वामहस्तिक बच्चे के संयोग 9% हैं।

यह मानते हुए कि $P(A) = P(B) = P(C) = P(D) = \frac{1}{4}$ है तथा L उस घटना को दर्शाता है जहाँ बच्चा वामहस्तिक हो।

उपरोक्त के आधार पर निम्न ज्ञात कीजिए :

(i) ज्ञात कीजिए : $P(L/C)$	1
(ii) ज्ञात कीजिए : $P(\bar{L}/A)$	1
(iii) (a) ज्ञात कीजिए : $P(A/L)$	2

अथवा

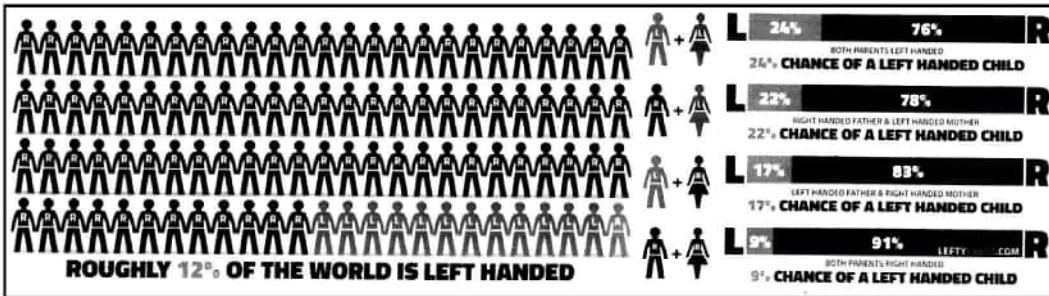
(b) प्रायिकता ज्ञात कीजिए कि एक यादृच्छया चुना गया बच्चा वामहस्तिक है जबकि दिया है माता-पिता में से मात्र एक वामहस्तिक है।	2
--	---



SECTION – E

This section comprises 3 source based case-based/passage based/integrated units of assessment questions of **4** marks each.

36. Recent studies suggest that roughly 12% of the world population is left handed.



Depending upon the parents, the chances of having a left handed child are as follows :

A : When both father and mother are left handed :
Chances of left handed child is 24%.

B : When father is right handed and mother is left handed :
Chances of left handed child is 22%.

C : When father is left handed and mother is right handed :
Chances of left handed child is 17%.

D : When both father and mother are right handed :
Chances of left handed child is 9%.

Assuming that $P(A) = P(B) = P(C) = P(D) = \frac{1}{4}$ and L denotes the event that child is left handed.

Based on the above information, answer the following questions :

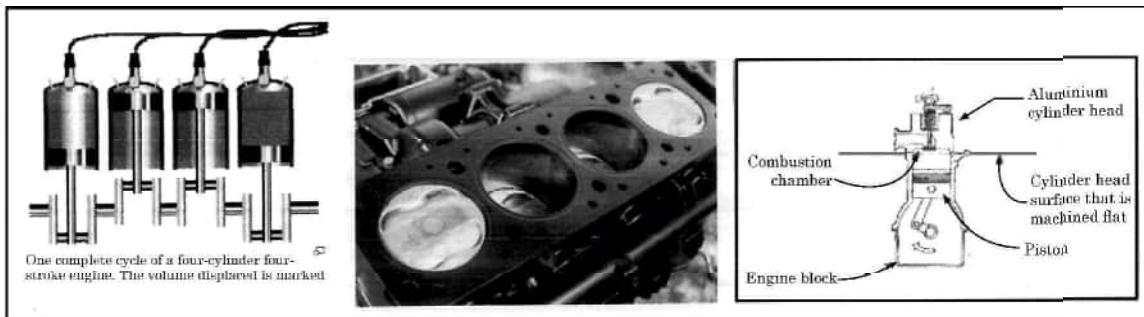
(i) Find $P(L/C)$	1
(ii) Find $P(\bar{L}/A)$	1
(iii) (a) Find $P(A/L)$	2

OR

(b) Find the probability that a randomly selected child is left handed given that exactly one of the parents is left handed. 2



37. एक पिस्टन इंजन में पिस्टनों के द्वारा बेलन में विस्थापित आयतन की माप को इंजन-विस्थापन कहा जाता है, जहाँ पिस्टन, बेलन के छिद्र में गति करता है।



75 π cm² क्षेत्रफल की धातु की शीट से, एक ऊपर से खुला वृत्तीय बेलन बनाया जाता है, जो बेलन के छिद्र का कार्य करता है।

उपरोक्त के आधार पर निम्न प्रश्नों के उत्तर दीजिए :

(i) यदि बेलन की त्रिज्या r cm तथा ऊँचाई h cm है तो बेलन के छिद्र के आयतन V को r में व्यक्त कीजिए। 1

(ii) $\frac{dV}{dr}$ ज्ञात कीजिए। 1

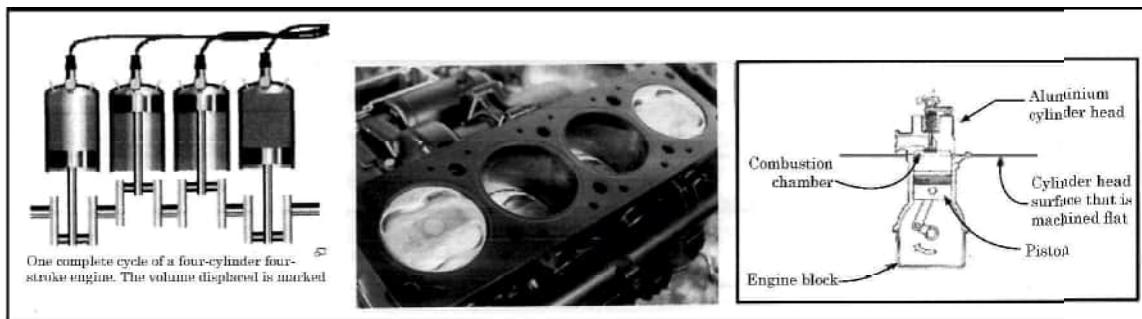
(iii) (a) बेलन की वह त्रिज्या ज्ञात कीजिए जब इसका आयतन अधिकतम हो। 2

अथवा

(b) बताइए कि अधिकतम आयतन के लिए, $h > r$ एक सत्य कथन है अथवा नहीं। अपने उत्तर का औचित्य दीजिए। 2



37. Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine. The piston moves inside the cylinder bore



The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area $75\pi \text{ cm}^2$.

Based on the above information, answer the following questions :

(i) If the radius of cylinder is $r \text{ cm}$ and height is $h \text{ cm}$, then write the volume V of cylinder in terms of radius r . 1

(ii) Find $\frac{dV}{dr}$. 1

(iii) (a) Find the radius of cylinder when its volume is maximum. 2

OR

(b) For maximum volume, $h > r$. State true or false and justify. 2



38. विद्युत वाहनों का प्रयोग अंत में वायु प्रदूषण पर नियंत्रण कर लेगा ।



विद्युत वाहनों का प्रयोग प्रति वर्ष बढ़ रहा है । किसी समय t पर प्रयोग में आ रहे विद्युत वाहनों का आकलन निम्न फलन V द्वारा दिया गया है :

$$V(t) = \frac{1}{5} t^3 - \frac{5}{2} t^2 + 25 t - 2$$

जहाँ t समय को निरूपित करता है तथा $t = 1, 2, 3, \dots$ क्रमशः वर्षों 2001, 2002, 2003, से संगत हैं ।

उपरोक्त के आधार पर निम्न प्रश्नों के उत्तर दीजिए :

(i) क्या वर्ष 2000 में प्रयोग हो रहे विद्युत वाहनों के आकलन में उपरोक्त फलन का प्रयोग किया जा सकता है ? औचित्य दीजिए । 2

(ii) सिद्ध कीजिए कि प्रदत्त फलन $V(t)$ एक वर्धमान फलन है । 2



38. The use of electric vehicles will curb air pollution in the long run.



The use of electric vehicles is increasing every year and estimated electric vehicles in use at any time t is given by the function V :

$$V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$$

where t represents the time and $t = 1, 2, 3, \dots$ corresponds to year 2001, 2002, 2003, respectively.

Based on the above information, answer the following questions :

(i) Can the above function be used to estimate number of vehicles in the year 2000 ? Justify. 2

(ii) Prove that the function $V(t)$ is an increasing function. 2



Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Senior School Certificate Examination, 2023
MATHEMATICS PAPER CODE 65/2/3

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc. may invite action under various rules of the Board and IPC."
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.
4	The Marking scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (✓) wherever answer is correct. For wrong answer CROSS 'X' be marked. Evaluators will not put right (✓) while evaluating which gives the impression that answer is correct, and no marks are awarded. This is most common mistake which evaluators are committing.
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note "Extra Question".
10	In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks _____ (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) must be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.

14	<p>Ensure that you do not make the following common types of errors committed by the Examiner in the past: -</p> <ul style="list-style-type: none"> ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	<p>While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.</p>
16	<p>Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.</p>
17	<p>The Examiners should acquaint themselves with the guidelines given in the "Guidelines for spot Evaluation" before starting the actual evaluation.</p>
18	<p>Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.</p>
19	<p>The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.</p>

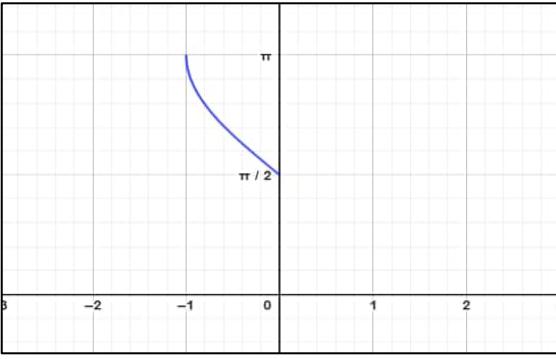
QUESTION PAPER CODE 65/2/3

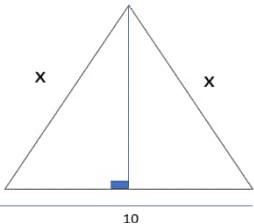
EXPECTED ANSWER/VALUE POINTS

SECTION A

	If $\int_0^{2\pi} \cos^2 x \, dx = k \int_0^{\pi/2} \cos^2 x \, dx$, then the value of k is	
4.	(A) 4 (B) 2 (C) 1 (D) 0	
Ans	(A) 4	1
5.	If $(a, b), (c, d)$ and (e, f) are the vertices of ΔABC and Δ denotes the area of ΔABC , then $\begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}^2$ is equal to (A) $2\Delta^2$ (B) $4\Delta^2$ (C) 2Δ (D) 4Δ	
Ans	(B) $4\Delta^2$	1
6.	The function $f(x) = x x $ is (A) continuous and differentiable at $x = 0$. (B) continuous but not differentiable at $x = 0$. (C) differentiable but not continuous at $x = 0$. (D) neither differentiable nor continuous at $x = 0$.	
Ans	(A) continuous and differentiable at $x = 0$	1
7.	... If $\tan\left(\frac{x+y}{x-y}\right) = k$, then $\frac{dy}{dx}$ is equal to (A) $\frac{-y}{x}$ (B) $\frac{y}{x}$ (C) $\sec^2\left(\frac{y}{x}\right)$ (D) $-\sec^2\left(\frac{y}{x}\right)$	
Ans	(B) $\frac{y}{x}$	1

16.	<p>The integrating factor of the differential equation $(1 - y^2) \frac{dx}{dy} + yx = ay, (-1 < y < 1)$ is</p> <p>(A) $\frac{1}{y^2 - 1}$ (B) $\frac{1}{\sqrt{y^2 - 1}}$ (C) $\frac{1}{1 - y^2}$ (D) $\frac{1}{\sqrt{1 - y^2}}$</p>	
Ans	<p>(D) $\frac{1}{\sqrt{1 - y^2}}$</p>	1
17.	<p>Unit vector along \vec{PQ}, where coordinates of P and Q respectively are $(2, 1, -1)$ and $(4, 4, -7)$, is</p>	
	<p>(A) $2\hat{i} + 3\hat{j} - 6\hat{k}$ (B) $-2\hat{i} - 3\hat{j} + 6\hat{k}$ (C) $\frac{-2\hat{i}}{7} - \frac{3\hat{j}}{7} + \frac{6\hat{k}}{7}$ (D) $\frac{2\hat{i}}{7} + \frac{3\hat{j}}{7} - \frac{6\hat{k}}{7}$</p>	
Ans	<p>(D) $\frac{2\hat{i}}{7} + \frac{3\hat{j}}{7} - \frac{6\hat{k}}{7}$</p>	1
18.	<p>Equation of a line passing through point $(1, 2, 3)$ and equally inclined to the coordinate axis, is</p> <p>(A) $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ (B) $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$ (C) $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{3}$ (D) $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1}$</p>	
Ans	<p>(D) $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1}$</p>	1

(Question Nos. 19 & 20 are Assertion-Reason based questions of 1 mark each)		
19.	Assertion (A) : Maximum value of $(\cos^{-1} x)^2$ is π^2 . Reason (R) : Range of the principal value branch of $\cos^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.	
Ans	(C) (A) is true but (R) is false	1
20.	Assertion (A) : If a line makes angles α, β, γ with positive direction of the coordinate axes, then $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$. Reason (R) : The sum of squares of the direction cosines of a line is 1.	
Ans	(A) Both (A) and (R) are true and (R) is the correct explanation of (A)	1
SECTION-B		
(Question nos. 21 to 25 are very short Answer type questions carrying 2 marks each)		
21.	If points A, B and C have position vectors $2\hat{i}, \hat{j}$ and $2\hat{k}$ respectively, then show that ΔABC is an isosceles triangle.	
Ans	$\vec{BA} = 2\hat{i} - \hat{j}$; $\vec{BC} = 2\hat{k} - \hat{j}$ $ \vec{BA} = \vec{BC} = \sqrt{5} \Rightarrow \Delta ABC$ is an isosceles triangle	1 1
22.	(a) Evaluate $\sin^{-1} \left(\sin \frac{3\pi}{4} \right) + \cos^{-1} (\cos \pi) + \tan^{-1} (1)$. OR (b) Draw the graph of $\cos^{-1} x$, where $x \in [-1, 0]$. Also, write its range.	
Ans	$\begin{aligned} \sin^{-1} \left(\sin \frac{3\pi}{4} \right) + \cos^{-1} (\cos \pi) + \tan^{-1} (1) &= \frac{\pi}{4} + \pi + \frac{\pi}{4} \\ &= \frac{3\pi}{2} \end{aligned}$ 	1 1/2 1/2
	Or Correct Graph $\text{Range: } \left[\frac{\pi}{2}, \pi \right]$	1 1

23.	<p>If equal sides of an isosceles triangle with fixed base 10 cm are increasing at the rate of 4 cm/sec, how fast is the area of triangle increasing at an instant when all sides become equal ?</p>	
Ans	<p>Let the equal side be 'x', then $\frac{dx}{dt} = 4 \text{ cm/s}$</p>  $A = 5\sqrt{x^2 - 25} \Rightarrow \frac{dA}{dt} = \frac{5x}{\sqrt{x^2 - 25}} \frac{dx}{dt}$ $\left. \frac{dA}{dt} \right _{x=10} = \frac{40}{\sqrt{3}} \text{ cm}^2/\text{s}$	$\frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$ $\frac{1}{2}$
24.	<p>Find the coordinates of points on line $\frac{x}{1} = \frac{y-1}{2} = \frac{z+1}{2}$ which are at a distance of $\sqrt{11}$ units from origin.</p>	
Ans	<p>General point on the curve is $P(k, 2k+1, 2k-1)$, $k \in \mathbb{R}$</p> $OP = \sqrt{11} \Rightarrow OP^2 = 11$ $\therefore k^2 + (2k+1)^2 + (2k-1)^2 = 11 \Rightarrow k = \pm 1$ <p>\therefore Coordinates of points are $(1, 3, 1)$ & $(-1, -1, -3)$</p>	$\frac{1}{2}$ 1 $\frac{1}{2}$
25.	<p>(a) If $y = \sqrt{ax+b}$, then prove that $y \left(\frac{d^2y}{dx^2} \right) + \left(\frac{dy}{dx} \right)^2 = 0$.</p> <p style="text-align: center;">OR</p> <p>(b) If $f(x) = \begin{cases} ax+b & ; 0 < x \leq 1 \\ 2x^2 - x & ; 1 < x < 2 \end{cases}$ is a differentiable function in $(0, 2)$, then find the values of a and b.</p>	

Ans	(a) $y = \sqrt{ax + b} \Rightarrow y^2 = ax + b$		
	Differentiate with respect to 'x', $2y \frac{dy}{dx} = a$		1
	Differentiate with respect to 'x', $2y \frac{d^2y}{dx^2} + 2 \left(\frac{dy}{dx} \right)^2 = 0 \Rightarrow y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right)^2 = 0$		1
	Or		
	(b) $f(x)$ is differentiable in $(0, 2) \Rightarrow f(x)$ is continuous on $(0, 2)$		$\frac{1}{2}$
	$\Rightarrow f(x)$ is continuous at $x = 1 \therefore \lim_{x \rightarrow 1^-} (ax + b) = \lim_{x \rightarrow 1^+} (2x^2 - x) \Rightarrow a + b = 1$		$\frac{1}{2}$
	Also, $f(x)$ is differentiable at $x = 1 \therefore L.H.D.(x = 1) = R.H.D.(x = 1)$		
	$\Rightarrow a = 4(1) - 1 \therefore a = 3 \& b = 1 - a = -2$		$\frac{1}{2} + \frac{1}{2}$

SECTION-C

(Question nos. 26 to 31 are short Answer type questions carrying 3 marks each)

26. The section comprises Short Answer (SA) type questions of 3 marks each.
Solve the following Linear Programming problem graphically :
Maximize : $Z = 3x + 3.5y$
subject to constraints : $x + 2y \geq 240$,
 $3x + 1.5y \geq 270$,
 $1.5x + 2y \leq 310$,
 $x \geq 0, y \geq 0$.

Ans		Correct Graph								
		2								
		1								
	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="width: 50%;">Corner points</th> <th style="width: 50%;">Value of Z</th> </tr> </thead> <tbody> <tr> <td>A (40,100)</td> <td>470</td> </tr> <tr> <td>B (140,50)</td> <td>595 (Max)</td> </tr> <tr> <td>C (20,140)</td> <td>550</td> </tr> </tbody> </table>	Corner points	Value of Z	A (40,100)	470	B (140,50)	595 (Max)	C (20,140)	550	
Corner points	Value of Z									
A (40,100)	470									
B (140,50)	595 (Max)									
C (20,140)	550									
	Max(Z) = 595 at x = 140 ; y = 50									

27.	<p>(a) Find $\int \frac{x+2}{\sqrt{x^2-4x-5}} dx$.</p> <p>OR</p> <p>(b) Evaluate $\int_{-a}^a f(x) dx$, where $f(x) = \frac{9^x}{1+9^x}$.</p>	
Ans	<p>(a) $\int \frac{x+2}{\sqrt{x^2-4x-5}} dx = \frac{1}{2} \int \frac{2x-4}{\sqrt{x^2-4x-5}} dx + 4 \int \frac{1}{\sqrt{(x-2)^2 - 3^2}} dx$ $= \sqrt{x^2-4x-5} + 4 \log x-2 + \sqrt{x^2-4x-5} + c$</p> <p>Or</p> <p>(b) $I = \int_{-a}^a \frac{9^x}{1+9^x} dx$ ----- (i)</p> <p>Replacing 'x' by $-a+a-x$</p> <p>$I = \int_{-a}^a \frac{9^{a-a-x}}{1+9^{a-a-x}} dx = \int_{-a}^a \frac{9^{-x}}{1+9^{-x}} dx = \int_{-a}^a \frac{1}{1+9^x} dx$ ----- (ii)</p> <p>Adding (i) & (ii) we get, $2I = \int_{-a}^a dx = x \Big _{-a}^a = 2a \Rightarrow I = a$</p>	2 1 1 1 1 1 1
28.	Find $\int e^{\cot^{-1} x} \left(\frac{1-x+x^2}{1+x^2} \right) dx$.	
Ans	<p>Put $\cot^{-1} x = t \therefore x = \cot t$ and $\frac{1}{1+x^2} dx = -dt$</p> <p>$\therefore \int e^{\cot^{-1} x} \left(\frac{1-x+x^2}{1+x^2} \right) dx = - \int e^t (1 - \cot t + \cot^2 t) dt = \int e^t (\cot t - \operatorname{cosec}^2 t) dt$ $= e^t \cot t + c$ $= x e^{\cot^{-1} x} + c$</p>	1 1 1/2 1/2
29.	Evaluate $\int_{\log \sqrt{2}}^{\log \sqrt{3}} \frac{1}{(e^x + e^{-x})(e^x - e^{-x})} dx$	
Ans	Let $I = \int_{\log \sqrt{2}}^{\log \sqrt{3}} \frac{1}{(e^x + e^{-x})(e^x - e^{-x})} dx = \int_{\log \sqrt{2}}^{\log \sqrt{3}} \frac{e^{2x}}{(e^{2x})^2 - 1} dx$	1

	<p>Put $e^{2x} = t \Rightarrow e^{2x} dx = \frac{1}{2} dt$, Upper limit = 3, Lower Limit = 2</p> $\therefore I = \frac{1}{2} \int_2^3 \frac{1}{t^2 - 1} dt = \frac{1}{4} \log \left \frac{t-1}{t+1} \right _2^3$ $\Rightarrow I = \frac{1}{4} \left[\log \frac{2}{4} - \log \frac{1}{3} \right] = \frac{1}{4} \log \frac{3}{2}$	$\frac{1}{2}$ 1 $\frac{1}{2}$
30.	<p>(a) Find the general solution of the differential equation :</p> $(xy - x^2) dy = y^2 dx.$ <p style="text-align: center;">OR</p> <p>(b) Find the general solution of the differential equation :</p> $(x^2 + 1) \frac{dy}{dx} + 2xy = \sqrt{x^2 + 4}$	
Ans	<p>(a) Given differential equation can be written as</p> $\frac{dy}{dx} = \frac{y^2}{xy - x^2} = \frac{\left(\frac{y}{x}\right)^2}{\frac{y}{x} - 1}$ <p>Put $y = ux, \frac{dy}{dx} = u + x \frac{du}{dx}$</p> $\therefore u + x \frac{du}{dx} = \frac{u^2}{u-1} \Rightarrow x \frac{du}{dx} = \frac{u}{u-1}$ <p>Separating the variables and integrating</p> $\int \left(1 - \frac{1}{u}\right) du = \int \frac{dx}{x} \Rightarrow u - \log u = \log x + c \Rightarrow \frac{y}{x} - \log \frac{y}{x} = \log x + c$ <p style="text-align: center;">Or</p> <p>Rewriting the given differential equation as:</p> $\frac{dy}{dx} + \frac{2x}{1+x^2} y = \frac{\sqrt{x^2 + 4}}{1+x^2}$ <p>Integrating factor = $e^{\int \frac{2x}{1+x^2} dx} = e^{\log(1+x^2)} = 1+x^2$</p> <p>$\therefore$ solution of the differential equation is</p> $y(1+x^2) = \int \frac{\sqrt{x^2 + 4}}{1+x^2} \cdot (1+x^2) dx + c$ $= \int \sqrt{x^2 + 4} dx + c$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $1 + \frac{1}{2}$

	$\therefore y(1+x^2) = \frac{x\sqrt{x^2+4}}{2} + 2\log x+\sqrt{x^2+4} + c$	1												
31.	<p>(a) Two numbers are selected from first six even natural numbers at random without replacement. If X denotes the greater of two numbers selected, find the probability distribution of X.</p> <p style="text-align: center;">OR</p> <p>(b) A fair coin and an unbiased die are tossed. Let A be the event, "Head appears on the coin" and B be the event, "3 comes on the die". Find whether A and B are independent events or not.</p>													
Ans	<p>(a) X: Greater of the two selected from first six even natural numbers</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>X</td><td>4</td><td>6</td><td>8</td><td>10</td><td>12</td></tr> <tr> <td>P(X)</td><td>$\frac{1}{15}$</td><td>$\frac{2}{15}$</td><td>$\frac{3}{15}$</td><td>$\frac{4}{15}$</td><td>$\frac{5}{15}$</td></tr> </table> <p style="text-align: center;">Or</p> <p>(b) $S = \text{Sample space} = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$</p> $\therefore P(A) = \frac{6}{12} = \frac{1}{2}, P(B) = \frac{2}{12} = \frac{1}{6}, P(A \cap B) = \frac{1}{12}$ $\Rightarrow P(A \cap B) = P(A) \cdot P(B) = \frac{1}{12}, \therefore A \text{ & } B \text{ are independent events.}$	X	4	6	8	10	12	P(X)	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$	1 2 1 $1\frac{1}{2}$ $\frac{1}{2}$
X	4	6	8	10	12									
P(X)	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$									
	SECTION-D (Question nos. 32 to 35 are Long Answer type questions carrying 5 marks each)													
32.	A function $f: [-4, 4] \rightarrow [0, 4]$ is given by $f(x) = \sqrt{16 - x^2}$. Show that f is an onto function but not a one-one function. Further, find all possible values of 'a' for which $f(a) = \sqrt{7}$.													
Ans	<p>Onto: Let $y = \sqrt{16 - x^2} \Rightarrow y \geq 0$</p> <p>Squaring we get, $x^2 = 16 - y^2 \Rightarrow x = \pm\sqrt{16 - y^2}$</p> <p>For each $y \in [-4, 4]$, 'x' is a real number, $\therefore 0 \leq y \leq 4 \Rightarrow R_f = [0, 4] = \text{Co-domain}$</p> <p style="text-align: center;">$\therefore 'f' \text{ is an onto function.}$</p> <p>One-One: $f(-1) = f(1) = \sqrt{15}$ but $-1 \neq 1$, $\therefore 'f' \text{ is not a one-one function.}$</p> <p>$f(a) = \sqrt{7} \Rightarrow \sqrt{16 - a^2} = \sqrt{7} \Rightarrow a = \pm 3$</p>	$\frac{1}{2}$ $\frac{1}{2}$ $1\frac{1}{2}$ $1\frac{1}{2}$ 1												

33.

(a) If $A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find AB and use it to solve the following system of equations :

$$x - 2y = 3$$

$$2x - y - z = 2$$

$$-2y + z = 3$$

OR

(b) If $f(\alpha) = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, prove that $f(\alpha) \cdot f(-\beta) = f(\alpha - \beta)$

Ans

$$(a) AB = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow B^{-1} = A$$

The given system of equations can be written as:

$$B^T \cdot X = C, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, C = \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}$$

$$X = (B^T)^{-1} \cdot C = (B^{-1})^T \cdot C = A^T \cdot C$$

$$\Rightarrow X = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}$$

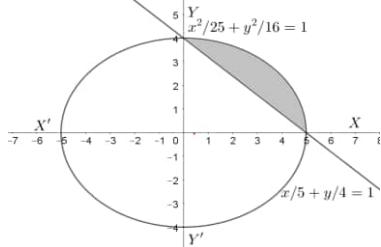
$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \therefore x = 1, y = -1, z = 1$$

Or

$$(b) LHS = f(\alpha)f(-\beta) = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \beta & \sin \beta & 0 \\ -\sin \beta & \cos \beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos \alpha \cos \beta + \sin \alpha \sin \beta & \cos \alpha \sin \beta - \sin \alpha \cos \beta & 0 \\ \sin \alpha \cos \beta - \cos \alpha \sin \beta & \sin \alpha \sin \beta + \cos \alpha \cos \beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

	$ \begin{aligned} &= \begin{bmatrix} \cos(\alpha-\beta) & -\sin(\alpha-\beta) & 0 \\ \sin(\alpha-\beta) & \cos(\alpha-\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \mathbf{f}(\alpha-\beta) = \text{RHS} \end{aligned} $	$1\frac{1}{2}$ $\frac{1}{2}$
34.	<p>(a) Find the equations of the diagonals of the parallelogram PQRS whose vertices are P(4, 2, -6), Q(5, -3, 1), R(12, 4, 5) and S(11, 9, -2). Use these equations to find the point of intersection of diagonals.</p> <p style="text-align: center;">OR</p> <p>(b) A line l passes through point (-1, 3, -2) and is perpendicular to both the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$. Find the vector equation of the line l. Hence, obtain its distance from origin.</p>	
Ans	<p>Equation of diagonal PR: $\frac{x-4}{8} = \frac{y-2}{2} = \frac{z+6}{11}$</p> <p>Equation of diagonal QS: $\frac{x-5}{6} = \frac{y+3}{12} = \frac{z-1}{-3}$</p> <p>General points on PR & QS are $(8k+4, 2k+2, 11k-6)$ and $(6t+5, 12t-3, -3t+1)$ for real numbers 'k' and 't' respectively.</p> <p>For point of intersection of PR and QS: $8k+4=6t+5, 2k+2=12t-3$</p> <p>Solving, we get $k=\frac{1}{2}, t=\frac{1}{2}$ ∴ The point of intersection is $\left(8, 3, -\frac{1}{2}\right)$</p> <p style="text-align: center;">Or</p> <p>(b) Let direction ratios of the required line be a, b, c Since it is perpendicular to the two given lines, $a+2b+3c=0$; $-3a+2b+5c=0$</p> <p>Solving together, $a=4k, b=-14k, c=8k$</p> <p>∴ Equation of line is: $\frac{x+1}{4k} = \frac{y-3}{-14k} = \frac{z+2}{8k} \Rightarrow \frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4}$</p> <p>Vector equation: $\vec{r} = -\hat{i} + 3\hat{j} - 2\hat{k} + \lambda(2\hat{i} - 7\hat{j} + 4\hat{k})$</p> <p>Distance from origin = $\frac{ (-\hat{i} + 3\hat{j} - 2\hat{k}) \times (2\hat{i} - 7\hat{j} + 4\hat{k}) }{ 2\hat{i} - 7\hat{j} + 4\hat{k} } = \frac{ -2\hat{i} + \hat{k} }{ 2\hat{i} - 7\hat{j} + 4\hat{k} } = \frac{\sqrt{5}}{\sqrt{69}} \text{ or } \sqrt{\frac{5}{69}}$</p>	1 1 1 $\frac{1}{2}$ $1\frac{1}{2}$ 1 1 1 1 1 1 1 1 1 1 1 1

35.	<p>Find the area of the smaller region bounded by the curves $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and $\frac{x}{5} + \frac{y}{4} = 1$, using integration.</p>	
Ans	 <p>Correct Figure</p> $\text{Required Area} = \frac{4}{5} \int_0^5 \sqrt{25-x^2} dx - \frac{4}{5} \int_0^5 (5-x) dx$ $= \frac{4}{5} \left(\frac{x}{2} \sqrt{25-x^2} + \frac{25}{2} \sin^{-1} \frac{x}{5} \right) \Big _0^5 + \frac{2}{5} (5-x)^2 \Big _0^5$ $= \frac{4}{5} \left(\frac{25\pi}{4} \right) - 10 = 5\pi - 10$	$1\frac{1}{2}$ $1\frac{1}{2}$ $1 + \frac{1}{2}$ $\frac{1}{2}$
36.	<p>Recent studies suggest that roughly 12% of the world population is left handed.</p>  <p>Depending upon the parents, the chances of having a left handed child are as follows :</p> <p>A : When both father and mother are left handed : Chances of left handed child is 24%.</p> <p>B : When father is right handed and mother is left handed : Chances of left handed child is 22%.</p> <p>C : When father is left handed and mother is right handed : Chances of left handed child is 17%.</p> <p>D : When both father and mother are right handed : Chances of left handed child is 9%.</p> <p>Assuming that $P(A) = P(B) = P(C) = P(D) = \frac{1}{4}$ and L denotes the event that child is left handed.</p> <p>Based on the above information, answer the following questions :</p> <p>(i) Find $P(L/C)$ (ii) Find $P(\bar{L}/A)$ (iii) (a) Find $P(A/L)$</p> <p style="text-align: center;">OR</p> <p>(b) Find the probability that a randomly selected child is left handed given that exactly one of the parents is left handed.</p>	

Ans	<p>(i) $P(L C) = \frac{17}{100}$</p> <p>(ii) $P(\bar{L} A) = 1 - P(L A) = 1 - \frac{24}{100} = \frac{76}{100}$ or $\frac{19}{25}$</p> <p>(iii) $P(A L) = \frac{\frac{1}{4} \times \frac{24}{100}}{\frac{1}{4} \times \frac{24}{100} + \frac{1}{4} \times \frac{22}{100} + \frac{1}{4} \times \frac{17}{100} + \frac{1}{4} \times \frac{9}{100}} = \frac{24}{72} = \frac{1}{3}$</p> <p>Or</p> <p>Probability that a randomly selected child is left-handed given that exactly one of the parents is left-handed.</p> $= P(L B \cup C) = \frac{22}{100} + \frac{17}{100} = \frac{39}{100}$	1 1 2 2
37.	<p>Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine. The piston moves inside the cylinder bore</p> <div data-bbox="198 1201 1325 1381"> <p>One complete cycle of a four-cylinder four-stroke engine. The volume displaced is marked.</p> <p>Combustion chamber</p> <p>Aluminium cylinder head</p> <p>Cylinder head surface that is machined flat</p> <p>Piston</p> <p>Engine block</p> </div> <p>The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area $75\pi \text{ cm}^2$.</p> <p>Based on the above information, answer the following questions :</p> <p>(i) If the radius of cylinder is $r \text{ cm}$ and height is $h \text{ cm}$, then write the volume V of cylinder in terms of radius r.</p> <p>(ii) Find $\frac{dV}{dr}$.</p> <p>(iii) (a) Find the radius of cylinder when its volume is maximum.</p> <p>OR</p> <p>(b) For maximum volume, $h > r$. State true or false and justify.</p>	

Ans	<p>(i) $\pi r^2 + 2\pi r h = 75\pi \Rightarrow h = \frac{75 - r^2}{2r}$, $\therefore V = \pi r^2 h = \frac{\pi}{2} (75r - r^3)$</p> <p>(ii) $\frac{dV}{dr} = \frac{\pi}{2} (75 - 3r^2)$</p> <p>(iii) $\frac{dV}{dr} = 0 \Rightarrow r = 5, \frac{d^2V}{dr^2} \Big _{r=5} = \frac{\pi}{2} (-6r) < 0 \therefore \text{volume is maximum when } r = 5$</p> <p>Or</p> <p>False,</p> <p>$\frac{dV}{dr} = 0 \Rightarrow r = 5, \frac{d^2V}{dr^2} \Big _{r=5} = \frac{\pi}{2} (-6r) < 0 \therefore \text{volume is maximum when } r = 5$</p> <p>As volume is maximum at $r = 5 \Rightarrow h = \frac{75 - 5^2}{2(5)} = 5 \Rightarrow h = r$</p>	1 1 2 $\frac{1}{2}$ 1 $\frac{1}{2}$
38.	<p>The use of electric vehicles will curb air pollution in the long run.</p>  <p>The use of electric vehicles is increasing every year and estimated electric vehicles in use at any time t is given by the function V :</p> $V(t) = \frac{1}{5} t^3 - \frac{5}{2} t^2 + 25t - 2$ <p>where t represents the time and $t = 1, 2, 3, \dots$ corresponds to year 2001, 2002, 2003, respectively.</p> <p>Based on the above information, answer the following questions :</p> <p>(i) Can the above function be used to estimate number of vehicles in the year 2000 ? Justify.</p> <p>(ii) Prove that the function $V(t)$ is an increasing function.</p>	
Ans	<p>(i) For the year 2000, $t = 0$ & $V(0) = -2$ and the number of vehicles cannot be negative \therefore the given function $V(t)$ cannot be used.</p> <p>(ii) $V'(t) = \frac{3}{5} t^2 - 5t + 25 = \frac{3}{5} \left[\left(t - \frac{25}{6} \right)^2 + \frac{875}{36} \right] > 0, \therefore V(t) \text{ is an increasing function.}$</p>	2 2